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Periodicty and Stability of Solutions of Rational Difference Systems

E. M. Elabbasy[1]* and S. M. Eleissawy[2]

[1]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, 35516, Egypt
[2]Department of Physics and Engineering Mathematics, Faculty of Engineering, Port-Said University, Egypt

Abstract

We study the stability character and periodic solutions of the following rational difference systems

$$x_{n+1}=\frac{x_{n-1}\pm 1}{y_nx_{n-1}\pm 1},\quad y_{n+1}=\frac{y_{n-1}\pm 1}{x_ny_{n-1}\pm 1},$$

where the initial values x_{-1},x_0,y_{-1},y_0 are nonzero real numbers. Some numerical examples are given to illustrate our results.

Keywords: Difference equation; Rational systems; Periodic solutions; Stability

Mathematics Subject Classification: 39A10

Introduction

In this paper we study stability character and periodic solutions for the following rational difference systems

$$x_{n+1}=\frac{x_{n-1}\pm 1}{y_nx_{n-1}\pm 1},\quad y_{n+1}=\frac{y_{n-1}\pm 1}{x_ny_{n-1}\pm 1},\tag{1}$$

where the initial values x_{-1},x_0,y_{-1},y_0 are nonzero real numbers .

Difference equation is a hot topic in that it is widely used to investigate equations arising in mathematical models describing real life situations such as population biology, probability theory, genetics and so on. Recently, rational difference equations appeals great interests. In particular, it is popular to study the system of two rational difference equations [1-4].

In [5], Çinar has obtained the positive solution of the difference equation system

$$x_{n+1}=\frac{1}{y_n},\quad y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}}.\tag{2}$$

Also, Çinar et al. [6] has obtained the positive solution of the difference equation system

$$x_{n+1}=\frac{m}{y_n},\quad y_{n+1}=\frac{py_n}{x_{n-1}y_{n-1}}.\tag{3}$$

In [7], Kurbanli et al. investigated the periodicity of the solutions of the system of difference equations

$$x_{n+1}=\frac{x_{n-1}+y_n}{y_nx_{n-1}-1},\quad y_{n+1}=\frac{y_{n-1}+x_n}{x_ny_{n-1}-1}.$$

In [8] Yalcinkaya et al. obtained a sufficient condition for the global asymptotic stability of the following system of difference equations

$$z_{n+1}=\frac{t_n+z_{n-1}}{t_nz_{n-1}+a},\quad t_{n+1}=\frac{z_n+t_{n-1}}{z_nt_{n-1}+a}.$$

Elsayed [9] has obtained the form of solutions for the rational difference system

$$x_{n+1}=\frac{x_{n-1}}{\pm 1+y_nx_{n-1}},\quad y_{n+1}=\frac{y_{n-1}}{\mp 1+x_ny_{n-1}}$$

Yang et al. [10] investigated the system of rational difference equations

$$x_n=\frac{a}{y_{n-p}},\quad y_{n+1}=\frac{by_{n-q}}{x_{n-q}y_{n-q}}.\tag{4}$$

Other related results on system of rational difference equations can be found in references [11-16].

Let I be some interval of real numbers and let $f,g:I\times I\to I$ be continuously differentiable functions. Then for all initial values $(x_k,y_k)\in I$, $k=-1,0,$ the system of difference equations [17].

$$x_{n+1}=f(x_{n-1},y_n),\quad y_{n+1}=g(x_n,y_{n-1}),\ n=0,1,2,...,\tag{5}$$

has a unique solution $\{(x_n,y_n)\}_{n=-1}^{\infty}$.

A point $(\bar{x},\bar{y})$ is called an equilibrium point of the system (5) if

$$\bar{x}=f(\bar{x},\bar{y})\text{ and }\bar{y}=g(\bar{x},\bar{y}).$$

Let $(\bar{x},\bar{y})$ be an equilibrium point of the system (5) [17].

