

# Partitioned Particle Swarm Optimization

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## Abstract

The particle swarm optimization (PSO) is a population-based optimization method inspired by flocking behavior of birds and human social interactions. So far, numerous modifications of PSO algorithm have been published, which make the PSO method more complex. Several improved PSO versions succeed in keeping the diversity of the particles during the searching process, but at the expense of convergence speed. This paper is aimed at increasing the rate of convergence and diversity of solutions in the population via two easy techniques:

(a) Applying improved acceleration coefficients

(b) Dividing search space into blocks. In particular, the second technique is efficient in the case of functions with optimal design variables situated in the one block. Hence, instead of proposing more complex variant of PSO, a simplified novel technique, called Partitioned Particle Swarm Optimizer (PPSO), has been proposed. In order to find optimal coefficients of this method, an extensive set of experiments were conducted. Experimental results and analysis demonstrate that PPSO outperforms nine well-known particle swarm optimization algorithms with regard to global search.

**Keywords:** Partitioned Particle Swarm Optimization (PPSO); Improved acceleration coefficients; Partitioning search space; Sub population

## Introduction

Particle Swarm Optimization (PSO) is one of the modern heuristic algorithms [1]. It was developed through simulation of simplified social systems, and has been found to be robust in solving nonlinear optimization problems [1,2]. PSO can generate good-quality solutions with stable convergence characteristic at a high speed [3,4]. Shi and Eberhart introduced a linearly decreasing inertia weight [5,6]. A fuzzy controller is used to adapt the inertia weight dynamically [7,8]. Clerc and Kennedy analyzed particle's trajectory and introduced a set of coefficients to control convergence tendencies [9]. Kennedy and Mendes investigated various population topologies and proposed the Von Neumann topological structure [10,11]. Parsopoulos and Vrahatis combined local and global best topologies and proposed Unified PSO (UPSO) [12]. Mendes, Kennedy and Neves proposed Fully Informed Particle Swarm (FIPS) which all of a particle's neighbors' previous best positions are considered to calculate the next positions [13]. By moving particles towards nearby particles with better fitness, Peram et al. proposed Fitness-Distance-Ratio based PSO (FDR-PSO) [14]. Van den Bergh and Engelbrecht, by using multiple swarms to optimize different components of the solution vector cooperatively, designed Cooperative Particle Swarm Optimizer (CPSO-H) [15]. A new method to maintain diversity in a standard generational evolutionary algorithm was proposed by creating subpopulations [16]. The notion of subpopulation is applied to the hybrid model based on the PSO and the standard genetic algorithm (GA) [17]. Comprehensive Learning Particle Swarm Optimizer (CLPSO) which performs well on multimodal problems is introduced [18]. In CLPSO each dimension of a particle may learn from the corresponding dimension of different particle's personal best solution. An elitist learning strategy is introduced, and by evaluating the population distribution and particle fitness, inertia weight and acceleration coefficients of Adaptive PSO (APSO) are controlled [19]. A mechanism is integrated in the velocity update formula of Cellular PSO to modify trajectories of particles to avoid being trapped in local

optimums [20]. In order to improve exploration ability, a random velocity is added to the velocity updating [21]. Inspired by the migratory behavior in the nature, the population is randomly partitioned into several sub-swarms, and some particles migrate from one complex to another to enhance the diversity of the population at periodic stage in the evolution [22]. By decomposition of the search space into subspaces which are probed by sub-swarms, a master-slave model for parallel cooperative micro-particle swarm optimization is introduced [23]. A PSO with Random Dimension Selection (PSORDS) algorithm by utilization of random dimension selection instead of stochastic coefficients is proposed [24]. While several PSO variants have been proposed, the complications were introduced in order to improve the PSO algorithm. Hence, a simplified novel technique is introduced in this paper. In this paper, to improve exploration of search space and escaping local minima, the particles are restricted to subspaces, and for increasing convergence, acceleration coefficients are improved. The rest of this paper is organized as follows. In the above section it gives a brief review on the original PSO algorithm. The above section introduces PPSO in details, followed by the other Sections analyzing the parameters used in PPSO. Experimental results and comparison study to verify the capability of PPSO are shown in the Section below. Finally, Conclusion concludes the paper.

## Basic Concept of Particle Swarm Optimization

James Kennedy and Russell C. Eberhart [1] originally proposed

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the PSO algorithm for optimization. PSO is a population-based search algorithm based on the simulation of the social behavior of birds within a flock. Although PSO was originally adopted to balance weights in neural networks [25], it soon became a popular global optimizer, mainly in problems which the decision variables were real numbers [26,27]. In PSO, particles are flown through hyper-dimensional search space. Changes to the position of particles within search space are based on social-psychological tendency of individuals to emulate success of the others. Let  $\vec{x}_i(t)$  denote position of particle  $i$ , at time step  $t$ . The position of  $\vec{x}_i(t)$  is then changed by adding a velocity  $\vec{v}_i(t)$  to the current position:

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \quad (1)$$

The velocity vector reflects the socially exchanged information and, in general, is defined in the following way:

$$\vec{v}_i(t+1) = W \vec{v}_i(t) + C_1 r_1 (\vec{x}_{pbest_i} - \vec{x}_i(t)) + C_2 r_2 (\vec{x}_{gbest} - \vec{x}_i(t)) \quad (2)$$

where  $r_1, r_2$  have random real values with uniform distribution in  $[0,1]$ .  $C_1$  is the cognitive learning factor and represents the attraction which a particle has toward its own success.  $C_2$  is the social learning factor and represents the attraction that a particle has toward the success of the entire swarm.  $W$  is the inertia weight, which is employed to control the effects of previous velocity on the current velocity.  $\vec{x}_{pbest_i}$  is the personal best position of the particle  $i$ .  $\vec{x}_{gbest}$  is the position of the best particle in the entire swarm.

### PPSO

In this section, a novel variant of PSO is proposed, which is improved by a new search technique and utilizing modified acceleration coefficients to update particle positions. Traditional PSO algorithms easily fall into stagnation, when no particle is able to discover a better position than its previous best position on several iterations. The aim of introducing subspaces is to restrict particles to keep diversity, and attempt to evade suboptimal convergence. Search space is divided into subspaces, and particles are forwarded to them to explore global minimums. This strategy allows particles to explore large areas on initial iterations, and in multimodal landscapes, particles can simultaneously locate multiple optima, quickly and precisely. Actually, particles forwarded to a region work as a team. Each team explores one subspace without sharing information with other teams and achieves the best global solution there. Each dimension is divided into  $R$  parts as follows:

$$x_{min}^1 = x_{min} \quad (3)$$

$$x_{min}^r = x_{max}^{r-1}, r = 2, \dots, R \quad (4)$$

$$x_{max}^r = x_{min}^r + \frac{x_{max} - x_{min}}{R}, r = 1, 2, \dots, R \quad (5)$$

Where  $R$  defined as the number of subspaces. If the search space is defined as  $[x_{min}, x_{max}]$ , the search bounds of  $r$ th subspace is defined by  $[x_{min}^r, x_{max}^r]$ .

### Search algorithm

As well as search space, population is divided in to  $R$  subpopulations. The number of members in a group is the integer part of the result of dividing the population number by  $R$ . If the remainder is not zero, the rest

of particles will be added randomly to groups. Each group has been sent to a special region. While the initial positions are distributed randomly, groups start exploring simultaneously for a maximum allowed number of iterations. By calculation of fitness values,  $\vec{x}_{pbest_{i,r}}$ , and  $\vec{x}_{gbest,r}$  of subspaces are determined, while there is no relationship among groups. Finally, personal best positions and the best particles are memorized as  $\vec{x}_{F,pbest_{i,r}}$  s and  $\vec{x}_{F,gbest,r}$  s.

In the transition step, all particles are initialized with uniform random distribution in the entire search space to have another opportunity to find global optimum. After determinate the number of iterations, personal best positions and the best particle are memorized as  $\vec{x}_{T,pbest_i}$  s and  $\vec{x}_{T,gbest}$ . In both of the first and the transition stages, if the global minimum in  $r$ th region is found or better  $\vec{x}_{gbest}$  after  $\epsilon$  iterations cannot be found,  $r$ th group will stop searching. Therefore, iteration number of the second stage may be increased. Comparing results of the first and the transition stages,  $\vec{x}_{gbest}$  for the second stage is selected. If  $\vec{x}_{gbest}$  is obtained from  $r$ th group, this group will be called *best r* group. In the second stage, in order to avoid being trapped in local minima and to discover better positions, particles can fly in the entire search space. Particles are initialized as the pseudo code shown in Figure 1.

