Journal of Lasers, Optics & Photonics
ISSN: 2469-410X

Rumyantsev et al., J Laser Opt Photonics 2015, 2:1
DOI: 10.4172/2469-410X.1000113

Research Article Open Access

Peculiarities of Light-Matter Coupling in Imperfect Lattice of Coupled Microresonators

Rumyantsev V*, Fedorov S and Sychanova M

A.A. Galkin Donetsk Institute for Physics and Engineering, National Academy of Sciences, Ukraine

Abstract

We study exciton-like electromagnetic excitations in non-ideal microcavity lattice with the use of the virtual crystal approximation. The effect of point defects (vacancies) on the excitation spectrum is being numerically modeled for a quasi-two-dimensional non-ideal binary microcavity supercrystal. The adopted approach permits to obtain the dispersion dependence of collective excitation frequencies and the energy gap width on defect concentrations in a microcavity lattice. Based on the representations of the ideal photonic structures, the non-ideal polaritonic crystal, which is a set of spatially ordered cavities containing atomic clusters, is considered too. The analytical expressions for polaritonic frequencies, effective mass and group velocities, as a function of corresponding quantum dots and vacancies concentrations is obtained.

Keywords: Microcavity supercrystal; Admixture elements; Virtual crystal approximation; Band gap width

Introduction

A number of recent theoretical and experimental works indicate that microcavity supercrystals may have interesting applications. Photonic structures and metamaterials are in the focus of interdisciplinary studies, which span laser physics, condensed matter physics, nanotechnology, chemistry and information science [1,2]. Many papers have been devoted to realization of light-emitting devices based on polariton crystals [3,4]. In this context, semiconductor microcavities represent quantum confined optical systems [5] featured by strong coupling of elementary crystal excitations (excitons) and the optical field. Photonic supercrystals can be built from spatially-periodic systems of coupled microcavities [6]. Semiconductor microcavities are widely used in optoelectronic devices nowadays [7,8]. Nanocavities in photonic crystals [9,10] represent a particular case of microcavities characterized by a discrete photonic spectrum. Nanocavities with embedded quantum dots have been used to demonstrate the strong light-matter coupling regime in Ref. [3,4,11] indicate also that chains of microcavities may be used for practical realization of quantum-information processing.

Here we study dispersions of localized electromagnetic excitations in an array of coupled microcavities which form a non-ideal supercrystal rich by point-like defects. The effect of point defects (vacancies) on the excitation spectrum is being numerically modeled. The adopted virtual crystal approximation (VCA) permits to obtain the dispersion law and the energy gap width of the considered quasiparticles and to analyze the dependence of their density of states on defect concentrations in a microcavity supercrystal.

Based on the representations of the ideal photonic structures, the non-ideal systems of this class - polaritonic crystal, which is a set of spatially ordered cavities containing atomic clusters, is considered too. Moreover, in this part of the work we believe that the spatial distribution of cavities (resonators) is translation invariant, and the atomic subsystem has randomly distributed defects (impurity atomic clusters: quantum dots or a vacancies). Numerical modeling of dependence of the dispersion of polaritons in this imperfect lattice of associated microresonators on impurity concentration is completed. The analytical expressions for polaritonic frequencies, effective mass and group velocities, as a function of corresponding quantum dots and vacancies concentrations, is obtained. It turned out that even with a small number of vacancies in the lattice (one position for a thousand resonators) weight polaritons increases by three orders of magnitude. These results enable to extend the possibility of creating a new class of functional materials - polaritonic crystal systems.

Exciton-like Electromagnetic Excitations in Non Ideal Coupled Microcavities Lattice

Unlike in Refs [3,11,12] devoted to coupled resonators with dopant atoms let us here pose a somewhat different problem. Namely, we intend to examine a 2D array of tunnel-coupled randomly distributed microresonators of different types at the total absence of an atomic subsystem (Figure 1). Each resonator is assumed to possess just one optical mode. We also account for the overlap of optical fields, which enables photons to move along the chain. Since the VCA consists in replacement of configurationally dependent Hamiltonian parameters with their averaged values, Hamiltonian of a "virtual" crystal $\langle H_{ph} \rangle$ in our case reads as follows:

$$\langle H_{ph} \rangle = \sum_{\mathbf{n}\alpha} \langle E_{\mathbf{n}\alpha} \rangle \Psi^{+}_{\mathbf{n}\alpha} \Psi_{\mathbf{n}\alpha} - \sum_{\mathbf{n}\alpha,\mathbf{m}\beta} \langle A_{\mathbf{n}\alpha\mathbf{m}\beta} \rangle \Psi^{+}_{\mathbf{n}\alpha} \Psi_{\mathbf{m}\beta} \,. \qquad (1)$$

