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# Polignac: New Conjecture

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**Abstract**

The intent of this essay is not to try to prove that the twin primes are infinite. We would just like to add another way so that others interested in Number Theory can help in elucidating this mystery.

The conjecture of Polignac states that each natural pair is equal to the difference of two primes; but this conjecture, it seems, has not yet been proven. However, we note that there is a certain correlation of that thesis with the foundations of our previous study, as proposed in "**Goldbach - New Conjecture**", which led to this monograph on twin primes.

## Introduction

Initially we will summarize the proposal equivalent to Goldbach's conjecture, which can be examined [1].

All natural >1 can be represented by the mean of two primes **p** and **q** equidistant from a natural **n**, through an integer index **k**, such that [2]:

**n 5(p 1 q) 4 2**, being

**p 5 n 2 k** and

**q 5 n 1 k**.

There is symmetry involving **n** and both primes **p** and **q** with amplitude [3]

3 **•••** **n** **•••** 23n 23.

We will use these concepts as a foundation for the study that we will present about the **twin primes**, the pairs (g, h), with [4]

| h 2 g |5 2.

So we have, within our formulation, for a given $k_g$ [5]:

g 5($p_g$ 1 $q_g$) 4 2,

$p_g$ 5(g 2 $k_g$),

$q_g$ 5 (g 1 $k_g$);

And for a given $k_h$:

h 5 ($p_h$ 1 $q_h$) 4 2,

$p_h$ 5 (h 2 $k_h$),

$q_h$ 5(h 1 $k_h$).

For example, below are some symmetries with the pair (71, 73) (Table 1).

According to the conjecture, both symmetries exist -individually, of course and therefore the index **k** behaves randomly [6,7], as seen in the examples with

$k_g$ 5 12 and $k_h$ 5 6 or

$k_g$ 5 18 and $k_h$ 5 36,

Without connection between **g** and **h**.

However, we observed the possibility of finding in each pair chosen for testing, many cases where index **k** could be unique, as seen in two other examples [8,9], with

$k_g$ 5 $k_h$ 5 30 or

$k_g$ 5 $k_h$ 5 66,

And there is a link between **g** and **h**.

This condition -**k 5 $k_g$ 5 $k_h$** -is the basis of this study and we are interested only when and if it can occur; in this situation [10].

For any pair **(g, h)** we can do

(g 2 k) 5p,

(g 1 k) 5q,

(h 2 k) 5 p 1 2,

(h 1 k) 5q 1 2.

Therefore we will have:

**g 5 (p 1 q) 4 2** and

**h 5 [ (p 12) 1 (q 1 2) ] 4 2.**

| Pair (g, h) | k | p | q |
|---|---|---|---|
| g 5 71<br>h 5 73 | $k_g$ 5 12 | $p_g$ 5 59 | $q_g$ 5 83 |
| | $k_h$ 5 6 | $p_h$ 5 67 | $q_h$ 5 79 |
| | $k_g$ 5 18 | $p_g$ 5 53 | $q_g$ 5 89 |
| | $k_h$ 5 36 | $p_h$ 5 37 | $q_h$ 5 109 |
| | $k_g$ 5 30 | $P_g$ 5 41 | $q_g$ 5 101 |
| | $k_h$ 5 30 | $p_h$ 5 43 | $q_h$ 5 103 |
| | $k_g$ 5 66 | $p_g$ 5 5 | $q_g$ 5 137 |
| | $k_h$ 5 66 | $p_h$ 5 7 | $q_h$ 5 139 |

**Table 1:** Symmetries pair of (71,73).

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Looking only for these solutions we had some success with several tests, which induced us to the theory that follows and to distinguish the twin primes in that $\mathbf{k_g \neq k_h}$ of others in that $\mathbf{k_g \neq k_h}$ we adopt the following concept.

## Identical twin primes

Are those in which at least one **k**, simultaneously, satisfies a pair **(g, h)**.

Therefore, among the symmetries of the previous examples, only the following identities can be considered **identical twin primes** (Table 2).

In iterative surveys with k, we were able to conduct symmetry in this way for identical twin primes of small magnitude, and we realized that it was possible to obtain them many times. In Table 3 we have the result of the first pairs.

However, as we can see in the table, we have already started with two pairs where we cannot obtain simultaneous symmetry and, later, we stop at pair (197, 199), also in the same situation; that is, there are impossible cases if we require **n>k**.

| Pair (g, h) | k | p | q |
|---|---|---|---|
| g 5 71<br>h 5 73 | 30 | $p_g$ 5 41 | $q_g$ 5 101 |
| | | $p_h$ 5 43 | $q_h$ 5 103 |
| | 66 | $p_g$ 5 5 | $q_g$ 5 137 |
| | | $p_h$ 5 7 | $q_h$ 5 139 |

**Table 2:** Symmetries of identical twin primes.

| Pairs | k | $p_g$ \| $p_h$ | $q_g$ \| $q_h$ |
|---|---|---|---|
| 3<br>5 | impossible | | |
| 5<br>7 | impossible | | |
| 11<br>13 | 6 | 5 | 17 |
| | | 7 | 19 |
| 17<br>19 | 12 | 5 | 29 |
| | | 7 | 31 |
| 29<br>31 | 12 | 17 | 41 |
| | | 19 | 43 |
| 41<br>43 | 30 | 11 | 71 |
| | | 13 | 73 |
| 59<br>61 | 42 | 17 | 101 |
| | | 19 | 103 |
| 71<br>73 | 30 | 41 | 101 |
| | | 43 | 103 |
| 101<br>103 | 90 | 11 | 191 |
| | | 13 | 193 |
| 107<br>109 | 90 | 17 | 197 |
| | | 19 | 199 |
| 137<br>139 | 132 | 5 | 269 |
| | | 7 | 271 |
| 149<br>151 | 42 | 107 | 191 |
| | | 109 | 193 |
| 179<br>181 | 168 | 11 | 347 |
| | | 13 | 349 |
| 191<br>193 | 90 | 101 | 281 |
| | | 103 | 283 |
| 197<br>199 | impossible | | |

**Table 3:** Symmetric survey of two identical twin primes.

At this point we will pause in our study of twin primes.