1. An equilibrium point $(\bar{x},\bar{y})$ is said to be stable if for any $\varepsilon>0$ there exist $\delta>0$ such that for every initial points (x_{-1},y_{-1}) and (x_0,y_0) for which $\|(x_{-1},y_{-1})-(\bar{x},\bar{y})\|+\|(x_0,y_0)-(\bar{x},\bar{y})\|<\delta$, the iterates (x_n,y_n) of

***Corresponding author:** E. M. Elabbasy, Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, 35516, Egypt, E-mail: emelabbasy@mans.edu.eg

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(x_{-1}, y_{-1}) and (x_0, y_0) satisfies $\|(x_n, y_n) - (\bar{x}, \bar{y})\| < \varepsilon$ for all $n > 0$. An equilibrium point $(\bar{x}, \bar{y})$ is said to be unstable if it is not stable. (By $\|.\|$ we denote the Euclidean norm in $\mathbb{R}^2$ given by $\|(x, y)\| = \sqrt{x^2 + y^2}$).

2. An equilibrium point $(\bar{x}, \bar{y})$ is said to be asymptotically stable if there exists $r > 0$ such that $(x_n, y_n) \to (\bar{x}, \bar{y})$ as $n \to \infty$ for all (x_{-1}, y_{-1}) and (x_0, y_0) that satisfies $\|(x_{-1}, y_{-1}) - (\bar{x}, \bar{y})\| + \|(x_0, y_0) - (\bar{x}, \bar{y})\| < r$.

Let $(\bar{x}, \bar{y})$ be an equilibrium point of a map $F = (f, g)$, where f and g are continuously differentiable functions at $(\bar{x}, \bar{y})$. The jacobian matrix of F at $(\bar{x}, \bar{y})$ is the matrix [17].

$$J_F(\bar{x}, \bar{y}) = \begin{bmatrix} \frac{\partial f}{\partial x}(\bar{x}, \bar{y}) & \frac{\partial f}{\partial y}(\bar{x}, \bar{y}) \\ \frac{\partial g}{\partial x}(\bar{x}, \bar{y}) & \frac{\partial g}{\partial y}(\bar{x}, \bar{y}) \end{bmatrix}.$$

The linear map $J_F(\bar{x}, \bar{y}) : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$J_F(\bar{x}, \bar{y}) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x}(\bar{x}, \bar{y})x + \frac{\partial f}{\partial y}(\bar{x}, \bar{y})y \\ \frac{\partial g}{\partial x}(\bar{x}, \bar{y})x + \frac{\partial g}{\partial y}(\bar{x}, \bar{y})y \end{bmatrix},$$

is called the linearization of the map F at $(\bar{x}, \bar{y})$.

A solution $\{(x_n, y_n)\}_{n=-1}^{\infty}$ of (1) is periodic if there exist a positive integer ω such that

$$(x_{n+\omega}, y_{n+\omega}) = (x_n, y_n), \quad n = 1, 2, \ldots,$$

and ω is called a period.

Theorem 1 (Linearized Stability Theorem)

Let $F = (f, g)$ be a continuously differentiable function defined on an open set I in $\mathbb{R}^2$, and let $(\bar{x}, \bar{y})$ in I be an equilibrium point of the map $F = (f, g)$ [17].

1. If all the eigenvalues of the Jacobian matrix $J_F(\bar{x}, \bar{y})$ have modulus less than one, then the equilibrium point $(\bar{x}, \bar{y})$ is asymptotically stable.

2. If at least one of the eigenvalues of the Jacobian matrix $J_F(\bar{x}, \bar{y})$ has modulus greater than one, then the equilibrium point $(\bar{x}, \bar{y})$ is unstable.