Particles in the *best r* group remain in their positions while others jump to new positions around the best solution, as follows:

$$\vec{x}_i(t+1) = \vec{x}_{gbest} + \overline{normrnd}(\mu, \sigma) \quad (6)$$

Where  $\overline{normrnd}(\mu, \sigma)$  is a vector randomly generated by normal distribution with a mean parameter  $\mu$  and a standard deviation parameter  $\sigma$ . Here,  $\mu$  and  $\sigma$  are set to 0 and 1 respectively [19]. Iteration number of last stage is defined as follows:

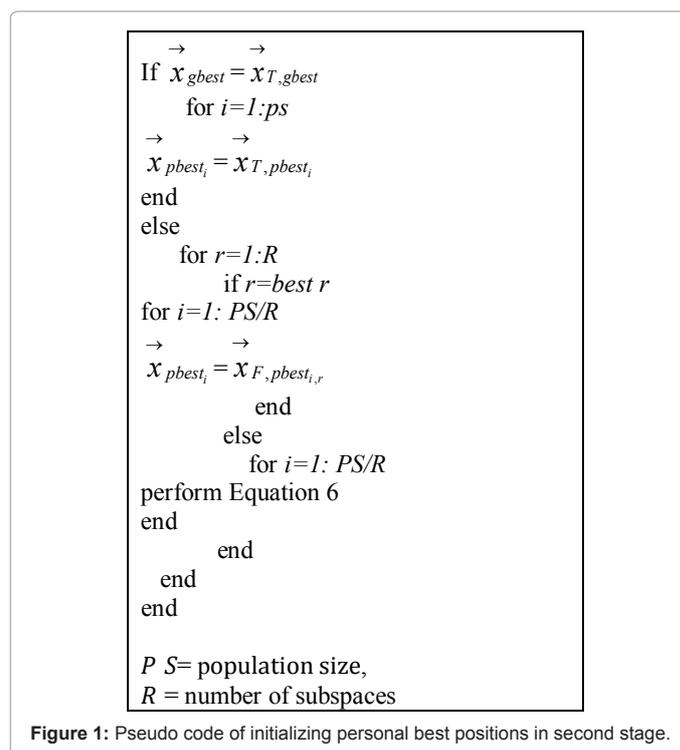


Figure 1: Pseudo code of initializing personal best positions in second stage.

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first stage:
partition search space into R subspaces
divide population into R subpopulations
for r=1:R
    perform PSO on rth subpopulation within rth subspace
        →
        determine  $x_{F,pbest,r}$  s and  $x_{F,gbest,r}$  s
    end
transition stage:
perform PSO on population within search space
    →
    determine  $x_{T,pbest}$  s and  $x_{T,gbest}$ 
second stage:
initialize  $x$  s as the pseudo code shown in Figure 1
perform PSO
    →
    determine  $x_{gbest}$ 

P S= population size,
R = number of subspaces
    
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Figure 2: Pseudo code of PPSO.

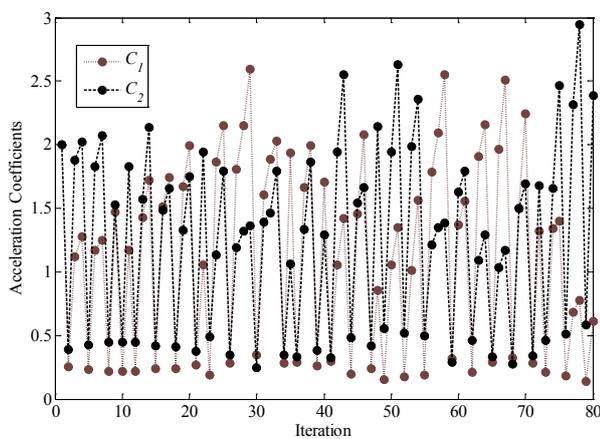


Figure 3: Acceleration coefficients characteristics.

$$\text{iterations of second stage} = \frac{FE - \sum_{i=1}^{R+1} NM(i) \times IT(i)}{PS} \quad (7)$$

where,  $FE$  is referred as the function evaluations,  $NM(i)$  as the number of members in  $i$ th group, and  $IT(i)$  as the number of iterations of  $i$ th group. The Pseudo code of PPSO is shown in Figure 2.

### Acceleration coefficients

In this paper, acceleration coefficients, social learning factor ( $C_2$ ) and cognitive learning factor ( $C_1$ ), are initialized to 2.0 and increased over iterations as follows [28]:

$$C_j(t+1) = C_j(t) + \delta_j, j=1,2 \quad (8)$$

Reference [28] proposed  $\delta_j$  as a random value in the interval [0.01 0.1]. The interval [0.05 0.1] is applied for  $\delta_j$  selection in [19]. In

order to find optimal interval for PPSO, an extensive set of experiments was conducted, and uniformly generated random values in interval [0.0250.80] is chosen. The sum of  $C_1$  and  $C_2$  should be clamped between 3.0 and 4.0, each of them should be set between 0 and 4 [28]. If the sum is larger than 4.0, the following expressions to set the coefficients are developed:

$$C_1(t+1) = \alpha_1 C_1(t) / (C_1(t) + C_2(t)) \quad (9)$$

$$C_2(t+1) = \alpha_2 C_2(t) / (C_1(t) + C_2(t)) \quad (10)$$

If the sum is smaller than 3.0 both of coefficients are magnified by:

$$C_1(t+1) = \alpha_3 C_1(t) / (C_1(t) + C_2(t)) \quad (11)$$

$$C_2(t+1) = \alpha_3 C_2(t) / (C_1(t) + C_2(t)) \quad (12)$$

In [19],  $\alpha_1$  and  $\alpha_2=4.0$  are considered. Experiments over our algorithm reveal that  $\alpha_1=0.5$ ,  $\alpha_2=0.8$ ,  $\alpha_3=3$  lead to perform well on most of test functions.

Figure 3 illuminates changes of acceleration coefficients over 80 iterations. Initially, both of coefficients are set equal to two. In the second iteration,  $C_2$  and  $C_1$  fall substantially to 0.39 and 0.25 respectively, so the effect of inertia weight increases. It can help particles explore new regions, and the ability to flee from local minima may be improved. After sharply growing  $C_2$  and  $C_1$  to 1.88 and 1.12, respectively, they steadily rose to 2.02 and 1.28, respectively. This results in stronger tendency toward search around  $x_{gbest}$ . Again the black line declines to 0.43 and the red line decreases to 0.23. In two iterations they grow to 2.08 and 1.25 respectively. However, both of the social and cognitive learning factors reveal similar trends, a small gap between them leads to discourage premature convergence. Therefore, there is a good balance between the ability to explore new regions and search a smaller region around  $x_{gbest}$ . On the other hand, on the first iterations,  $C_2$  is greater than  $C_1$ , so the tendency toward success of entire swarm is greater than individual's success. In contrast, from iteration 16 to 38, except iteration 22, the pattern changes with higher maximum values of  $C_1$ . These variations are another cause of the ability of algorithm to escape from local minima.

### Inertia weight

In this paper, adaptive inertia weight is used which is given in Equation (13) [19].

$$W(f) = \frac{1}{1 + 1.5e^{-2.6f}} \in [0.4, 0.9] \quad (13)$$

Where 0.4 and 0.9 are initial and final values of the inertia weight, respectively. The parameter  $f$ , which is called evolutionary factor, is defined by (14).

$$f = \frac{d_g - d_{\min}}{d_{\max} - d_{\min}} \in [0, 1] \quad (14)$$

$$d_i = \frac{1}{N-1} \sum_j \sqrt{\sum_{k=1}^D (x_i^k - x_j^k)^2} \quad (15)$$

Where  $d_i$  is the mean distance of particle  $i$  to all of the other particles in its group, which can be measured using a Euclidian metric (15).  $N$  and  $D$  are population size of a group and number of dimensions respectively. of the globally best particle is denoted as  $d_g$ . By comparing all  $d_i$ 's, the maximum and minimum distances,  $d_{\max}$  and  $d_{\min}$  are determined in a group, respectively.

### Analysis of PPSO

In order to get an insight of how parameters influence performance of PPSO, in the following sections the parameters of PPSO will be changed separately to gain the best ranges. All parameters of the PPSO are set as Table 1, except those will be noted. The mean results of 30 independent trials will be presented. An extensive set of experiments were conducted over different test functions, but a few of them are selected randomly to be shown in this paper.