Here angular brackets denote configurational averaging. Quantity $A_{\mathbf{n}\alpha\mathbf{m}\beta}$ defines the overlap of optical fields of the $\mathbf{n}\alpha$ -th and $\mathbf{m}\beta$ -th cavities and the transfer of the corresponding excitation, $\Psi^{+}_{\mathbf{n}\alpha}, \Psi_{\mathbf{n}\alpha}$ are bosonic creation and annihilation operators describing the photonic mode. Subscripts $\mathbf{n}$ and $\mathbf{m}$ are two-dimensional integer lattice vectors, α and β numerate sublattices, whose total number is σ. $E_{n\alpha} \equiv \hbar\omega_{n\alpha}$, where $\omega_{\mathbf{n}\alpha}$ is the frequency of photonic mode localized in the $\mathbf{n}\alpha$ -th site (cavity). Hamiltonian (1) is formally identical to the tight-

***Corresponding author:** Rumyantsev V, A.A Galkin Donetsk Institute for Physics and Engineering, National Academy of Sciences, Ukraine, Tel: (062)311-52-27; E-mail: vladimir.rumyantsev2011@yandex.ru

Received November 05, 2014; **Accepted** December 29, 2014; **Published** January 09, 2015

Citation: Rumyantsev V, Fedorov S, Sychanova M (2015) Peculiarities of Light-Matter Coupling in Imperfect Lattice of Coupled Microresonators. J Laser Opt Photonics 2: 113. doi:10.4172/2469-410X.1000113

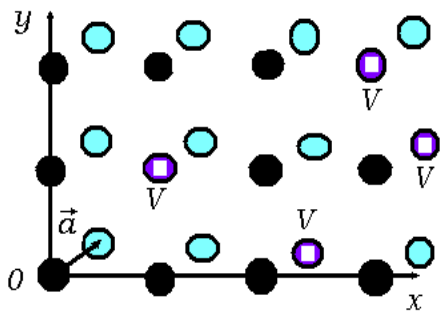

Figure 1: Schematic of a non-ideal two-dimensional two-sublattice system of microcavities for the case of a square Bravais lattice, "V" denotes vacancies.

binding excitonic Hamiltonian in a semiconductor crystal [13,14], for which reason the studied electromagnetic excitations can naturally be referred to as exciton-like. It is worth stressing that we discuss photonic super-crystal excitations and no electronic transitions are involved. Nevertheless, it will be seen below that the dispersion relations of purely electromagnetic crystal excitations in the system we study are quite similar to the Frenkel exciton bands in molecular crystals [13,15]. Here we consider a topologically ordered non-ideal lattice of microcavities with point-like defect: vacancies and non-typical microcavities. In such a system, Hamiltonian H_{ph} is no more translation invariant, hence the quantities $\omega_{\mathbf{n}\alpha}$ and $A_{\mathbf{n}\alpha\mathbf{m}\beta}$ are configurationally dependent. A convenient tool to study the quasiparticle excitation spectrum in a system with randomly distributed defects consists in configurational averaging of the solutions of corresponding Hamiltonians [16]. An averaged solution is translation invariant, hence the corresponding elementary excitation spectrum can be characterized by a wave vector **K**. A widespread method of computation of quasiparticle states in disordered media is the virtual crystal approximation (VCA) [16,17]. It proves sufficient to elucidate the transformations of elementary excitation spectra under varying defect concentrations. In what follows we rely on this method to compute and analyze the spectrum of electromagnetic excitations as well as the corresponding optical characteristics of the considered non-ideal super crystal.

Configurational averaging "restores" the translation invariance of the considered super crystal system. Eigenvalues of Hamiltonian (1) are found via its diagonalization by means of the Bogolyubov-Tyablikov transformation [13,14], and are ultimately found from the system of algebraic equations of the order σ:

$$\hat{L}(\mathbf{k})u_\lambda(\mathbf{k}) = E_\lambda(\mathbf{k})u_\lambda(\mathbf{k}). \tag{2}$$

$u_\lambda(\mathbf{k})$ are eigen functions of the $\sigma\times\sigma$ matrix $\hat{L}$ whose elements are expressed through the corresponding characteristics of the Hamiltonian (1):

$$\begin{aligned} L_{\alpha\beta} &= \langle E_{\mathbf{n}\alpha}\rangle\delta_{\alpha\beta} - \sum_{\mathbf{m}}\langle A_{\mathbf{n}\alpha\mathbf{m}\beta}\rangle\exp\left[i\mathbf{k}\left(\mathbf{r}_{\mathbf{n}\alpha}-\mathbf{r}_{\mathbf{m}\beta}\right)\right] = \\ &= \langle E_{\mathbf{n}\alpha}\rangle\delta_{\alpha\beta} - \sum_{\nu(\alpha),\mu(\beta)=1}^{s(\alpha)r(\beta)} A_{\alpha\beta}^{\nu(\alpha)\mu(\beta)}(\mathbf{k})C_\alpha^{\nu(\alpha)}C_\beta^{\mu(\beta)}, \end{aligned} \tag{3}$$