3. An equilibrium point $(\bar{x}, \bar{y})$ of the map $F = (f, g)$ is locally asymptotically stable if and only if every solution of the characteristic equation

$$\lambda^2 - trJ_F(\bar{x}, \bar{y})\lambda + \det J_F(\bar{x}, \bar{y}) = 0, \tag{6}$$

lies inside the unit circle, that is, if and only if

$$|trJ_F(\bar{x}, \bar{y})| < 1 + \det J_F(\bar{x}, \bar{y}) < 2. \tag{7}$$

4. An equilibrium point $(\bar{x}, \bar{y})$ of the map $F = (f, g)$ is a saddle point if the characteristic equation (6) has one root that lies inside the unit circle and one root that lies outside the unit circle if and only if

$$|trJ_F(\bar{x}, \bar{y})| > |1 + \det J_F(\bar{x}, \bar{y})| \text{ and } (trJ_F(\bar{x}, \bar{y}))^2 - 4\det J_F(\bar{x}, \bar{y}) > 0. \tag{8}$$

5. An equilibrium point $(\bar{x}, \bar{y})$ of the map $F = (f, g)$ is nonhyperbolic if at least one of the eigenvalues of the Jacobian matrix $J_F(\bar{x}, \bar{y})$ has modulus equal one.

6. The characteristic equation (6) has at least one root that lies on the unit circle if and only if

$$|trJ_F(\bar{x}, \bar{y})| = |1 + \det J_F(\bar{x}, \bar{y})| \tag{9}$$

or

$$\det J_F(\bar{x}, \bar{y}) = 1 \text{ and } trJ_F(\bar{x}, \bar{y}) \leq 2. \tag{10}$$

Main Results

The first system

$$x_{n+1} = \frac{x_{n-1} + 1}{y_n x_{n-1} + 1}, \quad y_{n+1} = \frac{y_{n-1} + 1}{x_n y_{n-1} + 1}$$

In this subsection, we study the stability of solutions of the difference system

$$x_{n+1} = \frac{x_{n-1} + 1}{y_n x_{n-1} + 1}, \quad y_{n+1} = \frac{y_{n-1} + 1}{x_n y_{n-1} + 1}, \tag{11}$$

where the initial values x_{-1}, x_0, y_{-1}, y_0 are nonzero real numbers such that $y_0 x_{-1} \neq -1$ and $x_0 y_{-1} \neq -1$.

Theorem 1: *System (11) has the unique positive equilibrium point $(\bar{x}, \bar{y}) = (1, 1)$ which is locally asymptotically stable.*

Proof: The equilibrium point of the system (11) satisfies the following system of equations

$$\bar{x} = \frac{\bar{x} + 1}{\bar{y}\bar{x} + 1}, \quad \bar{y} = \frac{\bar{y} + 1}{\bar{x}\bar{y} + 1}, \tag{12}$$

system (12) implies

$$\bar{y}\bar{x}^2 - 1 = 0, \tag{13}$$

$$\bar{x}\bar{y}^2 - 1 = 0, \tag{14}$$

from equations (13) and (14), the unique positive equilibrium point is $(1, 1)$.

The map F associated to system (11) is

$$F(x, y) = \begin{pmatrix} f(x, y) \\ g(x, y) \end{pmatrix} = \begin{pmatrix} \frac{x + 1}{yx + 1} \\ \frac{y + 1}{xy + 1} \end{pmatrix}. \tag{15}$$

The Jacobian matrix of F at the equilibrium point $(\bar{x}, \bar{y})$ is

$$J_F(\bar{x}, \bar{y}) = \begin{pmatrix} \frac{1 - \bar{y}}{(\bar{y}\bar{x} + 1)^2} & \frac{-(\bar{x}^2 + \bar{x})}{(\bar{y}\bar{x} + 1)^2} \\ \frac{-(\bar{y}^2 + y)}{(\bar{x}\bar{y} + 1)^2} & \frac{1 - \bar{x}}{(\bar{x}\bar{y} + 1)^2} \end{pmatrix}. \tag{16}$$