The twenty one benchmark functions, which represent challenges for optimization methods, are used. The name of function, search range of variables, location of global optimum ( $x^{opt}$ ), and corresponding optimum value  $f(x^{opt})$  are summarized in Table 2. Note that these test functions should be minimized.

The term  $\bar{v}_i(t)$  is limited to the range  $[-v_{max}, +v_{max}]$  which  $v_{max}$  is set to 20% of the search range [29]. If the  $d$ th dimension velocity violates this range, then it will be replaced by sign  $(v_i^d)v_{max}^d$ . The equation  $x_i^d = \min(x_{max}^d, \max(x_{min}^d, x_i^d))$  is used to prevent the particles

Number of iterations of first stages	500
Dimension of unfixed number functions	30
$\delta_j$	[0.0250,050]
$\alpha_1$	0.5
$\alpha_2$	0.8
$\alpha_3$	3
$\epsilon$	10
Population size	36
Total iterations	5000
Number of subspaces	6

Table 1: Suggested parameter values for PPSO.

NO.	Name	Search range	$(x^{opt})$	$f(x^{opt})$
1	Sphere	$[-100,100]^D$	$[0, 0, \dots, 0]$	0
2	Rosenbrock	$[-2.048,2.048]^D$	$[1, 1, \dots, 1]$	0
3	Quadric	$[-100,100]^D$	$[0, 0, \dots, 0]$	0
4	Griewank	$[-600,600]^D$	$[0, 0, \dots, 0]$	0
5	Weierstrass	$[0-0.5,0.5]^D$	$[0, 0, \dots, 0]$	0
6	Quartic	$[-1.28,1.28]^D$	$[0, 0, \dots, 0]$	0
7	Nonc_Rastr	$[-5.12,5.12]^D$	$[0, 0, \dots, 0]$	0
8	Rastrigin	$[-5.12,5.12]^D$	$[0, 0, \dots, 0]$	0
9	Beale	$[-4.5,4.5]^2$	$[3, 0.5]$	0
10	Bohachevsky1	$[-100,100]^2$	$[0,0]$	0
11	Bohachevsky2	$[-100,100]^2$	$[0,0]$	0
12	Bohachevsky3	$[-100,100]^2$	$[0,0]$	0
13	Booth	$[-10,10]^2$	$[1, 3]$	0
14	Branin	$[-5,10],[0,15]$	$[-\pi, 12.275], [\pi, 2.275], [9.42478, 2.475]$	0.397887
15	Easom	$[-100,100]^2$		-1
16	Hartmann1	$[0,1]^3$		-3.86278
17	Hump	$[-5,5]^3$	$[0.114614, 0.555649, 0.852547]$	-1.0316
18	Matyas	$[-10,10]^3$	$[0.0898, -0.7126], [-0.0898, 0.7126]$	0
19	Michalewics1	$[0, \pi]^2$	$[0,0]$	-1.8013
20	Michalewics2	$[0, \pi]^5$	$[2.2029055, 1.5707963], [2.2029055, 1.5707963, 1.2849916, 1.9230585, 1.7204698]$	-4.6876582
21	Needle-in-a-haystack	$[-5.12,5.12]^2$	$[0,0]$	-3600

Table 2: Test Functions.

moving out of the search bounds while the search range is defined as  $[x_{min}, x_{max}]$ . Number zero returns numbers less than  $10^{-324}$  in all of the following tables.

### Advantages of PPSO

In this section, modified acceleration coefficients and partitioning search space into blocks will be judged solely on their own merits. The adaptive inertia weight described in sections 4.3 and both of acceleration coefficients set to 2.05 is chosen. By partitioning search space without applying modified acceleration coefficients, effects of this technique on the performance of PPSO are revealed and vice versa. The results are presented in Tables 3 and 4. It can be seen that the easy technique of partitioning search space into blocks not only makes the searching ability stronger, but also increases the rate of convergence.

### Number of subspaces

In this section, the number of subspaces is determined in order to ensure sufficient diversity of particles (Table 4). It seems that, greater number of subspaces leads to greater probability that the algorithm is able to explore regions without attractions of local minima of other regions. On the other hand; the rate of convergence becomes slower if the number of subspaces increases. Furthermore, if dimensions of the global minimum are placed in different regions, increasing the number of subspaces cannot help to reach the global minimum. Therefore, there should be a balance. An exhaustive set of experiments are conducted with fixed number of particles in order to investigate the number of regions which contributes to good results. If the number of regions is

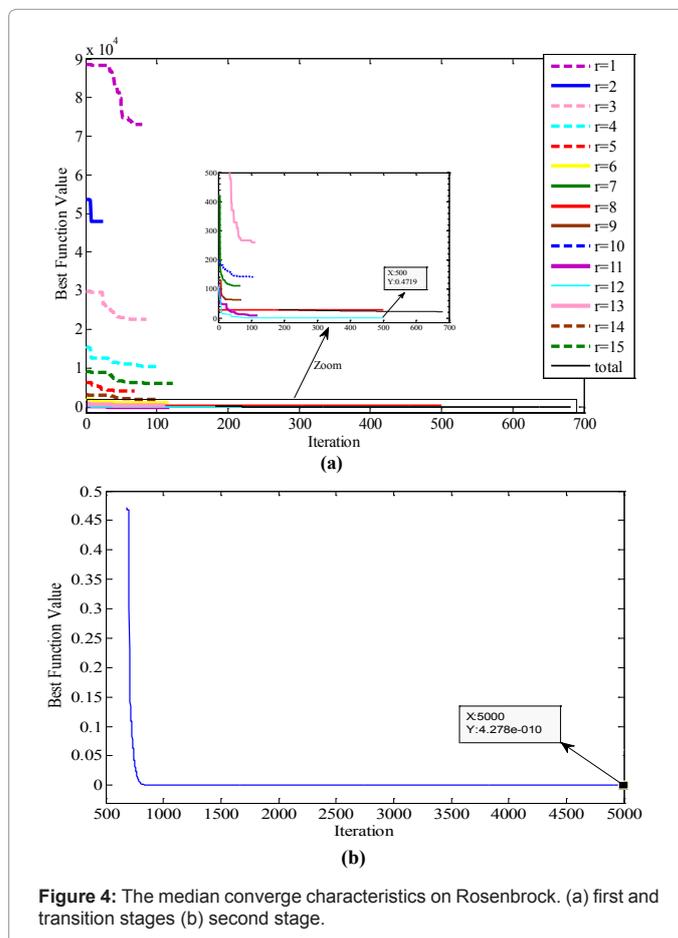


Figure 4: The median converge characteristics on Rosenbrock. (a) first and transition stages (b) second stage.