Here $C_\alpha^{\nu(\alpha)}$ and $C_\beta^{\mu(\beta)}$ are concentrations of the V-th and μ-th types of cavities, $\sum_{\nu(\alpha)} C_\alpha^{\nu(\alpha)} = 1$. $\mathbf{r}_{\mathbf{n}\alpha}$ being the radius-vector of the resonator located in the α th sublattice of the n th elementary cell. The solvability condition of the system (2)

$$\left\|\langle E_{\mathbf{n}\alpha}\rangle\delta_{\alpha\beta} - \hbar\omega_\lambda(\mathbf{k})\delta_{\alpha\beta} - A_{\alpha\beta}(\mathbf{k})\right\| = 0 \tag{4}$$

yields the dispersion law $\omega_\lambda(\mathbf{k})$ of electromagnetic excitations in the considered photonic super crystal.

Consider localized electromagnetic excitations in a two-sublattice ($\sigma = 2$) system of cavities. The left-hand side of Eq. (4) is then a second-order determinant, which being equated to zero gives the following dispersion of photonic excitations:

$$\omega_{1,2}(\mathbf{k}) = \frac{1}{2\hbar}\left\{L_{11}(\mathbf{k}) + L_{22}(\mathbf{k}) \pm \sqrt{\left[L_{11}(\mathbf{k}) - L_{22}(\mathbf{k})\right]^2 + 4L_{12}(\mathbf{k})L_{21}(\mathbf{k})}\right\} \tag{5}$$

Here $L_{11}(\mathbf{k}) = E_1 - A_{11}(\mathbf{k})$, $L_{22}(\mathbf{k}) = E_2 - A_{22}(\mathbf{k})$, $L_{12}(\mathbf{k}) = -A_{12}(\mathbf{k})$ and $L_{21}(\mathbf{k}) = -A_{21}(\mathbf{k})$ are the matrix elements of the operator $\hat{L}$.

To be more specific, let us consider a spectrum of electromagnetic excitations in a binary system where each sublattice contains only two types of cavities. In such a case, the quantities $\langle E_{\mathbf{n}\alpha}\rangle$ and $\langle A_{\mathbf{n}\alpha\mathbf{m}\beta}\rangle$ are given by $\langle E_{\mathbf{n}\alpha}\rangle = \sum_{\nu(\alpha)=1}^{2} E_\alpha^{\nu(\alpha)}C_\alpha^{\nu(\alpha)}$, $\langle A_{\mathbf{n}\alpha\mathbf{m}\beta}\rangle = \sum_{\nu(\alpha),\mu(\beta)=1}^{2} A_{\alpha\beta}^{\nu(\alpha)\mu(\beta)}(\mathbf{k})C_\alpha^{\nu(\alpha)}C_\beta^{\mu(\beta)}$.

Being applied to the supercrystal lattice of microcavities where the only defects are vacancies, these expressions take the form

$$\begin{aligned} &\langle E_{\mathbf{n}1}\rangle = E_1^{(1)}C_1^{(1)};\quad \langle A_{\mathbf{n}1\mathbf{m}1}\rangle = A_{11}^{(11)}(\mathbf{n}-\mathbf{m})C_1^{(1)}C_1^{(1)}, \\ &\langle E_{\mathbf{n}2}\rangle = E_2^{(1)}C_2^{(1)};\quad \langle A_{\mathbf{n}2\mathbf{m}2}\rangle = A_{22}^{(11)}(\mathbf{n}-\mathbf{m})C_2^{(1)}C_2^{(1)} \\ &\langle A_{\mathbf{n}1\mathbf{m}2}\rangle = A_{12}^{(11)}(\mathbf{n}-\mathbf{m})C_1^{(1)}C_2^{(1)},\ \langle A_{\mathbf{n}2\mathbf{m}1}\rangle = A_{21}^{(11)}(\mathbf{n}-\mathbf{m})C_2^{(1)}C_1^{(1)}, \end{aligned} \tag{6}$$

