

Peeping into the Collatz Conjecture

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Abstract

In the history of mathematics, there are number of conjectures, unsolved till date, Collatz Conjecture is one of them. Lothar Collatz introduced a conjecture in which a series of numbers get generated called hailstone numbers, a long chain of numbers, all ended to 1. This conjecture may be considered true if the sequence would not either enter a repeating cycle or increase without bound. In this paper a tangible solution is given that anticipate that no such repeated sequence is possible that increases without bound. Hence support the truthfulness of Conjecture. May be it helps the mathematicians to achieve a pragmatic mode.

Keywords: Collatz Conjecture • Repeated cycle • Successive division • Euclid's division

Introduction

Lothar Collatz a German mathematician in 1931 proposed a conjecture called Collatz Conjecture also known as $3n + 1$ conjecture [1]. This conjecture states that if we take any number 'n' and if this number is even, the next term will be the half of it; if the number is odd then the next number will be the 3 times the number plus one ($3n + 1$) which converts the number into even number, and the process goes continue till the sequence reached 1, the numbers generated by this way are also called hailstone numbers. The longest progression for initial starting number is less than 10 billion and 100 quadrillion calculated by R. E. Crandall [2] and Gary T. Leavens [3], several papers have outlined discoveries related towards the apparently unfeasible proofs. Perhaps the most famous recent development was made by Terence Tao, who showed that most orbits of the Collatz map attain almost bounded values [4]. An outstanding review of his paper was published in the College Mathematics Journal [5]. This major development illuminated one potential opportunity to proving the Collatz Conjecture. Even all these works, still this problem is open for further modification, the proof of the Collatz Conjecture seems persistent at this time. In fact Paul Erdos claimed that "mathematics may not be ready for such problems." [6], Jeffery Lagarias echoed this response and provided a scrupulous summary of numerous results concerning the conjecture [7].

Several manuscripts have outlined various, unproductive paths that mathematicians have taken to solve the conjecture [8-10]. Perhaps the proof of this mesmerizing conjecture is so required because the conjecture itself finds use in various applications. The Collatz Conjecture is used in high-uncertainty audio signal encryption [11], image encryption [12] dynamic software watermarking [13] and information discovery [14]. Although the lack of the proof does not exclude applications of the Collatz Conjecture, mathematical phenomenon related to the conjecture may be of interest in other applications, such as cryptography or information theory.

Method

Let 'N' be the set of all positive integers and 'n' be any number, $n \in N$, then Collatz define the following

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$$T(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n + 1 & \text{if } n \text{ is odd} \end{cases}$$

T(n) represents the terms in the sequence

Following cases may be possible:

- I. when the number is even and is in the form of (2^k) then successive division of this number ended with 1 [Euclid's division lemma].
- II. when the number is even and is not in the form of (2^k) then successive division ends with an odd number.
- III. when the number is odd (Figure 1).

Case II and III are ended/started with odd number then the next number will be $(3n + 1)$. If the number is same as in case I (2^k) then there is no problem and it ends to 1 other wise once again it comes an odd number and the same procedure is continue i.e. $(3n + 1)$.

Collatz Conjecture is considering a failure when the series increases without bound or the original number get repeated and the series enter a repeating cycle. Let us see whether it is possible or not. Let us consider a series of odd numbers and let 'j' denotes the serial number of the series, the numbers formed are $(3n + 1)$ and $\{(3n + 1)/2\}$ in Table -1. But the number $\{(3n + 1)/2\}$ can be written as $(2n - j)$. Now,

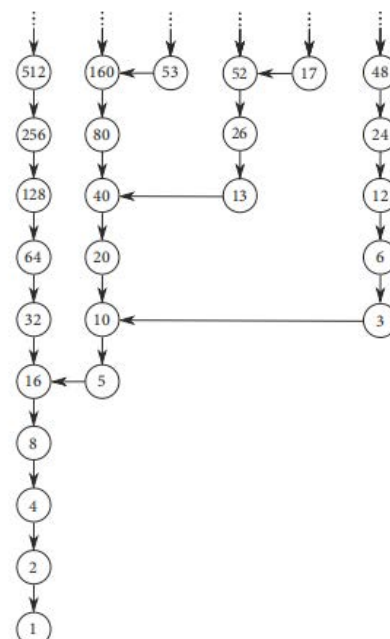


Figure 1. Successive division of number.

Table 1. Collatz Conjecture.

S. No.(J)	Possible odd numbers (n)	(3n + 1)	(3n + 1)/2
1	3	10	5
2	5	16	8
3	7	22	11
4	9	28	14
5	11	34	17
6	13	40	20
7	15	46	23
8	17	52	26

1. $(3n + 1) > 2n$

2. $(2n - j) < 2n \quad j = 1,2,3,\dots$

In both cases a new number never be "2n". Hence there is no chance for the number (n) get repeated. So the Collatz Conjecture is true (Table 1).

Conclusion

In this paper a tangible solution is given that anticipate that no such repeated sequence is possible that increases without bound. Hence support the truthfulness of Conjecture. May be it helps the mathematicians to achieve a pragmatic mode.

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