The value of the Jacobian matrix of F at the equilibrium point $(\bar{x}, \bar{y}) = (1, 1)$ is

$$J_F(1, 1) = \begin{pmatrix} 0 & \frac{-1}{2} \\ \frac{-1}{2} & 0 \end{pmatrix}. \tag{17}$$

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Then the characteristic equation about $(1,1)$ has the following form

$$\lambda^2 - p_1\lambda + q_1 = 0, \tag{18}$$

where

$$p_1 = trJ_F(1,1) = 0, \tag{19}$$

$$q_1 = \det J_F(1,1) = \frac{-1}{4}. \tag{20}$$

The result follows from Theorem 1.1 (iii) and the following relations

$$|p_1| - (1+q_1) = 0 - (1-\frac{1}{4}) = \frac{-3}{4} < 0,$$

and

$$q_1 = \frac{-1}{4} < 1.$$

Therefore, the equilibrium point $(1,1)$ is locally asymptotically stable.

Remark 1: *The following system*

$$x_{n+1} = \frac{x_{n-1}-1}{y_n x_{n-1}+1}, \quad y_{n+1} = \frac{y_{n-1}-1}{x_n y_{n-1}+1},$$

has the unique negative equilibrium point $(\bar{x},\bar{y}) = (-1,-1)$ which is locally asymptotically stable, where the initial values x_{-1}, x_0, y_{-1}, y_0 are nonzero real numbers such that $y_0 x_{-1} \neq -1$ and $x_0 y_{-1} \neq -1$.

The second system

$$x_{n+1} = \frac{x_{n-1}+1}{y_n x_{n-1}-1}, \quad y_{n+1} = \frac{y_{n-1}+1}{x_n y_{n-1}-1}$$

In this subsection, we study the solutions of the following system

$$x_{n+1} = \frac{x_{n-1}+1}{y_n x_{n-1}-1}, \quad y_{n+1} = \frac{y_{n-1}+1}{x_n y_{n-1}-1}, \tag{21}$$

where the initial values x_{-1}, x_0, y_{-1}, y_0 are nonzero real numbers such that $y_0 x_{-1} \neq 1$ and $x_0 y_{-1} \neq 1$.

Theorem 2: *Let* $x_{-1} = c, x_0 = d,\ y_{-1} = r,\ y_0 = h,$ *be nonzero real numbers such that* $y_0 x_{-1} \neq 1$ *and* $x_0 y_{-1} \neq 1$. *Let* $\{(x_n, y_n)\}_{n=-1}^{\infty}$ *be a solution of system (21). Then all solutions of system (21) are periodic with period ten and for* $n = 0,1,\ldots$

$$x_{10n+1} = \frac{c+1}{ch-1}, \quad y_{10n+1} = \frac{r+1}{rd-1},$$

$$x_{10n+2} = rd-1, \quad y_{10n+2} = ch-1,$$

$$x_{10n+3} = \frac{h+1}{ch-1}, \quad y_{10n+3} = \frac{d+1}{rd-1},$$

$$x_{10n+4} = r, \quad y_{10n+4} = c,$$

$$x_{10n+5} = h, \quad y_{10n+5} = d,$$

$$x_{10n+6} = \frac{r+1}{rd-1}, \quad y_{10n+6} = \frac{c+1}{ch-1},$$

$$x_{10n+7} = ch-1, \quad y_{10n+7} = rd-1,$$

$$x_{10n+8} = \frac{d+1}{rd-1}, \quad y_{10n+8} = \frac{h+1}{ch-1},$$

$$x_{10n+9} = c, \quad y_{10n+9} = r,$$

$$x_{10n+10} = d, \quad y_{10n+10} = h.$$

Proof: From equation (21), we see that

$$x_{n+1} = \frac{x_{n-1}+1}{y_n x_{n-1}-1}, \quad y_{n+1} = \frac{y_{n-1}+1}{x_n y_{n-1}-1},$$