Function	Algorithm	Simple Variant of PSO	PSO only with modified acceleration coefficients	PSO only with partitioning search space	PPSO
f <sub>1</sub>	Best	5.3980×10 <sup>-86</sup>	0	0	0
	Worst	3.2497×10 <sup>-87</sup>	2.3037×10 <sup>-137</sup>	4.8384×10 <sup>-94</sup>	1.5154×10 <sup>-136</sup>
	Mean	1.6472×10 <sup>-79</sup>	2.1460×10 <sup>-136</sup>	2.4192×10 <sup>-95</sup>	7.5770×10 <sup>-138</sup>
	Std. Dev.	7.2616×10 <sup>-79</sup>	6.0325×10 <sup>-136</sup>	1.0819×10 <sup>-94</sup>	3.3885×10 <sup>-137</sup>
f <sub>2</sub>	Best	1.8824×10 <sup>-1</sup>	7.3931×10 <sup>-1</sup>	1.9104×10 <sup>-8</sup>	6.9124×10 <sup>-12</sup>
	Worst	2.5290×10 <sup>-1</sup>	2.49751	1.2292×10 <sup>-4</sup>	1.5005×10 <sup>-6</sup>
	Mean	2.0009×10 <sup>-1</sup>	1.6701	2.7554×10 <sup>-5</sup>	2.0057×10 <sup>-7</sup>
	Std. Dev.	1.6533	4.342×10 <sup>-1</sup>	3.2341×10 <sup>-5</sup>	3.8164×10 <sup>-7</sup>
f <sub>3</sub>	Best	5.2318×10 <sup>-4</sup>	0	0	0
	Worst	4.0336×10 <sup>-2</sup>	3.5010×10 <sup>-32</sup>	4.3337×10 <sup>-35</sup>	1.5511×10 <sup>-46</sup>
	Mean	6.3514×10 <sup>-3</sup>	4.7657×10 <sup>-33</sup>	4.2690×10 <sup>-36</sup>	7.7560×10 <sup>-48</sup>
	Std. Dev.	9.4327×10 <sup>-3</sup>	1.0955×10 <sup>-32</sup>	1.3141×10 <sup>-35</sup>	3.4684×10 <sup>-47</sup>
f <sub>4</sub>	Best	0	0	0	0
	Worst	3.2020×10 <sup>-2</sup>	7.3960×10 <sup>-3</sup>	0	0
	Mean	9.6035×10 <sup>-3</sup>	7.3960×10 <sup>-4</sup>	0	0
	Std. Dev.	9.9730×10 <sup>-3</sup>	2.2764×10 <sup>-3</sup>	0	0
f <sub>5</sub>	Best	0	0	0	0
	Worst	2.4984	2.6221×10 <sup>-1</sup>	2.8242×10 <sup>-5</sup>	0
	Mean	6.6790×10 <sup>-1</sup>	6.1906×10 <sup>-2</sup>	1.4121×10 <sup>-6</sup>	0
	Std. Dev.	8.1019×10 <sup>-1</sup>	8.8027×10 <sup>-2</sup>	6.3151×10 <sup>-6</sup>	0
f <sub>6</sub>	Best	1.2633×10 <sup>-3</sup>	3.2819×10 <sup>-5</sup>	3.0071×10 <sup>-5</sup>	2.8664×10 <sup>-5</sup>
	Worst	4.7395×10 <sup>-3</sup>	7.5766×10 <sup>-1</sup>	2.9000×10 <sup>-3</sup>	0.00252×10 <sup>-4</sup>
	Mean	3.0865×10 <sup>-3</sup>	2.8834×10 <sup>-1</sup>	1.1975×10 <sup>-3</sup>	7.7829×1 <sup>-4</sup>
	Std. Dev.	9.4352×10 <sup>-4</sup>	2.4067×10 <sup>-1</sup>	9.4450×10 <sup>-4</sup>	7.6605×10 <sup>-4</sup>
f <sub>7</sub>	Best	1.4000×10 <sup>-1</sup>	0	0	0
	Worst	3.9000×10 <sup>-1</sup>	4.2000×10 <sup>-1</sup>	0	0
	Mean	2.6000×10 <sup>-1</sup>	1.3500×10 <sup>-1</sup>	0	0
	Std. Dev.	7.8001	1.6116×10 <sup>-1</sup>	0	0
f <sub>8</sub>	Best	1.5919×10 <sup>-1</sup>	0	0	0
	Worst	5.4722×10 <sup>-1</sup>	4.4773×10 <sup>-1</sup>	0	0
	Mean	3.5619×10 <sup>-1</sup>	8.1586	0	0
	Std. Dev.	1.1788×10 <sup>-1</sup>	1.6787×10 <sup>-1</sup>	0	0

Table 3: Merits of modified acceleration coefficients and partitioning search space on PSO.

Function	Number of Subspaces	2	3	6	7	10	11	14	15	17	18	19
		f <sub>2</sub>	Mean	4.0312	9.1836E-6	7.6532E-6	1.8902E-6	2.5992E-6	5.9704E-7	2.0540E-7	<b>4.6731E-8</b>	5.7236E-7
	Std. Dev.	3.0563	1.0948E-5	7.1094E-6	2.5823E-6	3.1969E-6	7.9011E-7	1.9815E-7	6.2346E-8	6.3662E-7	3.3057E-6	4.9099E-6
f <sub>6</sub>	Mean	0.0021984	0.0020557	0.0029927	0.0024364	0.0021393	0.0026192	0.0028811	<b>7.7829E-4</b>	0.0017587	0.0018858	0.0019014
	Std. Dev.	0.0015983	0.0023226	0.0019425	0.0019783	0.0016112	0.0024394	0.0024995	7.6605E-4	0.0013614	0.0017758	0.0007980

Table 4: Number of Subspaces in PPSO.

Function	N	0.163	0.111	0.081	0.053	0.020	0.010	0.005	0.002
		f <sub>2</sub>	Mean	8.3517E-8	<b>4.6731E-8</b>	1.00134E-7	6.3684E-8	5.2527E-8	2.1609E-7
	Std. Dev.	1.0213E-7	6.2346E-8	2.6681E-7	8.4437E-8	5.0510E-8	2.0105E-7	0.82835	0.75343
f <sub>6</sub>	Mean	0.0010682	<b>7.7829E-4</b>	0.0015144	0.0013360	0.0013307	0.0014006	0.0020807	0.0020584
	Std. Dev.	8.4955E-4	7.6605E-4	8.6131E-4	9.9126E-4	7.5878E-4	0.0011257	0.0012582	0.0013340

Table 5: Effects of ratio of number of iterations of first stage to number of iterations of second stage.

Function	δ <sub>1,2</sub>	[0.025, 0.05]	[0.05, 0.1]	[0.1, 0.2]	[0.2, 0.4]	[0.4, 0.8]	[0.8, 1]	[0.4, 1]	[0.025, 0.1]	[0.025, 0.2]	[0.025, 0.4]	[0.025, 0.8]	[0.025, 1]
		f <sub>2</sub>	Mean	2.0540×10 <sup>-7</sup>	2.0228×10 <sup>-7</sup>	3.9130×10 <sup>-7</sup>	2.4374×10 <sup>-7</sup>	1.1846×10 <sup>-5</sup>	1.1900×10 <sup>-4</sup>	8.1805×10 <sup>-6</sup>	5.9091×10 <sup>-7</sup>	1.1491×10 <sup>-7</sup>	7.2238×10 <sup>-8</sup>
	Std. Dev.	1.9815×10 <sup>-7</sup>	3.0964×10 <sup>-7</sup>	6.3112×10 <sup>-7</sup>	3.9810×10 <sup>-7</sup>	2.2811×10 <sup>-5</sup>	3.0265×10 <sup>-4</sup>	2.2520×10 <sup>-5</sup>	1.1365×10 <sup>-6</sup>	1.2335×10 <sup>-7</sup>	8.2123×10 <sup>-8</sup>	6.2346×10 <sup>-8</sup>	1.7269×10 <sup>-7</sup>
f <sub>6</sub>	Mean	1.2547×10 <sup>-3</sup>	1.7693×10 <sup>-3</sup>	1.3519×10 <sup>-3</sup>	1.0961×10 <sup>-3</sup>	1.1639×10 <sup>-3</sup>	1.2852×10 <sup>-3</sup>	1.0922×10 <sup>-3</sup>	1.2235×10 <sup>-3</sup>	1.2235×10 <sup>-3</sup>	1.6870×10 <sup>-3</sup>	<b>7.7829×10<sup>-4</sup></b>	1.3320×10 <sup>-3</sup>
	Std. Dev.	1.0330×10 <sup>-3</sup>	1.1993×10 <sup>-3</sup>	1.0320×10 <sup>-3</sup>	8.1700×10 <sup>-4</sup>	1.0881×10 <sup>-3</sup>	9.9044×10 <sup>-4</sup>	7.9003×10 <sup>-4</sup>	1.2530×10 <sup>-3</sup>	1.2530×10 <sup>-3</sup>	9.4670×10 <sup>-4</sup>	7.6605×10 <sup>-4</sup>	1.2088×10 <sup>-3</sup>

Table 6: Effects of parameters of acceleration coefficients.

increased, the number of particles of each group will be decreased. So there should be a balance between the number of subspaces and total number of particles to enhance good explorations in subspaces.

