Polignac's Conjecture with New Prime Number Theorem

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Abstract

There are infinitely many pairs of consecutive primes which differ by even number En. Let Po(N, En) be the number of Polignac Prime Pairs (which difference by the even integer En) less than an integer (N + En), Pei be taken over the odd prime divisors of the even integer En less than √(N + En), Pni be taken over the odd primes less than √(N + En) except Pei, Pi be taken over the odd primes less than √(N + En), then exists the formulas as follows:

Po(N, En) ≥ INT [N × (1 - 1/2) × ∏ (1 - 1/Pei) × ∏ (1 - 2/Pni)] - 1
≥ INT {Ctwin × Ke(N) × 2N/(Ln (N + En))^2} - 1
Po(N, 2) ≥ INT [0.660 × 1.000 × 2N/(Ln (N + 2))^2] - 1
∏ (Pi(Pi - 2)/(Pi - 1)^2) ≥ Ctwin=0.6601618158...
Ke(N)=∏ [(1 - 1/Pei)/(1 - 2/Pei)] = ∏ (Pei - 1)/(Pei - 2) ≥ 1
where -1 is except the natural integer 1.

Keywords: Twin prime, Polignac prime, Bilateral sieve method

Introduction

In number theory, Polignac's conjecture was made by Alphonse de Polignac in 1849 and states: For any positive even number En, there are infinitely many prime gaps of size En. In other words: There are infinitely many cases of two consecutive prime numbers with difference En [1].

The conjecture has not yet been proven or disproven for a given value of En. In 2013 an important breakthrough was made by Zhang Yitang who proved that there are infinitely many prime gaps of size En for some value of En<70,000,000 [2].

For En=6, it says there are infinitely many primes (p, p + 6). For En=4, it says there are infinitely many cousin primes (p, p + 4). For En=2, it is the twin prime conjecture that there are infinitely many twin primes (p, p + 2) as shown in Figure 1. For En=0, it is the new prime theorem.

The Polignac Prime of Even Integer

For any even integer En there exists a prime P for which the Polignac number Q = En + P is also prime. The Polignac Prime pairs shall be denoted by the representation En × P(Q = En + P) - P, where P and Q are primes and prime P[P ≤ Q] is a Polignac prime of even integer En. Looking at the Polignac partition a different way, we can look at the number of distinct representations (or Polignac primes) that exist for En.

For example, as noted at the beginning of this discussion:

4=01 - 03=(4+03) - 03; 4=07 - 03=(4+07) - 07; 4=11 - 07=(4+11) - 07;
4=13 - 07=(4+13) - 13; 4=17 - 11=(4+17) - 11;

where 3, 5, 11, 17, 29, 41, 59 and 71 are Polignac primes of even integer 2.

4=07 - 03=(4+03) - 03; 4=11 - 07=(4+11) - 07;
4=13 - 07=(4+13) - 13; 4=17 - 11=(4+17) - 11;

The Sieve Method about the Polignac Primes

Let En is an any even integer, Ci is a positive integer more than En, then exists the formulas as follows:

Po(N, En) ≥ INT [N × (1 - 1/2) × ∏ (1 - 1/Pei) × ∏ (1 - 2/Pni)] - 1
≥ INT {Ctwin × Ke(N) × 2N/(Ln (N + En))^2} - 1
Po(N, 2) ≥ INT [0.660 × 1.000 × 2N/(Ln (N + 2))^2] - 1
∏ (Pi(Pi - 2)/(Pi - 1)^2) ≥ Ctwin=0.6601618158...
Ke(N)=∏ [(1 - 1/Pei)/(1 - 2/Pei)] = ∏ (Pei - 1)/(Pei - 2) ≥ 1
where -1 is except the natural integer 1.

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Received July 04, 2016; Accepted November 08, 2016; Published November 11, 2016


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where the prime \( p_0 \) is a Polignac prime of even integer \( E_n \).

If both \( p_0 \) and \( E_n+p_0 \) are primes at the same time, \( p_0 \) and \( E_n+p_0 \) can not be divided by all primes more not large than \( \sqrt{N+E_n} \), then sieves out the positive integer \( C_i \); If both \( N+E_n \).

The Total of Representations of Even Integer

Let \( E_n \) is an any even integer, then exists the formula as follows:

\[
E_n=(E_n+C_i) - C_i
\]

(1)

where \( C_i \) and \( E_n+C_i \) are two positive integers more not large than \( N+E_n \).