where $C_1^{(1)} \equiv C_1$ is the cavity concentration in the first sublattice, $C_2^{(1)} \equiv C_2$ is the cavity concentration in the second sublattice, $C_{1(2)}^{(1),(2)} \equiv C_{1(2)}^V$ is vacancy concentration in the 1st and/or 2nd sublattices. Concentrations must obviously satisfy the relations $C_1^{(1)} + C_1^V = 1$, $C_2^{(1)} + C_2^V = 1$. In (6) matrix elements $A_{11}^{(11)} \equiv A_{11}$, $A_{22}^{(11)} \equiv A_{22}$, $A_{11}^{(12)} \equiv A_{12}$, $A_{22}^{(21)} \equiv A_{21}$, $A_{11}^{(12)} \equiv A_{12}$, $A_{22}^{(21)} \equiv A_{21}$ characterize the overlap of optical fields of cavities pertaining to the same sublattice but different cells (Figure 1). The energy spectrum of exciton-like electromagnetic excitations is defined by the type of the considered sublattices and the quantities $\langle E_{\mathbf{n}\alpha}\rangle$ and $\langle A_{\mathbf{n}\alpha\mathbf{m}\beta}\rangle$. Below we carry out a nearest-neighbor calculation for the case of a square Bravais lattice of period d [3]. Location of cavities is defined by the radius-vector $\mathbf{r}_{\mathbf{n}\alpha} = \mathbf{r}_\mathbf{n} + \mathbf{r}_\alpha$, hence their location in the zero elementary cell, r_n=0, is defined by vectors $\mathbf{r}_{01} = 0$ and $\mathbf{r}_{02} = \frac{d}{2}\vec{a}$, where $a \ll d$ (Figure 1). In the adopted approximation the matrix elements $A_{\alpha\beta}(\mathbf{k})$ can with reasonable accuracy be written as:

$$\begin{aligned} &A_{11}(\mathbf{k}) \simeq 2A_{11}(d)\left(\cos k_x d + \cos k_y d\right),\ A_{22}(\mathbf{k}) \simeq 2A_{22}(d)\left(\cos k_x d + \cos k_y d\right), \\ &A_{12}(\mathbf{k}) \simeq A_{12}(0)\exp\left(-i\vec{k}\cdot\vec{a}\right),\ A_{21}(\mathbf{k}) \simeq A_{21}(0)\exp\left(i\vec{k}\cdot\vec{a}\right). \end{aligned} \tag{7}$$

In (7) the overlap characteristic of optical fields $A_{11(22)}(d)$ defines the transfer probability of electromagnetic excitation between the nearest neighbors in the first (second) sublattice, and $A_{12(21)}(0)$ is the excitation transfer probability between cavities in the first (second) and second (first) sublattices in an arbitrary cell. Substitution of expressions (7) into Eq. (5) gives the dispersion law $\omega_{1,2}(\mathbf{k})$ for electromagnetic excitations (Figure 2). We performed calculation for modeling frequencies of resonance photonic modes in the cavities of the first and second sublattices $\omega_1 \equiv \langle E_{\vec{n}1}\rangle/\hbar = 6\cdot10^{15}\,Hz$ and

$\omega_2 \equiv \langle E_{n2} \rangle / \hbar = 8 \cdot 10^{14} Hz$ respectively and for the overlap parameters of resonator optical fields $A_{11}/2\hbar = 3 \cdot 10^{14} Hz$, $A_{22}/2\hbar = 5 \cdot 10^{13} Hz$ and $A_{12}/2\hbar \approx A_{21}/2\hbar = 5 \cdot 10^{13} Hz$. The lattice period was set equal to $d = 3 \cdot 10^{-7} m$. Figure 2 gives the example of surfaces depicting the dispersion dependence of collective excitation frequencies in the considered non-ideal microcavity lattice. Surfaces in Figure 2, plotted for $C_1^V = 0.9$ and $C_2^V = 0.4$. The presence of two dispersion branches $\omega_{1,2}(\mathbf{k})$ (see Eq. (5)) reflects a two-sublattice structure of the resonator system. For molecular crystals with two molecules in a cell an analogous occurrence of two branches in the dispersion law is referred to as the Davydov splitting of exciton zone [15] (Figures 2 and 3). Vacancies concentration dependence of the photonic gap width $\Delta\omega(C_1^V, C_2^V)$ in the studied microcavity supersystem. Concentration dependence of the energy gap width $\Delta\omega(C_1^V, C_2^V) \equiv \min_{\mathbf{k}} \left[\omega_+(\mathbf{k}, C_1^V, C_2^V) - \omega_-(\mathbf{k}, C_1^V, C_2^V) \right]$ is shown in Figure 3. The surface $\Delta\omega(C_1^V, C_2^V)$ is non-monotonic and turns to zero in a certain range of (C_1^V, C_2^V). Therefore in a certain region of $(C_1^V, C_2^V) \in (0,1)$ electromagnetic excitations pass unhindered through the binary two-sublattice microcavity system.