### Iteration number of first stage

Optimal maximum allowed number of iterations of the first stage

is associated with 0020 maximum allowed number of iterations of the second stage.

$$FE = F\_IT \times (NM \times R + PS) + S\_IT \times PS \tag{16}$$

where *F\_IT* and *S\_IT* are maximum allowed number of iterations of the first and second stages respectively. *PS* is population size. *NM* is

		$\alpha_1$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$f_2$	Mean		$4.0078 \times 10^{-6}$	$1.3823 \times 10^{-6}$	$2.3504 \times 10^{-7}$	$2.2685 \times 10^{-7}$	<b><math>4.6731 \times 10^{-8}</math></b>	$8.9373 \times 10^{-8}$	$1.6833 \times 10^{-7}$	$5.5808 \times 10^{-7}$	$3.0005 \times 10^{-6}$
	Std. Dev.		$6.4849 \times 10^{-6}$	$3.2109 \times 10^{-6}$	$4.0190 \times 10^{-7}$	$2.1163 \times 10^{-7}$	$6.2346 \times 10^{-8}$	$1.6042 \times 10^{-7}$	$3.7672 \times 10^{-7}$	$1.6830 \times 10^{-6}$	$4.5625 \times 10^{-6}$
$f_6$	Mean		$1.5318 \times 10^{-3}$	$1.5440 \times 10^{-3}$	$1.0815 \times 10^{-3}$	$9.9886 \times 10^{-4}$	<b><math>7.7829 \times 10^{-4}</math></b>	$1.3403 \times 10^{-3}$	$1.2665 \times 10^{-3}$	$1.0240 \times 10^{-3}$	$1.8199 \times 10^{-3}$
	Std. Dev.		$1.0490 \times 10^{-3}$	$9.9581 \times 10^{-4}$	$7.1119 \times 10^{-4}$	$1.0532 \times 10^{-3}$	$7.6605 \times 10^{-4}$	$1.2204 \times 10^{-3}$	$1.4148 \times 10^{-3}$	$9.3906 \times 10^{-4}$	$1.1178 \times 10^{-3}$

Table 7: Effects of parameter  $\alpha_1$

		$\alpha_1$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$f_2$	Mean		$2.4489 \times 10^{-6}$	$4.7842 \times 10^{-6}$	$1.4936 \times 10^{-6}$	$1.2971 \times 10^{-6}$	$5.0263 \times 10^{-7}$	$5.6400 \times 10^{-7}$	$1.6270 \times 10^{-7}$	<b><math>4.6731 \times 10^{-8}</math></b>	$2.7127 \times 10^{-7}$
	Std. Dev.		$2.8687 \times 10^{-6}$	$7.5940 \times 10^{-6}$	$2.2980 \times 10^{-6}$	$2.6730 \times 10^{-6}$	$6.0714 \times 10^{-7}$	$1.1954 \times 10^{-6}$	$2.9078 \times 10^{-7}$	$6.2346 \times 10^{-8}$	$6.0852 \times 10^{-7}$
$f_6$	Mean		$1.0179 \times 10^{-3}$	$1.2031 \times 10^{-3}$	$1.0798 \times 10^{-3}$	$1.0281 \times 10^{-3}$	$1.2706 \times 10^{-3}$	$1.1096 \times 10^{-3}$	$1.4781 \times 10^{-3}$	<b><math>7.7829 \times 10^{-4}</math></b>	$1.0770 \times 10^{-3}$
	Std. Dev.		$6.0484 \times 10^{-4}$	$1.0798 \times 10^{-3}$	$6.7911 \times 10^{-4}$	$6.3793 \times 10^{-4}$	$9.6123 \times 10^{-4}$	$9.9833 \times 10^{-4}$	$1.3642 \times 10^{-3}$	$7.6605 \times 10^{-4}$	$9.4116 \times 10^{-4}$

Table 8: Effects of parameter  $\alpha_2$

		$\alpha_1$	1.5	2	2.5	3	3.5	4	4.5	5
$f_2$	Mean		$1.0379 \times 10^{-2}$	$1.2946 \times 10^{-4}$	$1.7700 \times 10^{-7}$	<b><math>4.6731 \times 10^{-8}</math></b>	$6.5411 \times 10^{-3}$	1.3856	1.9446	1.8021
	Std. Dev.		$1.2764 \times 10^{-2}$	$2.3032 \times 10^{-4}$	$3.3328 \times 10^{-7}$	$6.2346 \times 10^{-8}$	$6.2742 \times 10^{-3}$	$7.125 \times 10^{-1}$	1.1841	$7.843 \times 10^{-1}$
$f_6$	Mean		$2.1145 \times 10^{-3}$	$2.2674 \times 10^{-3}$	$1.5028 \times 10^{-3}$	<b><math>7.7829 \times 10^{-4}</math></b>	$1.3546 \times 10^{-3}$	$1.9185 \times 10^{-3}$	$2.9698 \times 10^{-3}$	$2.1282 \times 10^{-3}$
	Std. Dev.		$2.7532 \times 10^{-3}$	$1.9325 \times 10^{-3}$	$1.5653 \times 10^{-3}$	$7.6605 \times 10^{-4}$	$7.9550 \times 10^{-4}$	$1.9474 \times 10^{-3}$	$2.6959 \times 10^{-3}$	$2.6912 \times 10^{-3}$

Table 9: Effects of parameter  $\alpha_3$

Algorithm	Year	Reference
PSO-w	1998	[5]
PSO-cf	2002	[9]
PSO-cf-local	2002	[10]
UPSO	2004	[12]
FDR	2003	[14]
FIPS	2004	[13]
CPSO-H	2004	[15]
CLPSO	2006	[18]
CPSO-outer	2010	[20]

Table 10: PSO algorithms used in the comparison.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	$6.16 \times 10^{-140}$	$4.79 \times 10^{-6}$	$3.91 \times 10^{-6}$	$8.98 \times 10^{-232}$	$4.69 \times 10^{-225}$	$7.34 \times 10^{-55}$	$4.55 \times 10^{-29}$	$3.58 \times 10^{-78}$	0	0
Best	$3.98 \times 10^{-131}$	$9.96 \times 10^{-6}$	$9.94 \times 10^{-6}$	$2.72 \times 10^{-225}$	$1.80 \times 10^{-241}$	$7.28 \times 10^{-52}$	$1.02 \times 10^{-24}$	$6.77 \times 10^{-75}$	0	0
Worst	$3.52 \times 10^{-132}$	$8.36 \times 10^{-6}$	$8.37 \times 10^{-6}$	$1.97 \times 10^{-226}$	$5.99 \times 10^{-243}$	$1.33 \times 10^{-52}$	$1.17 \times 10^{-25}$	$3.98 \times 10^{-76}$	0	0
Mean	$9.46 \times 10^{-132}$	$1.65 \times 10^{-6}$	$1.56 \times 10^{-6}$	0	0	$1.81 \times 10^{-52}$	$2.42 \times 10^{-25}$	$1.24 \times 10^{-75}$	0	0
Std. Dev.										
30	$1.13 \times 10^{-33}$	$7.37 \times 10^{-6}$	$7.38 \times 10^{-6}$	$1.82 \times 10^{-89}$	$1.41 \times 10^{-107}$	$1.59 \times 10^{-12}$	$4.20 \times 10^{-09}$	$3.27 \times 10^{-23}$	0 (27)	0 (27)
Best	$3.45 \times 10^{-28}$	$9.93 \times 10^{-6}$	$9.94 \times 10^{-6}$	$5.00 \times 10^{-85}$	$1.42 \times 10^{-95}$	$2.16 \times 10^{-11}$	$1.84 \times 10^{-07}$	$1.57 \times 10^{-21}$	$2.81 \times 10^{-69}$	$1.51 \times 10^{-136}$
Worst	$1.72 \times 10^{-29}$	$9.26 \times 10^{-6}$	$9.03 \times 10^{-6}$	$3.22 \times 10^{-86}$	$5.84 \times 10^{-97}$	$4.46 \times 10^{-12}$	$3.98 \times 10^{-8}$	$3.70 \times 10^{-22}$	$9.48 \times 10^{-71}$	$7.57 \times 10^{-138}$
Mean	$6.53 \times 10^{-29}$	$6.63 \times 10^{-7}$	$6.65 \times 10^{-7}$	$9.09 \times 10^{-89}$	$2.63 \times 10^{-96}$	$3.66 \times 10^{-12}$	$3.94 \times 10^{-8}$	$4.31 \times 10^{-22}$	$5.13 \times 10^{-70}$	$3.38 \times 10^{-137}$
Std. Dev.										

