Prime Distribution in Pythagorean Triples (1)

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Mini-Review

Using Jiang function we study the prime distribution in Pythagorean triples.

Pythagorean triples

\[ a^2 + b^2 = c^2, \quad (1) \]

In comprime integers must be of the form:

\[ a = x^2 - y^2, \quad b = 2xy, \quad c = x^2 + y^2, \quad (2) \]

Where \( x \) and \( y \) are comprime integers.

Theorem 1: From eqn. (2) we have,

\[ a = (x+y)(x-y) \quad (3) \]

Let \( x-y = 1 \) and \( a = x+y = P_1 \), we have,

\[ P_1^2 = (x+y)^2 = x^2 + y^2 + 2xy = c + b, \quad (4) \]

\[ 1 = (x-y)^2 = x^2 + y^2 - 2xy = c - b \quad (5) \]

From eqns. (4) and (5) we have,

\[ a = P_1, \quad b = \frac{P_1^2 - 1}{2}, \quad c = \frac{P_1^2 + 1}{2} = P_2 \quad (6) \]

There are infinitely many primes \( P_1 \) such that \( P_2 \) is a prime.

Proof: We have Jiang function \[ J_1(\omega) = \prod_{P \leq 2} (P - 1 - \chi(P)), \quad (7) \]

where \( \omega = \prod_{P \leq 2} P \), \( \chi(P) \) is the number of solutions of congruence

\[ q^2 + 1 = 0 \pmod{P}, \quad q = 1, \ldots, P - 1. \quad (8) \]

From (8) we have,

\[ \chi(P) = 1 + (-1)^{\frac{P-1}{2}} \quad (9) \]

Substituting (9) into (7) we have

\[ J_1(\omega) = \prod_{P \leq 2} (P - 2 - (-1)^{\frac{P-1}{2}}) \neq 0 \quad (10) \]

Since \( J_1(\omega) \neq 0 \), we prove that there are infinitely many primes \( P_1 \) such that \( P_2 \) is a prime.

We have the best asymptotic formula \[ \pi_2(N, 2) = \left\lfloor \frac{P_2 \leq N \text{ prime}}{\phi(N)} \right\rfloor \sim \frac{1 - \frac{P_1^2 - 1}{2}}{(P_1 - 1)^2} \left\lfloor \frac{N}{\log^2 N} \right\rfloor \quad (11) \]

where \( \phi(\omega) = \prod_{P \leq 2} (P - 1) \).

Theorem 2: Let \( x+y = P_1 \) and \( x-y = P_2 - 2 \), we have \( a = P_1(P_2 - 2) \) and,

\[ P_1^2 = (x+y)^2 = c + b, \quad (12) \]

\[ (P_2 - 2)^2 = (x-y)^2 = c - b \quad (13) \]

From eqns. (12) and (13) we have,

\[ a = P_1(P_2 - 2), \quad b = \frac{P_1^2 - (P_2 - 2)^2}{2}, \quad c = \frac{P_1^2 + (P_2 - 2)^2}{2} = P_2 \quad (14) \]

There are infinitely many primes \( P_1 \) such that \( P_2 \) is a prime.

Proof: We have Jiang function \[ J_1(\omega) = \prod_{P \leq 2} (P - 1 - \chi(P)), \quad (15) \]

Where \( \chi(P) \) is the number of solutions of congruence

\[ q^2 + (q-2)^2 = 0 \pmod{P}, \quad q = 1, \ldots, P-1. \quad (16) \]

From (16) we have,

\[ \chi(P) = 1 + \frac{P-1}{2} \quad (17) \]

Substituting (17) into (15) we have,

\[ J_1(\omega) = \prod_{P \leq 2} (P - 2 - (-1)^{\frac{P-1}{2}}) \neq 0 \quad (18) \]

Since \( J_1(\omega) \neq 0 \), we prove that there are infinitely many prime \( P_1 \) such that \( P_2 \) is a prime.

We have the best asymptotic formula \[ \pi_2(N, 2) = \left\lfloor \frac{P_2 \leq N \text{ prime}}{\phi(N)} \right\rfloor \sim \frac{1 - \frac{P_1^2 - 1}{2}}{(P_1 - 1)^2} \left\lfloor \frac{N}{\log^2 N} \right\rfloor \quad (19) \]

Theorem 3: Let \( x+y = 1 \) and \( a = x+y = P_2^2 \), we have,

\[ a = P_1^2, \quad b = \frac{P_1^4 - 1}{2}, \quad c = \frac{P_1^4 + 1}{2} = P_2 \quad (20) \]

There are infinitely many primes \( P_1 \) such that \( P_2 \) is a prime.

Proof: We have Jiang function \[ J_1(\omega) = \prod_{P \leq 2} (P - 1 - \chi(P)), \quad (21) \]

Where \( \chi(P) \) is the number of solutions of congruence,

\[ q^4 + 1 = 0 \pmod{P}, \quad q = 1, \ldots, P-1. \quad (22) \]

From (22) we have,

\[ \chi(P) = 4 \text{ if } 8 | P-1, \quad \chi(P) = 0 \text{ otherwise. } \quad (23) \]

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Since $J_1(\omega) \neq 0$, we prove that there are infinitely many prime $P_1$, such that $P_2$ is a prime.

We have the best asymptotic formula [1]:

$$\pi_1(N, 2) = \left| \left\{ P_1 \leq N : P_2 = \text{prime} \right\} \right| = \frac{J_1(\omega) N}{4\phi^2(\omega) \log^2 N}, \quad (24)$$

These results are in wide use in biological, physical and chemical fields.

References
1. Chun-Xuan Jiang, Jiang function $J_{\omega}(\omega)$ in prime distribution.