Polaritonic Crystal with the Atomic Subsystem Containing Vacancies

One way to create a polaritonic crystal is capture of two-level atoms in the photonic structure consisting of an array of tunnel-coupled microcavities (CROW) [18]. As in [19,20] we study 1D lattice

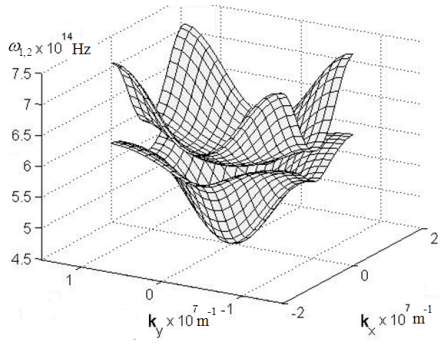

Figure 2: Dispersion $\omega_+(\mathbf{k}, C_1^V, C_2^V)$ of electromagnetic excitations in the non-ideal two-dimensional two-sublattice system of microcavities for $C_1^V = 0.9$, $C_2^V = 0.4$.

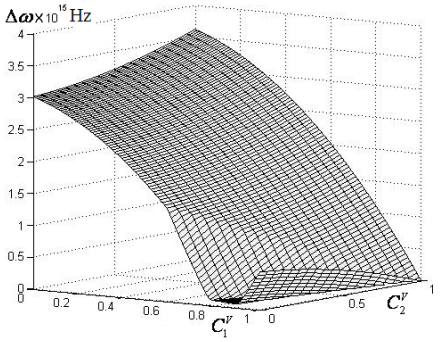

Figure 3: Vacancies concentration dependence of the photonic gap width $\Delta\omega(C_1^V, C_2^V)$ in the studied microcavity supersystem.

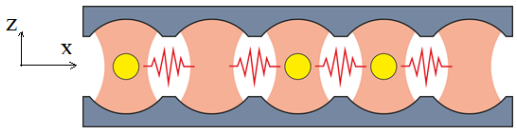

Figure 4: A schematic model of polariton crystal: microresonators lattice containing macroscopic ensembles of two-level atoms and interacting with resonators electromagnetic modes.

microcavities containing one optical mode, each of which interacts with a neighbor in the chain. Thus, each resonator contains the macroscopic cluster of ultracold two level atoms of the same type with the levels $|a\rangle$ and $|b\rangle$ interacting with the quantized electromagnetic field directed (along the *Z* axis) perpendicular to the chain, which oriented along the X axis (Figure 4). In such a configuration the overlap of optical field and the wave functions of atoms are taken into account and there is a possibility photon tunneling along the chain resonators (Figure 4). A schematic model of polariton crystal: microresonators lattice containing macroscopic ensembles of two-level atoms and interacting with resonators electromagnetic modes**.** The Hamiltonian H of the system considered is:

$$H = H_{at} + H_{ph} + H_{\text{int}} \tag{8}$$

Here H_{at} corresponds to an ensemble of two-level atoms (quantum dots) in the trap-resonator, H_{ph} corresponds to the propagation of the light field, H_{int} describes the atom-optical interaction in the cavity. Proceeding from the concepts developed, for example, in [4], the Hamiltonian H for ideal photonic structures can be reduced to the form:

$$H_{at} = \hbar \sum_{n=1}^{M} \left(\omega_{n,at}^{(a)} a_n^+ a_n + \omega_{n,at}^{(b)} b_n^+ b_n - \frac{\gamma_a}{2} (a_n^+ a_{n-1} + a_n^+ a_{n+1} + H.C.) - \right. \tag{9}$$
$$\left. - \frac{\gamma_b}{2} (b_n^+ b_{n-1} + b_n^+ b_{n+1} + H.C.) \right),$$

$$H_{ph} = \hbar \sum_{n=1}^{M} \left(\omega_{n,ph} \psi_n^+ \psi_n - \frac{\alpha}{2} (\psi_n^+ \psi_{n-1} + \psi_n^+ \psi_{n+1} + H.C. \right) \tag{10}$$

$$H_{\text{int}} = \hbar \sum_{n=1}^{M} \frac{g_n}{\sqrt{N}} \left(\psi_n^+ a_n^+ b_n + b_n^+ a_n \psi_n \right), \tag{11}$$