Table 11: Comparison of results on Sphere test function among ten PSOs.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	$1.03 \times 10^{-2}$	$2.05 \times 10^{-4}$	$4.70 \times 10^{-3}$	$5.70 \times 10^{-4}$	$5.41 \times 10^{-5}$	$4.08 \times 10^{-1}$	$3.80 \times 10^{-4}$	$1.16 \times 10^{-2}$	0 (26)	<b><math>7.64 \times 10^{-13}</math></b>
Best	3.99	3.99	$4.05 \times 10^{-2}$	$4.37 \times 10^{-3}$	$2.55 \times 10^{-2}$	$6.34 \times 10^{-1}$	$7.24 \times 10^{-2}$	3.48	3.99	<b><math>5.79 \times 10^{-9}</math></b>
Worst	1.09	$2.68 \times 10^{-1}$	$1.25 \times 10^{-2}$	$1.87 \times 10^{-3}$	$1.1 \times 10^{-3}$	$5.16 \times 10^{-1}$	$3.60 \times 10^{-2}$	1.40	$2.66 \times 10^{-1}$	<b><math>9.26 \times 10^{-10}</math></b>
Mean	$8.18 \times 10^{-1}$	1.01	$7.41 \times 10^{-3}$	$9.53 \times 10^{-4}$	$4.62 \times 10^{-3}$	$5.64 \times 10^{-2}$	$2.30 \times 10^{-2}$	1.07	1.01	<b><math>1.75 \times 10^{-9}</math></b>
Std. Dev.										
30	1.05	9.07	$2.41 \times 10^{+1}$	$1.40 \times 10^{+1}$	5.36	$2.38 \times 10^{+1}$	$1.37 \times 10^{-2}$	$1.22 \times 10^{+1}$	$1.05 \times 10^{-4}$	<b><math>6.04 \times 10^{-10}</math></b>
Best	4.38	$1.46 \times 10^{+1}$	$2.94 \times 10^{+1}$	$1.66 \times 10^{+1}$	8.98	$2.48 \times 10^{+1}$	$7.26 \times 10^{+1}$	$2.22 \times 10^{+1}$	3.46	<b><math>2.18 \times 10^{-7}</math></b>
Worst	2.07	$1.14 \times 10^{+1}$	$2.26 \times 10^{+1}$	$1.55 \times 10^{+1}$	6.76	$2.44 \times 10^{+1}$	$1.72 \times 10^{+1}$	$1.83 \times 10^{+1}$	1.01	<b><math>4.67 \times 10^{-8}</math></b>
Mean	1.20	1.95	2.26	$8.96 \times 10^{-1}$	1.34	$3.10 \times 10^{-1}$	$2.16 \times 10^{+1}$	3.59	$6.65 \times 10^{-1}$	<b><math>6.23 \times 10^{-8}</math></b>
Std. Dev.										

Table 12: Comparison of results on Rosenbrock test function among ten PSOs.

referred as the number of members in each group,  $FE$  as the number of function evaluations, and  $R$  as the number of subspaces. By substituting  $NM = \frac{PS}{R}$  in Equation (16), we have:

$$F\_IT = \frac{FE / PS - S\_IT}{2} \tag{17}$$

By choosing the value of  $F\_IT$  carefully, a configuration can be

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	5.88×10 <sup>-136</sup>	5.14×10 <sup>-6</sup>	3.63×10 <sup>-6</sup>	1.36×10 <sup>-234</sup>	1.47×10 <sup>-267</sup>	3.13×10 <sup>-53</sup>	1.57×10 <sup>-23</sup>	1.22×10 <sup>-78</sup>	0	0
Best	2.12×10 <sup>-128</sup>	9.91×10 <sup>-6</sup>	9.74×10 <sup>-6</sup>	4.80×10 <sup>-225</sup>	1.71×10 <sup>-258</sup>	4.85×10 <sup>-52</sup>	4.56×10 <sup>-19</sup>	1.08×10 <sup>-74</sup>	0	0
Worst	8.28×10 <sup>-130</sup>	7.81×10 <sup>-6</sup>	8.39×10 <sup>-6</sup>	1.61×10 <sup>-226</sup>	8.44×10 <sup>-260</sup>	3.13×10 <sup>-53</sup>	2.87×10 <sup>-20</sup>	4.58×10 <sup>-76</sup>	0	0
Mean	3.86×10 <sup>-129</sup>	1.34×10 <sup>-6</sup>	1.51×10 <sup>-6</sup>	0	0	8.84×10 <sup>-53</sup>	8.40×10 <sup>-20</sup>	1.97×10 <sup>-75</sup>	0	0
Std. Dev.										
30	3.16×10 <sup>-30</sup>	7.33×10 <sup>-6</sup>	8.06×10 <sup>-6</sup>	3.27×10 <sup>-86</sup>	1.53×10 <sup>-103</sup>	1.71×10 <sup>-10</sup>	1.27×10 <sup>-6</sup>	4.65×10 <sup>-21</sup>	0 (12)	0 (29)
Best	3.58×10 <sup>-27</sup>	9.96×10 <sup>-6</sup>	9.98×10 <sup>-6</sup>	7.61×10 <sup>-84</sup>	<b>1.62×10<sup>-92</sup></b>	8.54×10 <sup>-10</sup>	1.02×10 <sup>-5</sup>	9.03×10 <sup>-20</sup>	3.67×10 <sup>-51</sup>	<b>1.56×10<sup>-49</sup></b>
Worst	8.00×10 <sup>-28</sup>	8.99×10 <sup>-6</sup>	9.20×10 <sup>-6</sup>	1.91×10 <sup>-84</sup>	<b>2.70×10<sup>-93</sup></b>	6.02×10 <sup>-10</sup>	5.41×10 <sup>-6</sup>	3.41×10 <sup>-20</sup>	8.10×10 <sup>-52</sup>	<b>5.20×10<sup>-51</sup></b>
Mean	1.42×10 <sup>-27</sup>	1.00×10 <sup>-6</sup>	8.75×10 <sup>-7</sup>	2.82×10 <sup>-84</sup>	<b>6.60×10<sup>-93</sup></b>	2.55×10 <sup>-10</sup>	3.71×10 <sup>-6</sup>	2.31×10 <sup>-20</sup>	1.24×10 <sup>-51</sup>	<b>2.84×10<sup>-50</sup></b>
Std. Dev.										

Table 13: Comparison of results on Quadric test function among ten PSOs.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	9.86×10 <sup>-3</sup>	2.46×10 <sup>-2</sup>	7.40×10 <sup>-3</sup>	9.86×10 <sup>-3</sup>	7.40×10 <sup>-3</sup>	2.77×10 <sup>-10</sup>	9.86×10 <sup>-3</sup>	0 (6)	0	0
Best	1.50×10 <sup>-1</sup>	2.26×10 <sup>-1</sup>	6.89×10 <sup>-2</sup>	3.20×10 <sup>-2</sup>	1.06×10 <sup>-1</sup>	2.37×10 <sup>-2</sup>	1.25×10 <sup>-1</sup>	9.95×10 <sup>-11</sup>	0	0
Worst	5.05×10 <sup>-2</sup>	8.37×10 <sup>-2</sup>	3.41×10 <sup>-2</sup>	2.02×10 <sup>-2</sup>	5.53×10 <sup>-2</sup>	7.98×10 <sup>-3</sup>	3.95×10 <sup>-2</sup>	1.53×10 <sup>-11</sup>	0	0
Mean	2.61×10 <sup>-2</sup>	4.47×10 <sup>-2</sup>	1.65×10 <sup>-2</sup>	1.03×10 <sup>-2</sup>	2.28×10 <sup>-2</sup>	7.86×10 <sup>-3</sup>	2.49×10 <sup>-2</sup>	3.23×10 <sup>-11</sup>	0	0
Std. Dev.										
30	1.72×10 <sup>-2</sup>	1.72×10 <sup>-2</sup>	9.02×10 <sup>-6</sup>	6.40×10 <sup>-4</sup>	7.40×10 <sup>-3</sup>	2.55×10 <sup>-6</sup>	9.86×10 <sup>-3</sup>	0 (3)	0 (8)	0
Best	1.35×10 <sup>-1</sup>	1.18×10 <sup>-1</sup>	8.61×10 <sup>-2</sup>	6.23×10 <sup>-2</sup>	1.38×10 <sup>-1</sup>	4.94×10 <sup>-2</sup>	1.16×10 <sup>-1</sup>	8.00×10 <sup>-10</sup>	1.17×10 <sup>-1</sup>	0
Worst	7.06×10 <sup>-2</sup>	6.24×10 <sup>-2</sup>	3.35×10 <sup>-2</sup>	2.49×10 <sup>-2</sup>	7.26×10 <sup>-2</sup>	1.66×10 <sup>-2</sup>	5.71×10 <sup>-2</sup>	6.65×10 <sup>-11</sup>	1.52×10 <sup>-2</sup>	0
Mean	3.13×10 <sup>-2</sup>	2.39×10 <sup>-2</sup>	2.32×10 <sup>-2</sup>	1.61×10 <sup>-2</sup>	3.36×10 <sup>-2</sup>	1.38×10 <sup>-2</sup>	2.64×10 <sup>-2</sup>	1.91×10 <sup>-10</sup>	2.22×10 <sup>-2</sup>	0
Std. Dev.										