If \( C_i \) and \( E_n+C_i \) any one can be divided by the prime anyone more not large than \( \sqrt{N+E_n} \), then both the \( p_0 \) and \( E_n+p_0 \) are primes at the same time, where the prime \( p_0 \) is a Polignac prime of even integer \( E_n \).

The Bilateral Sieve Method of Even Prime 2

It is known that the number 2 is an even prime, and above arrangement from \((E_n+1, 1)\) to \((E_n+N, N)\) can be arranged to the form as follows:

\[
(E_n+1, 1), (E_n+2, 2), (E_n+3, 3), (E_n+4, 4), (E_n+5, 5), \ldots, (E_n+N, N).
\]

(2)

From the above arrangement we can obtain the formula about the total of Polignac numbers of even integer \( E_n \) as follows:

\[
C_i(N, E_n)=N=\text{Total of integers } C_i \text{ more not large than } N
\]

(3)

The Bilateral Sieve Method of Odd Prime 3

It is known that the number 3 is an odd prime, and above arrangement from \((E_n+1, 1)\) to \((E_n+N, N)\) can be arranged to the form as follows:

\[
(E_n+1, 1), (E_n+2, 2), (E_n+3, 3), (E_n+4, 4), (E_n+5, 5), \ldots, (E_n+N-XX< 3, N-XX< 3),
\]

(4)

\[
(E_n+2, 3), (E_n+5, 5), (E_n+8, 8), \ldots, (E_n+N-XX< 3, N-XX< 3),
\]

(5)

The number of integers \( C_i \) that both \( C_i \) and \( E_n+C_i \) can not be divided by odd prime 3 is \( \text{INT}(N \times (1/3)) \).

\[
\text{Si}(N, E_n, 3)=\text{INT}(N \times (1/3)), \text{Di}(N, E_n, 3)=\text{INT}(N \times (1/3))/N
\]

(6)

The number of integers \( C_i \) that both \( C_i \) and \( E_n+C_i \) can not be divided by odd prime 3 is \( \text{INT}(N \times (2/3)) \).

\[
\text{Ni}(N, E_n, 3)=\text{INT}(N \times (2/3)), \text{Di}(N, E_n, 3)=\text{INT}(N \times (2/3))/N
\]

(7)

The Bilateral Sieve Method of Odd Prime 5

It is known that the number 5 is an odd prime, and above arrangement from \((E_n+1, 1)\) to \((E_n+N, N)\) can be arranged to the form as follows:

\[
(E_n+1, 1), (E_n+5, 5), (E_n+10, 10), \ldots, (E_n+N-XX< 5, N-XX< 5),
\]

(8)

\[
(E_n+2, 2), (E_n+7, 7), (E_n+12, 12), \ldots, (E_n+N-XX< 5, N-XX< 5),
\]

(9)
(En+4, 4), (En+9, 09), (En+14, 14),..., (En+N-X<X< 5, N-X<X< 5), (En+5, 5), (En+10, 10), (En+15, 15),..., (En+N-X<X< 5, N-X<X< 5).

From the above arrangement we can known that:

If the even integer En can be divided by odd prime 5, then both the Ci and En+Ci can be or can not be divided by odd prime 5 at the same time.

The number of integers Ci that the Ci and En+Ci anyone can be divided by odd prime 5 is INT (N × (1/5)).

The number of integers Ci that both the Ci and En+Ci can not be divided by odd prime 5 is N-INT (N × (1/5))=INT [N-N × (1/5)]=INT[N × (1-1/5)].

The density of integers Ci that both Ci and En+Ci can not be divided by odd prime 5 (the ratio of the number of integers Ci that both Ci and En+Ci can not be divided by odd prime 5 to the total of integers Ci more not large than N) as follows:

Sei(N, En, 5)=INT (N × (1/5)), Ce(N, En, 5)=N-Sei(N, En, 5) \quad (10)

Dei(N, En, 5)=Ce(N, En, 5)/(N)=INT[N × (1-1/5)]/N \quad (11)

If the even integer En can not be divided by the odd prime 5, then both the Ci and En+Ci can not be divided by the odd prime 5 at the same time, that is the Ci and En+Ci only one can be divided or both the Ci and En+Ci can not be divided by the odd prime 5.