where the annihilation (creation) operators a_n (a_n^+), b_n (b_n^+) in equation (9) characterize the dynamical properties of atomic ensembles (atomic quantum modes) at the lower $|a\rangle$ and upper $|b\rangle$ levels in the *n*-site of array n=1,2…..M, $\hbar\omega_{n,at}^{(a)}$ and $\hbar\omega_{n,at}^{(b)}$ characterize the energy of atom sat the levels respectively. The coupling coefficients $\gamma_{a,b}$ are the nearest-neighbor hopping constants that depend on overlapping integrals of the atomic cloud wave function. Wave functions (these functions are real Wannier functions) are responsible for spatial distribution of ultracold *n*-site atoms under the so-called strong-bonding approximation [21]. The annihilation (creation) operators ψ_n (ψ_n^+) in (10) describe the temporal behaviour of a single photonic mode with frequency $\omega_{n,ph}$ located at the n-th cavity. The parameter α characterizes a spatial field overlapping between the neighboring cavities. The interaction of two-level atoms with the quantized electromagnetic field in equation (11) is considered under the rotating wave approximation and determined by the constant G_n. It is assumed in framework of the polarization crystal model [3] that all the cells are identical to each other and have the same number of atom N=N…., as well as the coefficients of atomic-optical

interaction in (11) are the same in all cells, that g=g1=g2=…..g_M. Let's consider the non-ideal system of this class-polaritonic structure–with atomic subsystem containing the impurity atom clusters. Moreover, the spatial distribution of traps resonators remains, as before, is translation invariant (with the lattice constant *l*) but the atomic subsystem contains randomly distributed foreign (relative to the ideal system) quantum dots (each cavity contains only one atomic complex of a certain sort). Therefore, the parameters of the problem related to the atomic subsystem are configuration-dependent variables. Numerical simulation of such systems can be completed within the VCA, which is to replace the configuration-dependent hamiltonian parameters H_{at} on their configuration averaged values. In this case, the configuration-dependent values are $\omega_{n,at}^{(a,b)}$, $\gamma^{(a,b)}$, g_n. After configuration averaging of these quantities we obtain:

$$\left\langle \omega_{n,at}^{(a,b)} \right\rangle = \sum_{\nu} \omega_{n\nu,at}^{(a,b)} C^{\nu}, \left\langle \gamma^{(a,b)} \right\rangle = \sum_{\nu,\mu} \gamma_{\nu\mu}^{(a,b)} C^{\nu} C^{\mu}, \left\langle g_n \right\rangle = \sum_{\nu} g^{\nu} C^{\nu} \qquad (12)$$

Here C^{ν} is the concentration of atomic clusters (quantum dots) type V, which satisfies the equality $\sum_{\nu} C^{\nu} = 1$; $\omega_{n\nu,at}^{(a,b)}$, $\gamma_{\nu\mu}^{(a,b)}$, g^{ν} are the characteristics relating to atomic components of V-type. The procedure for configuration averaging allowed to "restore" ([16]), the translational invariance of the Hamiltonian and, therefore, to use the scheme of calculations [3,12]. With the help of the Bogolyubov transformations Hamiltonian, which describes the light and dark polaritons [22], is reduced to a diagonal form:

$$H = \hbar \sum_{\bar{k}} \Omega_1(k, \{C^{\nu}\}) \Xi_{1,\bar{k}}^{+} \Xi_{1,\bar{k}} + \hbar \sum_{\bar{k}} \Omega_2(k, \{C^{\nu}\}) \Xi_{2,\bar{k}}^{+} \Xi_{2,\bar{k}} \qquad (13)$$

The annihilation operators $\Xi_{1,\bar{k}}$, $\Xi_{2,\bar{k}}$ in equation (13) characterize two types of quasi particles (due to the atom–field interaction) i.e. upper and lower branch polaritons, respectively. These quasi particles propagate along the X-direction of the periodical structure. The characteristic frequencies $\Omega_{1,2}$ in (13) define a dispersion relations, as well as the polaritonic band structure of the crystal.

To concretize the non-ideal polaritonic system, let's consider the case of the system containing only one type of atoms with the concentration C_1 and vacancies with concentration C_v $C_1 + C_v = 1$. In this case we obtain:

$$\Omega(k, C_v) \equiv \Omega_2(k, C_v) = \frac{1}{2}\left[\omega_{at}(k, C_v) + \omega_{ph}(k) - \omega_R(k, C_v) \right]. \qquad (14)$$

Here the dispersion relations for the atomic and photonic subsystems in a neighbor-hood dot Kl = 0 defined as follows:

$$\omega_{at}(k, C_v) \simeq \left[\omega^{ba} + \gamma k^2 l^2 \right] - C_v \left[\omega^{ba} + \gamma k^2 l^2 \right], \quad \omega^{ba} = \omega_{n,at}^{(b)} - \omega_{n,at}^{(a)} - 2\gamma, \qquad (15)$$

$$\omega_{ph}(k) \simeq \omega_L + \alpha k^2 l^2, \quad \omega_L = \omega_{n,ph} - 2\alpha, \qquad (16)$$

and the Rabi frequency $\omega_R(k, C_v)$ and atomic-optical detuning are:

$$\omega_R(k, C_v) = \sqrt{4g^2 (1 - C_v)^2 + \delta^2}, \delta \equiv \delta(k, C_v) = \omega_{ph}(k) - \omega_{at}(k, C_v) \qquad (17)$$