Table 14: Comparison of results Griewank test function among ten PSOs.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	0	2.33×10 <sup>-6</sup>	5.48×10 <sup>-6</sup>	0 (14)	0	0 (5)	2.13×10 <sup>-14</sup>	0	0	0
Best	0	1.41×10 <sup>-1</sup>	9.97×10 <sup>-6</sup>	1.37	0	4.10×10 <sup>-6</sup>	6.23×10 <sup>-9</sup>	0	0	0
Worst	0	9.89×10 <sup>-3</sup>	8.52×10 <sup>-6</sup>	1.06×10 <sup>-1</sup>	0	2.34×10 <sup>-5</sup>	3.98×10 <sup>-10</sup>	0	0	0
Mean	0	3.46×10 <sup>-2</sup>	1.13×10 <sup>-6</sup>	2.79×10 <sup>-1</sup>	0	8.15×10 <sup>-5</sup>	1.17×10 <sup>-9</sup>	0	0	0
Std. Dev.										
30	7.11×10 <sup>-15</sup>	4.21×10 <sup>-6</sup>	6.26×10 <sup>-6</sup>	7.77	7.11×10 <sup>-15</sup>	1.25×10 <sup>-4</sup>	7.11×10 <sup>-15</sup>	0	1.32×10 <sup>-1</sup>	0
Best	2.25×10 <sup>-3</sup>	1.65×10 <sup>-2</sup>	9.98×10 <sup>-6</sup>	1.26×10 <sup>-1</sup>	1.50	1.96×10 <sup>-4</sup>	3.96×10 <sup>-9</sup>	0	3.00	0
Worst	9.53×10 <sup>-4</sup>	6.94×10 <sup>-4</sup>	8.80×10 <sup>-6</sup>	9.06	3.20×10 <sup>-1</sup>	1.62×10 <sup>-4</sup>	2.48×10 <sup>-10</sup>	0	1.91	0
Mean	1.02×10 <sup>-3</sup>	3.07×10 <sup>-3</sup>	8.06×10 <sup>-6</sup>	1.85	6.63×10 <sup>-1</sup>	3.00×10 <sup>-5</sup>	7.66×10 <sup>-10</sup>	0	1.38	0
Std. Dev.										

Table 15: Comparison of results on Weierstrass test function among ten PSOs.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	1.51×10 <sup>-4</sup>	3.20×10 <sup>-4</sup>	1.27×10 <sup>-4</sup>	4.19×10 <sup>-4</sup>	1.07×10 <sup>-4</sup>	1.36×10 <sup>-4</sup>	7.65×10 <sup>-4</sup>	3.37×10 <sup>-4</sup>	7.35×10 <sup>-6</sup>	1.67×10 <sup>-5</sup>
Best	2.02×10 <sup>-3</sup>	2.88×10 <sup>-3</sup>	8.27×10 <sup>-4</sup>	3.39×10 <sup>-3</sup>	1.95×10 <sup>-3</sup>	1.00×10 <sup>-3</sup>	6.02×10 <sup>-3</sup>	1.54×10 <sup>-3</sup>	7.36×10 <sup>-4</sup>	7.63×10 <sup>-4</sup>
Worst	7.77×10 <sup>-4</sup>	1.36×10 <sup>-3</sup>	3.66×10 <sup>-4</sup>	1.70×10 <sup>-3</sup>	4.75×10 <sup>-4</sup>	5.31×10 <sup>-4</sup>	2.97×10 <sup>-3</sup>	7.05×10 <sup>-4</sup>	1.99×10 <sup>-4</sup>	3.21×10 <sup>-4</sup>
Mean	4.10×10 <sup>-4</sup>	6.15×10 <sup>-4</sup>	1.68×10 <sup>-4</sup>	8.36×10 <sup>-4</sup>	3.79×10 <sup>-4</sup>	2.25×10 <sup>-4</sup>	1.40×10 <sup>-3</sup>	2.61×10 <sup>-4</sup>	1.87×10 <sup>-4</sup>	2.57×10 <sup>-4</sup>
Std. Dev.										
30	1.37×10 <sup>-2</sup>	2.09×10 <sup>-3</sup>	4.00×10 <sup>-3</sup>	3.36×10 <sup>-3</sup>	1.70×10 <sup>-3</sup>	4.02×10 <sup>-3</sup>	9.68×10 <sup>-3</sup>	2.51×10 <sup>-3</sup>	6.05×10 <sup>-5</sup>	2.86×10 <sup>-5</sup>
Best	4.41×10 <sup>-2</sup>	9.71×10 <sup>-3</sup>	6.89×10 <sup>-3</sup>	1.59×10 <sup>-2</sup>	5.37×10 <sup>-3</sup>	5.74×10 <sup>-3</sup>	2.26×10 <sup>-2</sup>	8.45×10 <sup>-3</sup>	4.45×10 <sup>-4</sup>	2.52×10 <sup>-3</sup>
Worst	2.68×10 <sup>-2</sup>	6.27×10 <sup>-3</sup>	5.09×10 <sup>-3</sup>	1.01×10 <sup>-2</sup>	3.28×10 <sup>-3</sup>	4.79×10 <sup>-3</sup>	1.44×10 <sup>-2</sup>	5.44×10 <sup>-3</sup>	1.54×10 <sup>-4</sup>	7.78×10 <sup>-4</sup>
Mean	1.18×10 <sup>-2</sup>	2.62×10 <sup>-3</sup>	1.17×10 <sup>-3</sup>	4.99×10 <sup>-3</sup>	1.07×10 <sup>-3</sup>	6.35×10 <sup>-4</sup>	4.84×10 <sup>-3</sup>	1.91×10 <sup>-3</sup>	1.10×10 <sup>-4</sup>	7.66×10 <sup>-4</sup>
Std. Dev.										

Table 16: Comparison of results on Quatric test function among ten PSOs.

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	0 (3)	6.12×10 <sup>-6</sup>	7.72×10 <sup>-8</sup>	0 (4)	0 (4)	4.65×10 <sup>-9</sup>	0	0	0	0
Best	3.00	9.00	3.00	7.00	2.00	2.24	0	0	0	0
Worst	3.67×10 <sup>-1</sup>	4.67	2.00×10 <sup>-1</sup>	3.19	4.67×10 <sup>-1</sup>	6.52×10 <sup>-1</sup>	0	0	0	0
Mean	7.18×10 <sup>-1</sup>	2.09	7.61×10 <sup>-1</sup>	1.67	6.81×10 <sup>-1</sup>	7.48×10 <sup>-1</sup>	0	0	0	0
Std. Dev.										
30	7.00	2.30×10 <sup>-1</sup>	4.00	6.50×10 <sup>-1</sup>	7.00	5.62×10 <sup>-1</sup>	3.33×10 <sup>-10</sup>	4.80×10 <sup>-13</sup>	2.50×10 <sup>-1</sup>	0
Best	3.90×10 <sup>-1</sup>	7.10×10 <sup>-1</sup>	1.90×10 <sup>-1</sup>	1.00	1.70×10 <sup>-1</sup>	7.61×10 <sup>-1</sup>	2.04×10 <sup>-8</sup>	7.12×10 <sup>-12</sup>	1.25×10 <sup>-2</sup>	0
Worst	1.70×10 <sup>-1</sup>	4.15×10 <sup>-1</sup>	1.87×10 <sup>-1</sup>	7.96×10 <sup>-1</sup>	1.08×10 <sup>-1</sup>	6.23×10 <sup>-1</sup>	5.39×10 <sup>-9</sup>	2.57×10 <sup>-12</sup>	7.52×10 <sup>-1</sup>	0
Mean	1.18×10 <sup>-1</sup>	1.92×10 <sup>-1</sup>	5.20	1.59×10 <sup>-1</sup>	4.40	7.21	7.49×10 <sup>-9</sup>	2.61×10 <sup>-12</sup>	3.52×10 <sup>-1</sup>	0
Std. Dev.										

Table 17a: Comparison of results on Non Continuous Rastrigin test function among ten PSOs.

found which maintains the balance between  $F_{IT}$  and  $F_{IT}$ . The ratio of  $F_{IT}$  to  $S_{IT}$  is defined as N:1 Table 5, which lists the results gained by different values of parameter N, makes it clear that which value should be set to achieve a better performance.

Table 5 represents that there is no much difference between high numbers of iterations of the first stage. Therefore,  $\epsilon$  is set to 10. On the other hand, because of incomplete search in subspaces, few numbers of iterations of the first stage led to poor performance.