The number of integers Ci that the Ci and En+Ci anyone can be divided by the odd prime 5 is INT (N × (2/5)).

The number of integers Ci that both the Ci and En+Ci can not be divided by odd prime 5 (the ratio of the number of integers Ci that both Ci and En+Ci can not be divided by odd prime 5 to the total of integers Ci more not large than N) as follows:

Sn(N, En, 5)=INT (N × (2/5)), Cn(N, En, 5)=N-Sn(N, En, 5) \quad (12)

Dn(N, En, 5)=Cn(N, En, 5)/(N)=INT[N × (1-2/5)]/N \quad (13)

The Sieve Function of Bilateral Sieve Method

Let En is an even integer, then exists the formula as follows:

En=(En + Ci) - Ci \quad (14)

where Ci is the natural integer less than N.

In terms of the above formula we can obtain the array as follows:

(En+1, 1), (En+2, 2), (En+3, 3), (En+4, 4), (En+5, 5),..., (En+N, N).

Let Pi be an odd prime less than \sqrt{N+En}, then the above arrangement can be arranged to the form as follows:

(En+1, 1), (En+Pi+1, Pi+1),..., (En+N-X<X< Pi, N-X<X< Pi), (En+2, 2), (En+Pi+2, Pi+2),..., (En+N-X<X< Pi, N-X<X< Pi), (En+3, 3), (En+Pi+3, Pi+3),..., (En+N-X<X< Pi, N-X<X< Pi), (En+Pi, Pi), (En+2Pi, 2Pi),..., (En+N-X<X< Pi, N-X<X< Pi).

If the even integer En can be divided by the odd prime Pei, then both the Ci and En+Ci can be or can not be divided by the odd prime Pei at the same time.

The number of integers Ci that the Ci and En+Ci anyone can be divided by the odd prime Pei is INT (N × (1/Pei)).

The number of integers Ci that both the Ci and En+Ci can not be divided by odd prime Pei is N-INT (N × (1/Pei))=INT [N-N × (1-Pei)]=INT[N × (1-1/Pei)].

The density of integers Ci that both the Ci and En+Ci can not be divided by odd prime Pei (or the ratio of the number of integers Ci that both the Ci and En+Ci can not be divided by the odd prime Pei to the total of integers Ci more not large than N) as follows:

Se(N, En, Pei)=INT (N × (1/Pei)), Ce(N, En, Pei)=N-Se(N, En, Pei) \quad (15)

De(N, En, Pei)=Ce(N, En, Pei)/(N)=INT[N × (1-1/Pei)]/N \quad (16)

If the even integer En can not be divided by the odd prime Pni, then both the Ci and En+Ci can not be divided by the odd prime Pni at the same time, that is the Ci and En+Ci only one can be divided or both the Ci and En+Ci can not be divided by the odd prime Pni.

The number of integers Ci that the Ci and En+Ci anyone can be divided by the odd prime Pni is INT(N × (2/Pni)).

The number of integers Ci that both the Ci and En+Ci can not be divided by odd prime Pni (the ratio of the number of integers Ci that both the Ci and En+Ci can not be divided by the odd prime Pni to the total of integers Ci more not large than N) as follows:

Sn(N, En, Pni)=INT(N × (2/Pni)), Cn(N, En, Pni)=N-Sn(N, En, Pni) \quad (17)

Dn(N, En, Pni)=Cn(N, En, Pni)/(N)=INT[N × (1-2/Pni)]/N \quad (18)

Let Po(N, En) be the number of Polignac Prime Pairs (which difference by the even integer En) less than an integer (N+En), Pei be taken over the odd integer divisors of the even integer En less than \sqrt{N+En}, Pni be taken over the odd primes less than \sqrt{N+En} except Pei, Pi be taken over the odd primes less than \sqrt{N+En}, then exists the formulas as follows:

Po(N,En) ≥ INT[N × Di(N,En,2) × |DDe(N,En,Pei) × |Dn(N,En,Pni)|-1

=INT [N × (1-1/2) × Π (1-1/Pei) × Π (1-2/Pni)] - 1 \quad (19)

where -1 is except the natural integer 1.