In expression (15) appeared term $C_v \omega^{ba}$, which is extremely large in comparison, for example, with the average frequency of transition for rubidium D-line which is $\omega^{ba} \approx 2\pi \times 382 THz$. This term, even for a very small concentration of vacancies ($C_v \sim 10^{-4}$) amounts to several *THz*. For comparison, the parameter α characterizing the photon tunneling, is from a few hundred *GHz* to *THz*, and the parameter of atomic-optic communication g consists of from tens to hundreds of *GTz*. It are these parameters, α and g, even in the absence of vacancies are the main contributors to the features of the dispersion curve. The expression for the mass of polaritons has the form:

$$m_2(C_v) = \frac{2 m_{at}(C_v) m_{ph} \omega_R(C_v)}{\left(m_{at}(C_v) + m_{ph} \right) \omega_R(C_v) - \left(m_{at}(C_v) - m_{ph} \right) \Delta(C_v)}, \qquad (18)$$

where

$$\omega_R(C_v) \equiv \omega_R(k, C_v)\big|_{k=0} = \sqrt{\left[\Delta(C_v) \right]^2 + 4g^2 (1 - C_v)^2} \qquad (19)$$

In formulas (21) and (22): $\Delta(C_v) \equiv \delta(k, C_v)\big|_{k=0} = \Delta + C_v \omega^{ba}$, $\Delta \equiv \delta(k, C_v)\big|_{\substack{k=0 \\ C_v=0}} = \omega_L - \omega^{ba}$, $m_{ph} = \hbar / 2\alpha l^2$, $m_{at}(C_v) = \hbar / 2\gamma (1 - C_v) l^2 = M_{at} / (1 - C_v)$.

Figure 5 shows the dependence of the mass m_2 of the polariton on concentration of vacancies C_v introduced by the method described above. In this case, we used the following numerical values of the parameters: the size of the resonators is $l = 3\mu m$, the effective mass of the atoms in the lattice without vacancies is $M_{at} = 1.44 \times 10^{-25} kg$, the effective mass of the photon is $m_{ph} = 2.8 \times 10^{-36} kg$, the atom-optical the detuning $\Delta = 0$ is independent on the wave vector, the average frequency of the rubidium D-line is $\omega^{ba} = 2\pi \times 382 THz$, the parameter of atomic-optic coupling is $g = 2\pi \times 12.2 GHz$. In the case of a lattice without vacancies the mass of polaritons is $m_2 \approx 2m_{ph} \approx 5.6 \times 10^{-35}$.

Analysis of the graph shows that even for small number of vacancies in the lattice (one vacancy on 10^{-4} of resonators) the mass of polaritons increases by an order. The last circumstance testifies an essential role vacancies in effectively reducing of excitation velocity jumping between cavities.

Conclusion

A number of recent experimental works indicate that microcavity supercrystals may have interesting applications, in particular for creating of optical clockworks of unprecedented accuracy [23-25]. In [26] authors considered spectrum of exciton-like electromagnetic excitations in a quasi-2D binary lattice of coupled microcavities. Here we have used the virtual crystal approximation to model the effect of lattice point defects (vacancies) on the spectrum and the photonic gap width dependence on the vacancies concentration for the specific case. Based on the representations of the ideal photonic structures, the non-ideal systems of this class-polaritonic crystal, which is a set of spatially ordered cavities containing atomic clusters, is considered in the paper. The cflculations show that even a minute presence of vacancies (one

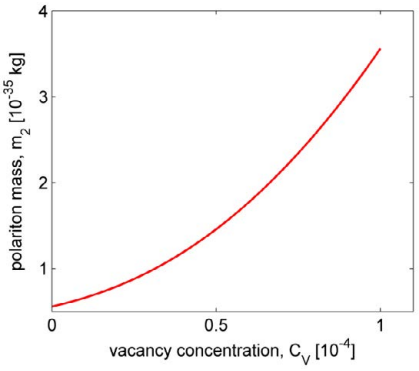

Figure 5: The dependence of polaritonic mass m_2 on vacancies C_v in the imperfect superlattice of coupled microresonators.

per 10^4 of resonators) increases the mass of polaritons by an order. This illustrates the crucial role of vacancies in reducing the effective excitation velocity in chains of microcavities. Obtained here dispersions of electromagnetic excitations are noticeably more complex than those of primitive lattices. This complexity is due to the non-ideality of the structure and to the presence of two sublattices. The latter entails multiple manifestations in experimentally observable integral characteristics of optical processes. Evaluation of excitation spectra in more complex photonic systems requires the use of more sophisticated computational methods. Depending on particular cases such can be the one- or multiple-node coherent potential method [16], the averaged T-matrix method [27] as well as their various modifications. Our study contributes to the modeling of novel functional materials with controllable propagation of electromagnetic excitations.