Let's revisit the original conjecture considering what would happen if we could expand the symmetry to negative values, that is, if we could make **k>n** possible.

Without restriction for **k**, one immediately observes symmetry with infinite amplitude.

Similarly, as in the initial conjecture, equalities are maintained:

**n 5 (p 1 q) 4 2**, being

**p 5 n 2 k** and

**q 5 n 1 k**;

Where are primes:

**| p | and q**.

Note that **any** integers can now be obtained, and that, in particular:

**n 5 0** with any primes, for p 1q 5 0;

**n 5 1** with any pairs of twin primes;

**n < 0** it is a reflection of **n>0**.

The search iteration can be obtained as follows:

For n even:

k 5 1, 3, 5, ••• ∞.

For n odd:

k 5 2, 4, 6, ••• ∞.

But, let's return to our study, when we have **identical twin primes**.

The proposition assumes the bond between twin primes **g** and **h**, when and if

$\mathbf{k\ 5\ k_g\ 5\ k_h}$.

And, except for the pair (3, 5), we have the iteration of k boils down to:

k 5 6, 12, 18, ••• ∞

Until simultaneously appear the primes:

| p | and q;

| p 1 2 | and q 1 2.

In summary, we have:

$p_g$ 5 (g 2 k),

$q_g$ 5 (g 1 k),

$p_h$ 5 (g 2 k 1 2) and

$q_h$ 5 (g 1 k 1 2).

Without restriction for **k** let's see those impossible identities of Table 4.

Interesting; it is possible to obtain symmetry.

In addition, among the set of the first 1048576 odd primes we have:

3199 identities representing the identical twin primes (5, 7);

1669 identities for (197, 199).

Curiously, even with infinite amplitude, there is only one identity

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| Pairs | k | $p_g$ \| $p_h$ | $q_g$ \| $q_h$ |
|---|---|---|---|
| 3<br>5 | 8 | 25 | 11 |
| | | 23 | 13 |
| 5<br>7 | 12 | 27 | 17 |
| | | 25 | 19 |
| 197<br>199 | 630 | 2433 | 827 |
| | | 2431 | 829 |

**Table 4:** Impossible identities without restriction of k.

| Pair | k | $p_g$ \| $p_h$ | $q_g$ \| $q_h$ |
|---|---|---|---|
| 41<br>43 | 30 | 111 | 71 |
| | | 213 | 73 |
| | 18000 | 217959 | 18041 |
| | | 217957 | 18043 |
| | 1008000 | 21007959 | 1008041 |
| | | 21007957 | 1008043 |
| | 2070000 | 22069959 | 2070041 |
| | | 22069957 | 2070043 |
| | 2163000 | 22162959 | 2163041 |
| | | 22162957 | 2163043 |
| | 3894000 | 23893959 | 3894041 |
| | | 23893957 | 3894043 |
| | 4092000 | 24091959 | 4092041 |
| | | 24091957 | 4092043 |
| | 5010000 | 25009959 | 5010041 |
| | | 25009957 | 5010043 |

**Table 5:** Identities illustrate twin primes for (41, 43).

for (3, 5), with k 5 8, and it is an exercise for the reader to demonstrate the fact.

Hint: other twin primes are of the form (6m 2 1, 6m 1 1) for some natural m and therefore, g ≡ 2 (mod 3) and h ≡ 1 (mod 3).

For symmetry of pairs of identical twin primes it is necessary that, in general, more than one coincidence occurs for **g** and **h** -in isolation —and such that at some point, for identical **k** values, we find equidistant primes.

For 12484 first pairs of twins primes, with the same set of primes already mentioned, we found multiple identities intended, the lowest number being 1035 for (1302017, 1302019) and the highest value was 9468 for (180179, 180181).

To illustrate: among 2188 identities for (41, 43) we selected some cases (Table 5):

So it seems that being infinite amplitude, with infinite prime numbers, it is impossible to determine for each chosen pair how many representations result in identical twin primes, excluding, as already mentioned, the pair (3, 5) with a single identity.

However, one remaining question remains: can all twin primes be identified as identical? That is: are sets equivalents?

Then, reiterating, if

(g, h) are identical twin primes, we have:

g 5 (p 1 q) 4 2,

h 5 [ (p 1 2) 1 (q 1 2) ] 4 2

And as a consequence, are also twin primes the pairs:

**(p, p 1 2)** and

**(q, q 1 2)**.

Therefore, under these conditions, each pair of identical twin primes leads to other twin primes, however not necessarily identical!

But by exploring the previous question:

❖ If we could ensure that all twin primes can also be identical

**and**

❖ If there were one last pair of identical twin primes **($g_u$, $h_u$)**.

It would mean that the last pair of identical twin primes would forward to the another pair of identical twin primes of greater magnitude, which would be an incongruity.

## Conclusion

If it were so, forcibly, the twin primes numbers would be infinite.

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