The Polignac Prime Theorem

From above we can obtain that:

Let Po(N, En) be the number of Polignac Prime Pairs (which difference by the even integer En) less than an integer (N+En), Pei be taken over the odd prime divisors of the even integer En less than \sqrt{N+En}, Pni be taken over the odd primes less than \sqrt{N+En} except Pei, Pi be taken over the odd primes less than \sqrt{N+En}, then exists the formulas as follows:

Po(N,En) ≥ INT[N × Di(N,En,2) × |DDe(N,En,Pei) × |Dn(N,En,Pni)|-1

=INT [N × (1-1/2) × Π (1-1/Pei) × Π (1-2/Pni)] - 1 \quad (20)

Apply the Prime Number Theorem as follows:
Let $p_i(N)$ be the number of primes less than or equal to $N$, $p_i(3 \leq p_i \leq p_m)$ be taken over the odd primes less than $\sqrt{N}$, then exists the formula as follows:

$$p_i(N) \geq \text{INT}\left\{\frac{N}{1-1/p_i}\right\} - 1$$

(21)

$$\geq \text{INT}\left\{\frac{N}{(1-1/p_i) \times (1-1/p)}\right\} - 1 \geq \text{INT}\left\{\frac{N}{L(n)(N)}\right\} - 1$$

(22)

$$p_i(N) = 2p_i(N) - 1 \geq 2\times \text{Ctw}n = 0.6601618158...$$

(23)

$$K(n) = \frac{1}{((1-1/p)(1-1/p))} \geq 1$$

(24)

From the above and the formula (20) we can obtain the formula as follows:

$$p(N) = \text{INT}\left\{\frac{N}{1-1/p} \times \sum\left\{\frac{1}{1-1/p}\right\} - 1\right\} - 1$$

(25)

$$\geq \text{INT}\left\{\frac{N}{(1-1/p)(1-1/p)}\right\} - 1 \geq \text{INT}\left\{\frac{N}{1-1/p}\right\} - 1$$

(26)

$$\geq \frac{N}{1-1/p} \times (\frac{N}{1-1/p})^2 - 2 \geq 1$$

(27)

$$K(N) = \prod\left\{\frac{1-1/Pe_i}{1-2/Pe_i}\right\} = \prod\left\{\frac{Pe_i-1}{Pe_i-2}\right\} \geq 1$$

(24)

When the number $N \to \infty$, we can obtain the formula as follows:

$$p(N) = \text{INT}\left\{\frac{N}{1-1/p} \times \text{Ctw}n = 0.6601618158...\right\} - 1$$

(28)

$$\geq 0.660 \times 1.000 \times \text{Ctw}n = 0.6601618158...$$

(29)

The above formula expresses that there are infinitely many pairs of Polignac primes which differ by every even number $En$.

When the En=2, then there are infinitely many twin primes.

**Every Even Integer Greater than Four Can be Expressed as a Sum of Two Odd Primes**

Every even integer greater than four can be expressed as a sum of two odd primes, and exists the formula as follows:

$$p(N) = \text{INT}(Kp_c \times \text{Ctw}n = \frac{N}{(p_c)\times(\frac{N}{2})}) - 1 \geq \text{INT}(\frac{N}{(\frac{N}{2})}) > 1$$

where the $p(N)$ be the number of primes $P$ with $N$-$P$ primes, or, equivalently, the $p(N)$ be the number of ways of writing $N$ as a sum of two primes, the $N$ be the even integer greater than 30000.

**The proof method of Goldbach's conjecture**

The Goldbach's Conjecture is one of the oldest unsolved problems in Number Theory. In its modern form, it states that every even integer greater than two can be expressed as a sum of two primes.

Let $N$ be an even integer greater than 2, and let $N=(N-Gn)+Gn$, with $N-Gn$ and $Gn$ prime numbers, the $Gp(Gp \leq N/2)$ be a Goldbach Prime of even integer $N$. Let $Gp(N)$ be the number of Goldbach Primes of even integer $N$. The number of ways of writing $N$ as a sum of two prime numbers, when the order of the two primes is important, is thus $Gp(N) \geq 2Gp(N)/2$ when $N/2$ is not a prime and is $Gp(N) = 2Gp(N)-1$ when $N/2$ is a prime. The Goldbach's Conjecture states that $Gp(N) > 0$, or, equivalently, that $Gp(N) > 0$, for every even integer $N$ greater than two.

We known that the Goldbach's Conjecture is true for every even integer $N$ greater than 30000, therefore, we only need to prove that the Goldbach's Conjecture is true for every even integer $N$ greater than 30000, that is: $Gp(N) \geq 30000$.