References

1. Cai W, Shalaev V (2010) Optical Metamaterials: Fundamentals and Applications, Springer.
2. Razeghi M (2010) Technology of Quantum Devices, Springer.
3. Alodjants AP, Barinov IO, Arakelian SM (2010) Strongly localized polaritons in an array of trapped two-level atoms interacting with a light field. J Phys B: At Mol Opt Phys 43.
4. Sedov ES, Alodjants AP, Arakelian SM, Lin YY, Lee RK (2011) Phys Rev A 84: 013813.
5. Joannopoulos JD, Johnson SG, Winn JN, Meade RD (2008) Photonic Crystals. Molding the Flow of Light, Second Edition, Princeton University Press: Princeton.
6. Vahala KJ (2003) Optical microcavities. Nature 424**:** 839-846.
7. Kaliteevskii MA (1997) Tech Phys Lett *2*3: 120-121.
8. Golubev VG, Dukin AA, Medvedav AV, Pevtson AB, Selkin AV, et al. (2003) Splitting of resonant optical modes in Fabry-Perot microcavities. Semiconductors 37: 832-837.
9. Lee RK, Painter O, Kitzke B, Scherer A, Yariv A (2000) J Opt Soc Am B 17: 629-633.
10. Vučković J, Loncar M, Mabuchi H, Scherer A (2001) Design of photonic crystal microcavities for cavity QED. Phys Rev E 65.
11. Englund D (2010) Resonant Excitation of a Quantum Dot Strongly Coupled to a Photonic Crystal Nanocavity. Phys Rev Lett 104.
12. Alodjants AP, Rumyantsev VV, Fedorov SA, Proskurenko MV (2014) Polariton dispersion dependence on concentration of admixture in iperfect superlative of coupled microresonators. Functional Materials 21: 211.
13. Agranovich VM (1968) Theory of Excitons Nauka: Moscow.
14. Kittel C (1987) Quantum theory of solids, Wiley: New York.
15. Davydov AS (1962) Theory of molecular excitons, McGraw Hill: New York.
16. Ziman JM (1979) Models of disorder: The theoretical physics of homogeneously disordered systems, Cambridge University Press, Cambridge.
17. Rumyantsev VV, Fedorov SA (2007) Transformation of the polariton spectrum of nonideal topologically ordered superlattices. Optics and Spectroscopy 102: 68-72.
18. Yang ZS, Kwong NH, Binder R, Smirl AL (2005) Stopping, storing, and releasing light in quantum-well Bragg structures. J Opt Soc Am B 22: 2144.
19. Aoki T, Dayan B, Wilcut E, Bowen WP, Parkins AS, et al. (2006) Observation of strong coupling between one atom and a monolithic microresonator. Nature 443: 671-673.
20. Hartman MJ, Brandao F, Plenio MB (2006) Strongly interacting polaritons in coupled arrays of cavities. Nature physics 2: 849-855.
21. Smerzi A, Trombettoni A, Kevrekidis PG, Bishop AR (2002) Tricontinuous Cubic Structures in ABC/A/C Copolymer and Homopolymer Blends. Physical Review Letters 89: 205502.
22. Turukhin AV, Sudarshanam VS, Shahriar MS, Musser JA, Ham BS (2002) Physical Review Letters 88**.**
23. Del'Haye PD, Schliesser1 A, Arcizet1 O, Wilken T, Holzwarth R (2007) Optical frequency comb generation from a monolithic microresonator. Nature 450: 1214-1217.
24. Hou D, Ning B, Wu J, Wang Z, Zhao J (2013) Demonstration of a stable erbium-fiber-laser-based frequency comb based on a single rubidium atomic resonator. J Appl Phys Lett 102: 151104.
25. Papp SB, Beha K, Haye PD, Quinlan F, Lee H (2014) Microresonator frequency comb optical clock. Optica 1 10-14.
26. Rumyantsev V, Fedorov S, Gumennyk K, Sychanova M, Kavokin A (2014) Nature Sci Rep 4.
27. Los VF (1987) Projection operator method in the theory of disordered systems. I. Spectra of quasiparticles. Theor Math Phys 73: 1076-1088.