$$x_{n+2} = \frac{x_n+1}{y_{n+1}x_n-1} = \frac{x_n+1}{x_n\left(\frac{y_{n-1}+1}{x_n y_{n-1}-1}\right)-1} = x_n y_{n-1}-1,$$

$$y_{n+2} = \frac{y_n+1}{x_{n+1}y_n-1} = \frac{y_n+1}{y_n\left(\frac{x_{n-1}+1}{y_n x_{n-1}-1}\right)-1} = y_n x_{n-1}-1,$$

$$x_{n+3} = \frac{x_{n+1}+1}{y_{n+2}x_{n+1}-1} = \frac{\frac{x_{n-1}+1}{y_n x_{n-1}-1}+1}{(y_n x_{n-1}-1)\left(\frac{x_{n-1}+1}{y_n x_{n-1}-1}\right)-1} = \frac{y_n+1}{y_n x_{n-1}-1},$$

$$y_{n+3} = \frac{y_{n+1}+1}{x_{n+2}y_{n+1}-1} = \frac{\frac{y_{n-1}+1}{x_n y_{n-1}-1}+1}{(x_n y_{n-1}-1)\left(\frac{y_{n-1}+1}{x_n y_{n-1}-1}\right)-1} = \frac{x_n+1}{x_n y_{n-1}-1},$$

$$x_{n+4} = \frac{x_{n+2}+1}{y_{n+3}x_{n+2}-1} = \frac{(x_n y_{n-1}-1)+1}{\left(\frac{x_n+1}{x_n y_{n-1}-1}\right)(x_n y_{n-1}-1)-1} = y_{n-1},$$

$$y_{n+4} = \frac{y_{n+2}+1}{x_{n+3}y_{n+2}-1} = \frac{(y_n x_{n-1}-1)+1}{\left(\frac{y_n+1}{y_n x_{n-1}-1}\right)(y_n x_{n-1}-1)-1} = x_{n-1},$$

$$x_{n+5} = \frac{x_{n+3}+1}{y_{n+4}x_{n+3}-1} = \frac{\left(\frac{y_n+1}{y_n x_{n-1}-1}\right)+1}{(x_{n-1})\left(\frac{y_n+1}{y_n x_{n-1}-1}\right)-1} = y_n,$$

$$y_{n+5}=\frac{y_{n+3}+1}{x_{n+4}y_{n+3}-1}=\frac{\left(\frac{x_n+1}{x_ny_{n-1}-1}\right)+1}{(y_{n-1})\left(\frac{x_n+1}{x_ny_{n-1}-1}\right)-1}=x_n,$$

$$x_{n+6}=\frac{x_{n+4}+1}{y_{n+5}x_{n+4}-1}=\frac{y_{n-1}+1}{x_ny_{n-1}-1}=y_{n+1},$$

$$y_{n+6}=\frac{y_{n+4}+1}{x_{n+5}y_{n+4}-1}=\frac{x_{n-1}+1}{y_nx_{n-1}-1}=x_{n+1},$$

$$x_{n+7}=\frac{x_{n+5}+1}{y_{n+6}x_{n+5}-1}=\frac{y_n+1}{x_{n+1}y_n-1}=y_{n+2},$$

$$y_{n+7}=\frac{y_{n+5}+1}{x_{n+6}y_{n+5}-1}=\frac{x_n+1}{y_{n+1}x_n-1}=x_{n+2},$$

$$x_{n+8}=\frac{x_{n+6}+1}{y_{n+7}x_{n+6}-1}=\frac{y_{n+1}+1}{x_{n+2}y_{n+1}-1}=y_{n+3},$$

$$y_{n+8}=\frac{y_{n+6}+1}{x_{n+7}y_{n+6}-1}=\frac{x_{n+1}+1}{y_{n+2}x_{n+1}-1}=x_{n+3},$$

