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Representation Theory in Mathematics

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Introduction

Karin Erdmann, an Oxford mathematician, is an expert in homological algebra and representation theory, particularly modular representation theory. The role of vector spaces in algebraic systems is examined by representation theory. One can use linear algebra to analyse "abstract" algebraic systems when the vector spaces are finite-dimensional since this enables one to directly describe the elements of the algebraic system via matrices. Through collective activities, symmetry can be studied in this way. Studying irreversible processes is another option. An obvious framework for this is provided by algebras and their representations.

Description

There are symmetry in both mathematics and science. Understanding all the potential causes of an abstract collection of symmetries is the goal of representation theory. The structure of electron orbitals was partially explained by representation theory in the nineteenth century, and the foundation of quantum chromodynamics is representation theory from the 1920s. The Langlands program, a series of conjectures that have shaped a significant portion of number theory over the past forty years, is centered on p-adic representation theory [1].

One essential issue is expressing all the continuous symmetries of a finite-dimensional geometry, the irreducible unitary representations of each Lie group. This is equivalent to identifying all of a quantum-mechanical system's finite-dimensional symmetries. We've come a long way with this significant issue, thanks in large part to the efforts of the highly qualified academics at MIT. String theory, statistical mechanics, integrable systems, tomography, and many other branches of mathematics and their applications all depend on the representation theory of infinite-dimensional groups and supergroups. Vertex algebras, quantum groups, infinite-dimensional Lie algebras, representations of real and p-adic groups, Hecke algebras, and symmetric spaces are among the topics of study for this group [2].

Scalars commute with everything in algebra, which is a ring that also doubles as a vector space. Path algebras are a significant construction: Take a coefficient field K and a directed graph Q, sometimes known as a quiver. The vector space over K with all pathways having basis in Q is then the path algebra KQ. In the resulting algebra, the product of two basic elements is either their concatenation, if one exists, or zero, if there is no such thing as a concatenation. In other words, if we start with a group, we automatically have algebra; we then take the vector space with the group's basis labelled and extend the group multiplication to a ring structure [3].

The representations of groups when the coefficients are included in the complex numbers have been researched for a very long time and have various uses. The algebras and their representations are substantially more challenging to comprehend when coefficients, for instance, are in the integers modulo 2. The representations have a "finite type" for some groupings. Although they have "infinite type" virtually invariably, these are well understood. These are often "wild," meaning there is no chance of a classification of the representations, with the exception of a few exceptional "tame" occurrences. For module 2 arithmetic and when the symmetry is based on dihedral, semidihedral, or quaternion 2-groups, the identical circumstances precisely occur. When n is a power of two, dihedral 2-groups are symmetries of normal n-gons [4].

Such tame circumstances were categorized (some time ago) by looking at these group symmetries in the larger setting of algebras. It was just found out that this is a little portion of a much larger cosmos. In particular, surface triangulations can be used to create algebras, in which the triangulations from the group setting appear as special instances.

The crystal in the Andrew Wiles Building's north wing, which houses Oxford Mathematics, can be seen as a triangulation of a surface with boundary. The reader will have to draw the quiver. We build algebras from the path algebra of such a quiver by enforcing explicit relations that resemble the triangulation. Despite the fact that the quiver can be arbitrary big and intricate, the algebras are simple to describe. These are what we refer to as weighted surface algebras. Together with A. Skowronski, we created this. We demonstrate how these algebras give group representations a more comprehensive setting [5].

Starting from the fact that weighted surface algebras generalize group algebras with quaternion symmetry, these algebras are periodic of period four (with one exception).The relationships that resemble triangles can be degenerated, making the algebraic sum of two arrows around a triangle equal to zero. Multiple new algebras are created as a result. The resulting algebras closely resemble group algebras with dihedral symmetry when all such relations are degenerated. We get algebras that share characteristics of group algebras with semidihedral symmetry if we degenerate relations around some but not all triangles. On these, work is already underway.

Conclusion

The classification of algebraic varieties, particularly the birational classification, and the theory of moduli, which takes into account how algebraic varieties change when one changes the coefficients of the defining equations, are among the areas of research that our group is interested in. One potential approach to classification is provided by the Minimal Model Program. Hodge theory, which connects the topology of an algebraic variety with harmonic functions, is another field of active research. One of the seven Clay Millennium Problems with a $1,000,000 reward is the Hodge Hypothesis. Active fields that have linkages to theoretical high energy particle physics, particularly string theory, include the study of the derived category, Calabi-Yau manifolds, and mirror symmetry. A number of faculty members in our department have made noncommutative algebraic geometry, a generalization with connections to representation theory, an important and active area of study. New study into algorithmic strategies for resolving polynomial problems has been stimulated by the development of high-speed computers, with numerous intriguing practical applications.

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