**TWO: The Sieve Method about the Goldbach Primes**

Let $N$ be an even integer greater than 30000, then the even integer $N$ can be expressed to the form as follows:

$$N=(N - Gn) + Gn, Gn \leq N/2$$

(1)

where $Gn$ be the positive integer no greater than $N/2$.

**Sieve method**

Let $N-Gn$ and $Gn$ are two positive integers, if $N-Gn$ and $Gn$ any one can be divisible by the prime $P$, then sieve the positive integer $Gn$; if both the $N-Gp$ and $Gp$ can not be divisible by the all primes no greater than $\sqrt{N}$, then both the $N-Gp$ and $Gp$ are primes at the same time, the prime $Gp$ be called the Goldbach Prime of even integer $N$.

**Theorem 1:** Let $p_c$ be an odd prime factor of even integer $N$ and no greater than $\sqrt{N}$, then the ratio of the number of integers $Gp$ that both the $N-Gp$ and $Gp$ can not be divisible by the prime $P$ to the total of integers $N$ no greater than $N/2$ is follows:

$$R(N,p_c) = \frac{\text{INT}(N/2 - N/2/p_c) = \text{NIT}(N/2 - N/2/p_c)/(N/2)}{N/2}$$

(2)

**Proof:** Because $p_c$ is an odd prime factor of even integer $N$, therefore, both the $N-Gn$ and $Gn$ can or can not be divisible by prime $P$ at the same time, then the number of integers $N$ that the $N-Gn$ and $Gn$ any one can be divisible by the prime $P$ is INT$(N/2/p_c)$, the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the prime $P$ is $\text{INT}(N/2 - N/2/p_c)$, the ratio of the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the prime $P$ to the total of integers $N$ no greater than $N/2$ is follows:

$$R(N,p_c) = \frac{\text{INT}(N/2 - N/2/p_c) = \text{NIT}(N/2 - N/2/p_c)/(N/2)}{N/2}$$

(2)

**Theorem 2:** Let $p_n$ be an odd prime factor of even integer $N$ and no greater than $\sqrt{N}$, then the ratio of the number of integers $Gp$ that both the $N-Gn$ and $Gn$ can not be divisible by the prime $P$ to the total of integers $N$ no greater than $N/2$ is follows:

$$R(N,p_n) = \frac{\text{INT}(N/2 - N/2/p_n) = \text{NIT}(N/2 - N/2/p_n)/(N/2)}{N/2}$$

(2)

**Proof:** Because the $p_n$ is an odd prime factor of even integer $N$, therefore, both the $N-Gn$ and $Gn$ can or can not be divisible by prime $P$ at the same time, then the number of integers $N$ that the $N-Gn$ and $Gn$ any one can be divisible by the prime $P$ is INT$(N/2/p_n)$, the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the prime $P$ is $\text{INT}(N/2 - N/2/p_n)$, the ratio of the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the prime $P$ to the total of integers $N$ no greater than $N/2$ is follows:

$$R(N,p_n) = \frac{\text{INT}(N/2 - N/2/p_n) = \text{NIT}(N/2 - N/2/p_n)/(N/2)}{N/2}$$

(2)

**Theorem 3:** The integer 2 is an even prime factor of even integer $N$, the ratio of the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the even prime 2 to the total of integers $N$ no greater than $N/2$ is follows:

$$R(N,2) = \frac{\text{INT}(N/2 - N/2/2) = \text{NIT}(N/2 - N/2/2)/(N/2)}{N/2}$$

(2)

**Proof:** Because the 2 is an even prime factor of even integer $N$, therefore, both the $N-Gn$ and $Gn$ can be divisible or can not be divisible by the even prime 2 at the same time, then the number of integers $Gn$ that the $N-Gn$ and $Gn$ any one can be divisible by the even prime 2 is $\text{INT}(N/2/2)$, the number of integers $Gn$ that both the $N-Gn$ and $Gn$ can not be divisible by the even prime 2 is $\text{INT}(N/2 - N/2/2)/N/2$.
\[ \text{INT}(N/2/2) = \text{INT}[N/2 - N/2/2], \] the ratio of the number of integers \( N_\text{Gn} \) that both the N-Gn and Gn can not be divisible by the even prime 2 to the total of integers Gn no greater than N/2 is follows:

\[ R(N,2) = \frac{\text{INT}[N/2) - \text{INT}(N/2/2)]}{(N/2)}/(N/2) = \text{INT}[N/2 - N/2/2]/(N/2) \tag{4} \]