$$x_{n+9}=\frac{x_{n+7}+1}{y_{n+8}x_{n+7}-1}=\frac{y_{n+2}+1}{x_{n+3}y_{n+2}-1}=y_{n+4},$$

$$y_{n+9}=\frac{y_{n+7}+1}{x_{n+8}y_{n+7}-1}=\frac{y_{n+2}+1}{y_{n+3}y_{n+2}-1}=x_{n+4},$$

$$x_{n+10}=\frac{x_{n+8}+1}{y_{n+9}x_{n+8}-1}=\frac{y_{n+3}+1}{x_{n+4}y_{n+3}-1}=y_{n+5}=x_n,$$

$$y_{n+10}=\frac{y_{n+8}+1}{x_{n+9}y_{n+8}-1}=\frac{x_{n+3}+1}{y_{n+4}x_{n+3}-1}=x_{n+5}=y_n.$$

For $n=0$ the result holds for the given solutions. Now suppose $n>0$ that and our assumption holds for $n-1$. That is;

$$x_{10n-9}=\frac{c+1}{ch-1},\quad y_{10n-9}=\frac{r+1}{rd-1},$$

$$x_{10n-8}=rd-1,\quad y_{10n-8}=ch-1,$$

$$x_{10n-7}=\frac{h+1}{ch-1},\quad y_{10n-7}=\frac{d+1}{rd-1},$$

$$x_{10n-6}=r,\quad y_{10n-6}=c,$$

$$x_{10n-5}=h,\quad y_{10n-5}=d,$$

$$x_{10n-4}=\frac{r+1}{rd-1},\quad y_{10n-4}=\frac{c+1}{ch-1},$$

$$x_{10n-3}=ch-1,\quad y_{10n-3}=rd-1,$$

$$x_{10n-2}=\frac{d+1}{rd-1},\quad y_{10n-2}=\frac{h+1}{ch-1},$$

$$x_{10n-1}=c,\quad y_{10n-1}=r,$$

$$x_{10n}=d,\quad y_{10n}=h.$$

It follows that

$$x_{10n+1}=\frac{x_{10n-1}+1}{y_{10n}x_{10n-1}-1}=\frac{c+1}{ch-1},\quad y_{10n+1}=\frac{y_{10n-1}+1}{x_{10n}y_{10n-1}-1}=\frac{r+1}{rd-1},$$

$$x_{10n+2}=\frac{x_{10n}+1}{y_{10n+1}x_{10n}-1}=rd-1,\quad y_{10n+2}=\frac{y_{10n}+1}{x_{10n+1}y_{10n}-1}=ch-1,$$

$$x_{10n+3}=\frac{x_{10n+1}+1}{y_{10n+2}x_{10n+1}-1}=\frac{h+1}{ch-1},\quad y_{10n+3}=\frac{y_{10n+1}+1}{x_{10n+2}y_{10n+1}-1}=\frac{d+1}{rd-1},$$

$$x_{10n+4}=\frac{x_{10n+2}+1}{y_{10n+3}x_{10n+2}-1}=r,\quad y_{10n+4}=\frac{y_{10n+2}+1}{x_{10n+3}y_{10n+2}-1}=c,$$

$$x_{10n+5}=\frac{x_{10n+3}+1}{y_{10n+4}x_{10n+3}-1}=h,\quad y_{10n+5}=\frac{y_{10n+3}+1}{x_{10n+4}y_{10n+3}-1}=d,$$

$$x_{10n+6} = \frac{x_{10n+4}+1}{y_{10n+5}x_{10n+4}-1} = \frac{r+1}{rd-1},\ y_{10n+6} = \frac{y_{10n+4}+1}{x_{10n+5}y_{10n+4}-1} = \frac{c+1}{ch-1},$$

$$x_{10n+7} = \frac{x_{10n+5}+1}{y_{10n+6}x_{10n+5}-1} = ch-1,\ y_{10n+7} = \frac{y_{10n+5}+1}{x_{10n+6}y_{10n+5}-1} = rd-1,$$

