

Prime Counting Function $\pi(n)$

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Abstract

We have created a formula to calculate the number of primes less than or equal to any given positive integer 'n'. It is denoted by $\pi(n)$.

This is a fundamental concept in number theory and it is difficult to calculate. A prime number can be divided by 1 and itself.

Therefore the set of primes (2,3,5,7,11,13,17,...). The Prime Counting Function was conjectured the end of the 18th century by Gauss and by Legendre to be approximately $x/\ln(x)$

But in this paper we are presenting the real formula, by applying the modern approach that is applying the basic concept of set theory.

Keywords: Integer • Prime • Positive

Introduction

The main problem in number theory is to understand the distribution of prime numbers. Let $\pi(n)$, denote the Primes Counting Function defined as the number of primes less than or equal to 'n'. Many Mathematicians had worked hard and tried to create the formula for Prime Counting Function $\pi(n)$. A good numbers of deep problem in analytic number theory can be expressed in terms of the Prime Counting Function $\pi(n)$. For example, the Riemann Hypothesis, so Gauss and Legendre approximation solution $x/\ln(x)$ in the sense that the statement is the prime number theorem So till now there is no formula for the Prime Counting Function $\pi(n)$ as you see from the end of 18th century to till now. In this formula we are presenting the real formula and it's prove (examine) by taking examples we find that the formula which I have created is absolutely correct [1-7].

Method

We were trying to create the formula for the Prime Counting Function $\pi(n)$ (Figure 1).

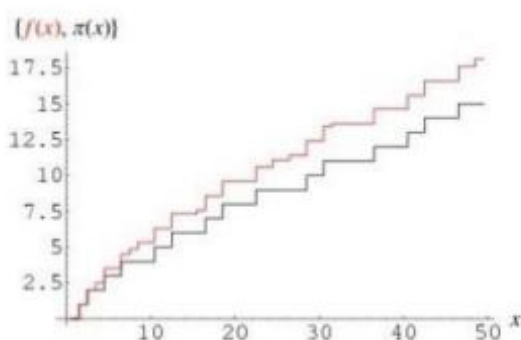


Figure 1. Riemann Prime Counting Function.

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Figure in number theory in Mathematics than we observe the figures often times. The set containing the prime numbers (2,3,5,7,11,13,17,19,23,29,31) we observed that there is no distinct common gaps between two serial prime numbers, that is we cannot find out any common interval to the primes. How can we formulate the Prime Counting Function $\pi(n)$, we were so worked hard and hard To formulate it, as it is originally a basic concept of Number theory (arithmetic). We have done the formula to the Prime Counting Function $\pi(n)$, we can give lecture and demonstration to our students in a very understanding and simple way to "the Prime Counting Function $\pi(n)$."

Proof

In number theory, we introduce one new formula to calculate the number of primes to any given positive integer 'n', by applying a basic concept of set theory to that number theory. We know that there is no such prime less than positive integer 1, as smallest prime is

2. So by keeping it in our mind, let's start Let $\pi(n)$ =number of primes less than or equal to the positive integer n.

Therefore, $\pi(1) = 0$;

Now we can introduce the formula for $\pi(n)$, as mentioned below

$$\pi(n) = 1 + n \{Z \setminus \{A \cup B \cup C \cup D \dots\}\}$$

Where, Z=the set consists of all the positive odd integers less than or equal to n, which are greater than 2.

A=the set consists of all the positive multiples of the prime 3, which are greater than 3 and less than or equal to n.

B=the set consists of all the positive multiples of the prime 5, which are greater than 5 and less than or equal to n.

C= the set consists of all the positive multiples of the prime 7, which are greater than 7 and less than or equal to n.

D=the set consists of all the positive multiples of the prime 11, Which are greater than 11, and less than or equal to n..... And so on.

Therefore, for $n=2$; $\pi(2) = 1 + n \{Z \setminus \{A\}\}$, as there is no Odd positive integer less than or equal to 2.

That is, $\pi(2) = 1 + 0 = 1$.