Three: The Number of Goldbach Primes of Even Integer

Let \( G_{p}(N) \) be the number of Goldbach primes of even integer N, let \( G_{p}(N,P_n) \) be the number of Goldbach primes no greater than \( \sqrt{N} \), then exists the formulas as follows:

\[ G_{p}(N) = \text{INT} \left\{ \frac{N}{2} \times R(N,2) \times \prod R(N, P_{c_i}) \times \prod R(N, P_{n_i}) \right\} + G_{p}(N,P_{n_i}) - 1 (\text{if } N-1 \text{ prime}) \]

\[ = \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/P_{c_i}) \times \prod (1-2/P_{n_i}) \right\} + G_{p}(N,P_{n_i}) - 1 (\text{if } N-1 \text{ prime}) \tag{5} \]

Let \( P(N) \) be the number of primes less than an integer N, then, be the formula as follows:

\[ P(N) = \text{INT} \left\{ N \times (1-1/P_1) \times (1-1/P_2) \times \cdots \times (1-1/P_m) \right\} + P(N) \cdot P(V(N)) \cdot 1 \]

\[ P(N) = \text{Ps}ha(N) = L_i(N) - 1/2 \times L_i(N^{0.5}) \]

\[ P(N \geq 2 \times 10^{10}) \geq 2/(1+\sqrt{1-4/Ln(N)}) \times N/Ln(N) \geq N/(Ln(N)-1) \]

\[ P(N \geq 2 \times 10^{12}) = \text{INT} \left\{ (N-1) \times 2 \times \prod (1-1/P_i) \right\} \geq N/Ln(N) \tag{6} \]

The Proof of Goldbach’s Conjecture

Theorem 4: Every even integer greater than 30000 can be expressed as a sum of two odd primes.

Proof: According to the formula (5),

We can obtain the formula as follows:

\[ G_{p}(N)+1 \geq \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/P_{c_i}) \times \prod (1-2/P_{n_i}) \right\} \]

\[ = \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (P_{c_i}-1)/(P_{c_i}-2) \times \prod (1-2/P_{c_i}) \times \prod (1-1/P_{n_i}) \right\} \]

\[ = \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/(P_{c_i}-1)^2) \times \prod (1-1/P_{n_i})^2 \right\} \]

\[ = \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/(P_{c_i}-1)^2) \times \prod (1-1/P_{n_i})^2 \right\} \]

\[ = \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/(P_{c_i}-1)^2) \times \prod (1-1/P_{n_i})^2 \right\} \]

\[ \geq \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/(P_{c_i}-1)^2) \times \prod (1-1/P_{n_i})^2 \right\} \]

Apply the formula (6), we can obtain the formula as follows:

\[ \text{Gp}(N \mid N \geq 30000) \geq \text{INT} \left\{ \frac{N}{2} \times (1-1/2) \times \prod (1-1/P_{c_i}) \times \prod (1-1/P_{n_i}) \right\} \]

\[ \geq \text{INT} \left\{ K_{p} \times C_{twin} \times N/Ln(N)^2 \right\} - 1 \geq \text{INT} \left\{ 0.66016 \times N/Ln(N)^2 \right\} - 1 \]

\[ \geq \text{INT} \left\{ 0.66016 \times (30000)/Ln(30000)^2 \right\} - 1 = \text{INT} \left\{ 186.355 \ldots \right\} - 1 = 185 \]

\[ > 1 \] \tag{8}

From above formula (8) we can obtain that:

Every even integer greater than 30000 can be expressed as a sum of two odd primes.

Conclusion

For every even integer \( E_n \) there are infinitely many pairs of Polignac primes which difference by \( E_n \).

When the \( E_n = 0 \), we can obtain New Prime Number Theorem: Let \( P(N) \) be the number of primes less than or equal to \( N \), for any real number \( N \), the New Prime Number Theorem can be expressed by the formula as follows: \( P(N) = R(N) + K \times (L_i(N) - R(N)) \), \( 1 > K > -1 \). The Goldbach’s Conjecture is a Complete Correct Theorem.

References