$$x_{10n+8} = \frac{x_{10n+6}+1}{y_{10n+7}x_{10n+6}-1} = \frac{d+1}{rd-1},\ y_{10n+8} = \frac{y_{10n+6}+1}{x_{10n+7}y_{10n+6}-1} = \frac{h+1}{ch-1},$$

$$x_{10n+9} = \frac{x_{10n+7}+1}{y_{10n+8}x_{10n+7}-1} = c,\ y_{10n+9} = \frac{y_{10n+7}+1}{x_{10n+8}y_{10n+7}-1} = r,$$

$$x_{10n+10} = \frac{x_{10n+8}+1}{y_{10n+9}x_{10n+8}-1} = d,\ y_{10n+10} = \frac{y_{10n+8}+1}{x_{10n+9}y_{10n+8}-1} = h.$$

Hence, the proof is completed.

Remark 2: *Let* $x_{-1} = c, x_0 = d,\ y_{-1} = r,\ y_0 = h,$ *be nonzero real numbers such that* $y_0 x_{-1} \neq 1$ *and* $x_0 y_{-1} \neq 1$. *Let* $\{(x_n, y_n)\}_{n=-1}^{\infty}$ *be a solution of system*

$$x_{n+1} = \frac{x_{n-1}-1}{y_n x_{n-1}-1},\ y_{n+1} = \frac{y_{n-1}-1}{x_n y_{n-1}-1}. \qquad (22)$$

Then all solutions of system (22) are periodic with period ten and for $n = 0,1,\ldots$

$$x_{10n+1} = \frac{c-1}{ch-1},\ y_{10n+1} = \frac{r-1}{rd-1},$$

$$x_{10n+2} = 1-rd,\ y_{10n+2} = 1-ch,$$

$$x_{10n+3} = \frac{h-1}{ch-1},\ y_{10n+3} = \frac{d-1}{rd-1},$$

$$x_{10n+4} = r,\ y_{10n+4} = c,$$

$$x_{10n+5} = h,\ y_{10n+5} = d,$$

$$x_{10n+6} = \frac{r-1}{rd-1},\ y_{10n+6} = \frac{c-1}{ch-1},$$

$$x_{10n+7} = 1-ch,\ y_{10n+7} = 1-rd,$$

$$x_{10n+8} = \frac{d-1}{rd-1},\ y_{10n+8} = \frac{h-1}{ch-1},$$

$$x_{10n+9} = c,\ y_{10n+9} = r,$$

$$x_{10n+10} = d,\ y_{10n+10} = h.$$

Numerical Examples

In this section, we give some numerical simulations supporting our theoretical analysis via the software package Matlab 7.13. These examples represent the periodicity and stablity of solutions of two dimensional systems of rational difference equations (1).

Example 1

Consider the difference system:

$$x_{n+1} = \frac{x_{n-1}+1}{y_n x_{n-1}+1},\ y_{n+1} = \frac{y_{n-1}+1}{x_n y_{n-1}+1}, \qquad (23)$$

with the initial conditions $x_{-1} = -0.5,\ x_0 = -2,\ y_{-1} = 0.3,\ y_0 = -1.7$ System (23) has local asymptotic stability of the equilibrium point (1,1) (Figure 1).

Example 2

Consider the difference system:

$$x_{n+1} = \frac{x_{n-1}+1}{y_n x_{n-1}-1},\ y_{n+1} = \frac{y_{n-1}+1}{x_n y_{n-1}-1}, \qquad (24)$$

with the initial conditions $x_{-1} = 2,\ x_0 = -5,\ y_{-1} = 3,\ y_0 = 4$. The solution of (24) is periodic with period 10. (Figure 2).