And $\pi(3) = 1 + n \{Z \setminus \{A\}\}$, here $Z = \{3\}$ and $A = \{3\}$

$$= 1 + n \{Z \setminus \{A\}\}$$

$$= 1 + 1 = 2$$

Which is correct, as the number of primes less than or Equal to 3 are 2 and 3 that is the number of primes 2. Now for n =15, that is $\pi(15) = 1 + n[Z \setminus (A \cup B \cup C)]$

Here, Z =the set consists of all positive odd integers less than or equal to 15 and which are greater than 2.

$$Z = \{3, 5, 7, 9, 11, 13, 15\}$$

A=the set consists of all the positive multiples of the prime 3, which are greater than 3 and less than or equal to 15.

$$A = \{6, 9, 12, 15\}$$

B=the set consists of all the positive multiples of the prime 5, which are greater than 5 and less than or equal to 15.

$$B = \{10, 15\}$$

C=the set consists of all the positive multiples of the prime 7, which are greater than 7 and less than or equal to 15. $C = \{14\}$

$$\text{Therefore, } A \cup B \cup C = \{6, 9, 10, 12, 14, 15\} \quad Z \setminus (A \cup B \cup C) = \{3, 5, 7, 9, 11, 13, 15\} \setminus \{6, 9, 10, 12, 14, 15\} = \{3, 5, 7, 11, 13\}$$

$$\text{Thus, } n\{Z \setminus (A \cup B \cup C)\} = n\{3, 5, 7, 11, 13\} = 5$$

$$\text{Therefore, } \pi(15) = 1 + n\{Z \setminus (A \cup B \cup C)\} = 1 + 5 = 6$$

Which is correct, as the prime numbers less than or equal to 15 Are 2, 3, 5, 7, 11, 13; that is 6 .

Now for n =100, that is $\pi(100) = ?$ Here, $Z = \{3, 5, 7, \dots, 95, 97, 99\}$

A = the set consists of multiples of the prime 3. $= \{6, 9, 12, \dots, 93, 96, 99\}$

B = the set consists of all the multiples of the prime 5. $= \{10, 15, 20, \dots, 90, 95, 100\}$

C = the set consists of all the positive multiples of the prime 7. $= \{14, 21, 28, \dots, 84, 91, 98\}$

D = the set consists of all the positive multiples of the prime 11 $= \{22, 33, 44, \dots, 77, 88, 99\}$

E=the set consists of all the positive multiples of the prime 13 $= \{26, 39, 52, 65, 78, 91\}$

F=the set consists of all the positive multiples of the prime 17 $= \{34, 51, 68, 85\}$

G=the set consists of all the positive multiples of the prime 19 $= \{38, 57, 76, 95\}$

H=the set consists of all the positive multiples of the prime 23 $= \{46, 69, 92\}$

I=the set consists of all the positive multiples of the prime 29 $= \{58, 87\}$

J=the set consists of all the positive multiples of the prime 31. $= \{62, 93\}$

K=the set consists of all the positive multiples of the prime 37. $= \{74\}$

L=the set consists of all the positive multiples of the prime 41 $= \{82\}$

M=the set consists of all the positive multiples of the prime 43. $= \{86\}$

N=the set consists of all the positive multiples of the prime 47 $= \{94\}$.

$$(A \cup B \cup \dots \cup M \cup N) = \{6, 9, 10, 12, 14, 15, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 33, 34, 35, 36, 38, 39, 40, 42, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 65, 66, 68, 69, 70, 72, 74, 75, 76, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100\}$$

$$\text{Thus, } Z \setminus (A \cup B \cup C \cup \dots \cup M \cup N) = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$$

$$\text{So that, } n\{Z \setminus (A \cup B \cup C \cup \dots \cup M \cup N)\} = 24$$

$$\text{Therefore, } \pi(100) = 1 + n\{Z \setminus (A \cup B \cup C \cup \dots \cup M \cup N)\} = 1 + 24 = 25. \#$$

Conclusion

The Prime Counting Function $\pi(n)$ has many applications in Number Theory and its related to one of the famous problems in Mathematics, for example the Riemann Hypothesis because the Prime Counting Function is related to Riemann's Function and it has many thousands of applications across science and Mathematics.

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