

Remark on “Tripled Coincidence Point Theorem for Compatible Maps in Fuzzy Metric Spaces”

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Abstract

In this note, we point out and rectify an error in a recently published paper “PP Murthy, Rashmi, VN Mishra, Tripled Coincidence Point Theorem For Compatible Maps In Fuzzy Metric Spaces, Electronic Journal of Mathematical Analysis and Applications, Vol. 4(2) July 2016, pp. 96-106”.

Keywords: Fuzzy metric space; Hadžić type t-norm; Compatible mappings; Tripled coincidence points.

MSC (2010): Primary: 47H10; Secondary: 54H25.

In ref. [1], the authors showed the existence of tripled coincidence points for the pair of compatible mappings in the setup of complete fuzzy metric spaces with Hadžić type t-norm. Authors accompanied the main result with the help of a suitable example. The reader should consult [1] for terms not specifically defined in this note.

Remark 1

The authors in [1] claimed that Example 20 supports Theorem 17. In Theorem 17, the t-norm considered is the Hadžić type t-norm but in Example 20, the t-norm * is defined by a*b=ab for all a, b ∈ [0, 1], which is actually not a Hadžić type t-norm. Hence, we conclude that the Example 20 does not support Theorem 17 in [1].

We now rectify Example 17 as follows:

Example 2: Let X=[-1, 1] and a*b=min{a, b} for all a, b ∈ [0,1]. Let for all t>0 and x, y ∈ X, M(x, y, t)=e^{-(x-y)/t}. Then (X, M,*) is a complete fuzzy metric space such that M(x, y, t) → 1 as t → ∞, for all x, y ∈ X and * being the Hadžić type t-norm.

Let us define the mappings g: X → X and F: X × X → X respectively by g(x)=x² and F(x, y, z) = $\frac{x^2+y^2+z^2}{4}$ for x, y, z ∈ X.

Then, F(X × X) ⊂ g(X), the pair (F, g) of the mappings is compatible and g is continuous.

Let $\phi(t) = \frac{3}{4}t$ for t > 0, then $\phi \in \Phi$.

Next, we verify inequality (3.2) of Theorem 17 in [1], that is

$$M(F(x, y, z), F(u, v, w), \phi(t)) \geq M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t) \tag{1}$$

for all x, y, z, u, v, w ∈ X, t > 0.

If the inequality (1) does not hold, then for some t > 0 and x, y, z, u, v, w ∈ X, we have

$$M(F(x, y, z), F(u, v, w), \phi(t)) < M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t),$$

that is,

$$e^{-\left(\frac{F(x,y,z)-F(u,v,w)}{\phi(t)}\right)} < \min \left\{ e^{-\left(\frac{g(x)-g(u)}{t}\right)}, e^{-\left(\frac{g(y)-g(v)}{t}\right)}, e^{-\left(\frac{g(z)-g(w)}{t}\right)} \right\}$$

that is,

$$e^{-\left(\frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t}\right)} < \min \left\{ e^{-\left(\frac{x^2-u^2}{t}\right)}, e^{-\left(\frac{y^2-v^2}{t}\right)}, e^{-\left(\frac{z^2-w^2}{t}\right)} \right\},$$

that is,

$$e^{-\left(\frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t}\right)} < e^{-\left(\frac{y^2-v^2}{t}\right)},$$

$$e^{-\left(\frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t}\right)} < e^{-\left(\frac{y^2-v^2}{t}\right)},$$

$$\text{and } e^{-\left(\frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t}\right)} < e^{-\left(\frac{z^2-w^2}{t}\right)},$$

that is,

$$\left| \frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t} \right| > \frac{|x^2-u^2|}{t},$$

$$\left| \frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t} \right| > \frac{|y^2-v^2|}{t},$$

$$\left| \frac{(x^2+y^2+z^2)-(u^2+v^2+w^2)}{3t} \right| > \frac{|z^2-w^2|}{t}.$$

and, Combining the above three last inequalities, we get $|x^2-u^2| + |y^2-v^2| + |z^2-w^2| > |x^2-u^2| + |y^2-v^2| + |z^2-w^2|$, which is impossible for $x^2 < u^2, y^2 > v^2$ and $z^2 < w^2$. Hence, (1) holds.

Thus all the conditions of Theorem 17 in [1] are satisfied. Hence on applying Theorem 17 [1], we obtain that (0,0,0) is the coupled coincidence point of the mappings F and g.

References

1. Murthy PP, Rashmi, Mishra VN (2016) Tripled Coincidence Point Theorem for Compatible Maps in Fuzzy Metric Spaces. Electronic Journal of Mathematical Analysis and Applications 4: 96-106.

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