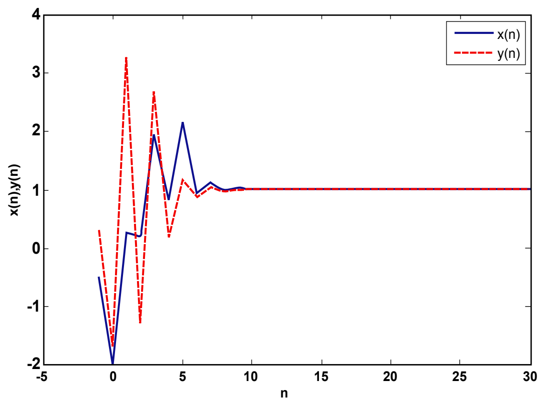

Figure 1: System (23) has local asymptotic stability of the equilibrium point.

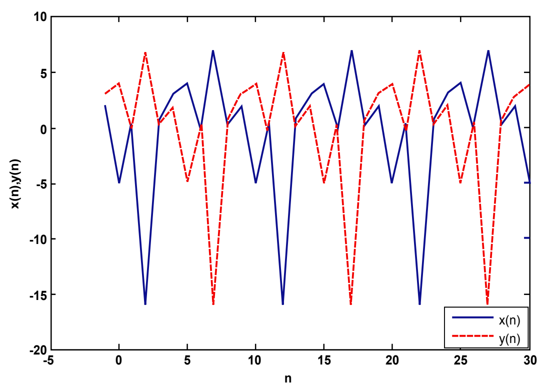

Figure 2: The solution of (24) is periodic with period 10.

References

1. Elabbasy EM, El-Metwally H, Elsayed EM (2008) On the solutions of a class of difference equations systems. Demonstratio Mathematica 41: 109-122.
2. Elsayed EM (2010) On the solutions of a rational system of difference equations. Fasciculi Mathematici 45: 25-36.
3. Gari -Demirovi M, Nurkanovi M (2011) Dynamics of an anti-competitive two dimensional rational system of difference equations. Sarajevo Journal of Mathematics 7: 39-56.
4. Yuan Z, Huang L (2004) All solutions of a class of discrete-time systems are eventually periodic. Appl Math Comput 158: 537-546.
5. Çinar C (2004) On the positive solutions of the difference equation system. Appl Math Comput 158: 303-305.
6. Çinar C, Yalçinkaya I, Karatas R (2005) On the positive solutions of the difference equation system. Journal Institute of Mathematics and Computer Sciences 18: 135-136.
7. Kurbanli AS, Çinar C, Simsek D (2011) On the periodicity of solutions of the system of rational difference equations. Applied Math 2: 410-413.
8. Yalcinkaya, Çinar C (2010) Global asymptotic stability of a system of two nonlinear difference equations. Fasciculi Mathematici 43: 171-180.
9. Elsayed EM (2012) Solutions of rational difference systems of order two. Math Comput Model 55: 378-384.
10. Yang Y, Chen L, Shi Y (2011) On solutions of a system of rational difference equations. Acta Math Univ Comenianae 50: 63-70.
11. Çinar C, Yalçinkaya I (2004) On the positive solutions of the difference equation system. Int Math J 5: 525-527.
12. Çinar C, Yalçinkaya I (2005) On the positive solutions of the difference equation system. Journal Institute of Mathematics and Computer Sciences 18: 91-93.
13. Çinar C, Yalçinkaya I (2004) On the positive solutions of the difference equation system. International Mathematical Journal 5: 517-519.
14. Kurbanli AS (2011) On the behavior of solutions of the system of rational difference equations. Discrete Dyn Nat Soc.
15. Kurbanli S, Çinar C, Yalcinkaya I (2011) On the behavior of positive solutions of the system of rational difference equations. Mathematical and Computer modeling 53: 1261-1267.
16. Özban YA (2007) On the system of rational difference equations. Appl Math Comput 188: 833-837.
17. Kulenovic MRS, Merino O (2002) Discrete Dynamical Systems and Difference Equations with Mathematica. CRC Press Company.