Function	Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
Sphere	10	0	0	0	0	0	0	0	0	100	100
	30	0	0	0	0	0	0	0	0	90	90
Rosenbrock	10	0	0	0	0	0	0	0	0	86.7	0
	30	0	0	0	0	0	0	0	0	0	0
Quadric	10	0	0	0	0	0	0	0	0	100	100
	30	0	0	0	0	0	0	0	0	40	96.7
Griewank	10	0	0	0	0	0	0	0	20	100	100
	30	0	0	0	0	0	0	0	10	26.7	100
Weierstrass	10	100	0	0	46.7	100	16.7	0	100	100	100
	30	0	0	0	0	0	0	0	100	0	100
Quartic	10	0	0	0	0	0	0	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0
Non Continues Rastrigin	10	10	0	0	13.3	13.3	0	100	100	100	100
	30	0	0	0	0	0	0	0	0	0	100
Rastrigin	10	16.7	0	0	0	16.7	13.3	100	100	100	100
	30	0	0	0	0	0	0	0	0	10	100
Totally winning	10	1	0	0	0	1	0	2	3	6	6
	30	0	0	0	0	0	0	0	1	0	4

Table 17b: Successful rates of ten PSOs (problems with 10 and 30 dimensions).

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
10	0 (5)	$9.95 \times 10^{-1}$	$9.95 \times 10^{-1}$	$9.95 \times 10^{-1}$	0 (5)	0 (4)	0	0	0	0
Best	2.98	$1.29 \times 10^{-1}$	9.95	8.95	3.98	$9.95 \times 10^{-1}$	0	0	0	0
Worst	1.45	5.27	3.45	5.49	1.72	$1.01 \times 10^{-1}$	0	0	0	0
Mean	$8.70 \times 10^{-1}$	2.88	1.95	2.46	1.11	$3.03 \times 10^{-1}$	0	0	0	0
Std. Dev.										
30	$1.29 \times 10^{-1}$	$4.68 \times 10^{-1}$	$2.89 \times 10^{-1}$	$5.17 \times 10^{-1}$	$2.09 \times 10^{-1}$	$5.66 \times 10^{-1}$	$2.07 \times 10^{-8}$	$1.07 \times 10^{-14}$	0 (3)	0
Best	$3.68 \times 10^{-1}$	$9.35 \times 10^{-1}$	$6.17 \times 10^{-1}$	$9.95 \times 10^{-1}$	$3.38 \times 10^{-1}$	$8.51 \times 10^{-1}$	$7.12 \times 10^{-8}$	$7.66 \times 10^{-13}$	$1.08 \times 10^{-2}$	0
Worst	$2.89 \times 10^{-1}$	$2.59 \times 10^{-1}$	$4.03 \times 10^{-1}$	$6.63 \times 10^{-1}$	$2.77 \times 10^{-1}$	$7.30 \times 10^{-1}$	$1.46 \times 10^{-8}$	$2.12 \times 10^{-13}$	$5.00 \times 10^{-1}$	0
Mean	7.54	$1.70 \times 10^{-1}$	$1.20 \times 10^{-1}$	$1.45 \times 10^{-1}$	4.30	9.70	$1.92 \times 10^{-8}$	$2.18 \times 10^{-13}$	$2.49 \times 10^{-1}$	0
Std. Dev.										

Table 18a: Comparison of results on Rastrigin test function among ten PSOs.

Function	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
Beale	100	100	100	100	80	100	100	100	100	100
Bohachevsky1	100	100	100	100	100	100	100	100	100	100
Bohachevsky2	100	100	100	100	100	100	100	100	100	100
Bohachevsky3	100	100	100	100	100	100	100	100	100	100
Booth	100	100	100	100	100	100	100	100	100	100
Branin	100	100	100	100	100	100	100	100	100	100
Easom	100	100	100	100	100	100	100	100	100	100
Hartmann1	100	100	100	100	100	100	100	100	100	100
Hump	100	100	100	100	100	100	100	100	100	100
Matyas	0	100	100	100	100	87.5	100	0	100	100
Michalewics1	100	100	100	100	100	100	100	100	100	100
Michalewics2	90	25	63.33	100	33.33	90	100	100	88.33	100
Needle-in-a-haystack	6.67	-	60	33.3	40	26.67	20	0	0	100
Totally winning	10	11	11	12	10	10	12	10	11	13

Table 18b: Successful rates of ten PSOs (problems with fixed number of dimensions).

The convergence curves of Rosenbrock function is shown in Figures 4a and 4b). If each dimension of global minimum is situated in one region, it is clear that algorithm cannot reach the best solution in restricted areas. In order to find global minimum, search area should be extended to the entire search space in the second stage. If search space is divided into 15 regions, each dimension of global minimums of Hartmann1 and Michalewick 2 functions will be placed in different regions. The convergence curves of these functions are shown in Figures 5a and 5b). These figures clearly indicate that global minimum would not be found via searching in subspaces.

### Parameters of acceleration coefficients

To access the sensitivity of  $\delta_{1,2}$ , different values of  $\delta_{1,2}$  are tested. Results presented in Table 6 makes it clear that the interval [0.025, 0.8] is suitable. The magnitudes of  $a_1$ ,  $a_2$  and  $a_3$  are discussed in this section. From Tables 7-9, it is found that the best results are achieved when parameters  $a_1$ ,  $a_2$  and  $a_3$  are set equal to 0.5, 0.8, and 3, respectively.

### Numerical Experiments and Discussion

To evaluate the performance of the PPSO, nine famous PSO

Dimension	PSO-w	PSO-cf	PSO-cf-local	UPSO	FDR	FIPS	CPSO-H	CLPSO	CPSO-outer	PPSO
2	<b>-3600</b>	-	<b>-3600</b>	<b>-3600</b>	<b>-3600</b>	<b>-3600</b>	<b>-3600</b>	-3598.9	<b>-3600</b>	<b>-3600</b>
Best	-2729.1	-	-2734.9	-2732	-2748.8	-2748.8	-2748.8	-2748.8	<b>-3600</b>	<b>-3600</b>
Worst	-2798.1	-	-3269.8	-3105.3	-3089.3	-2975.8	-2919	-29681	<b>-3600</b>	<b>-3600</b>
Mean	218.0172	-	417.5431	412.3334	424.1387	382.8577	-346.3078	355.7935	<b>0</b>	<b>0</b>
Std. Dev.										

Table 19: Comparison of results on Needle-in-a-haystack test function among ten PSOs.

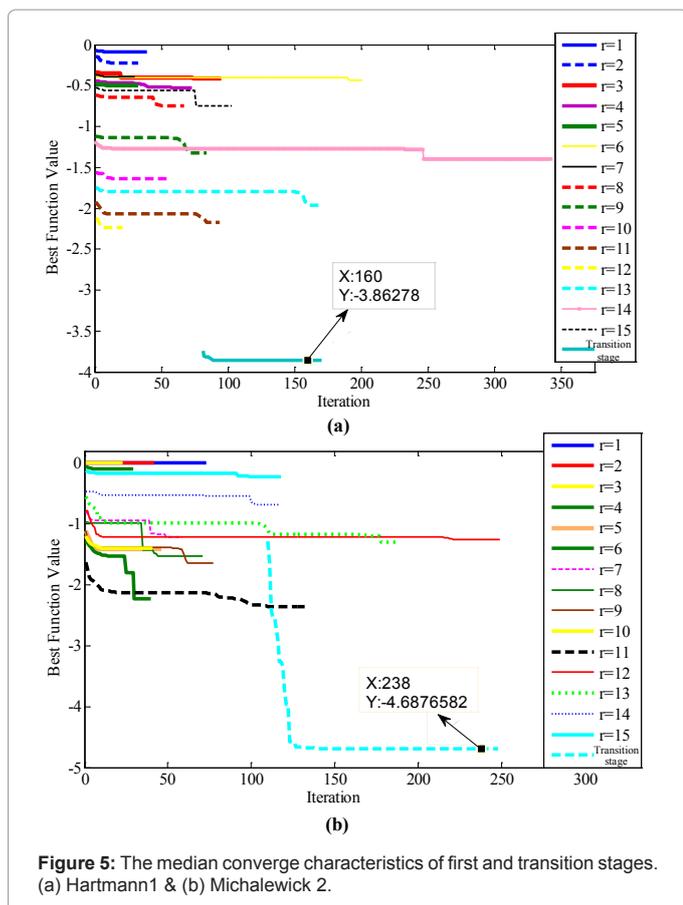


Figure 5: The median converge characteristics of first and transition stages. (a) Hartmann 1 & (b) Michalewicz 2.

algorithms are used, as detailed in Table 10. In all of the tests, the parameter configurations of these PSO variants are set according to their corresponding references, and the algorithm configuration of the PPSO is shown in Table 2. For the first eight functions which are commonly used in different articles and one function which is randomly chosen from the rest of Table 1, the worst, the best, mean and standard deviation of fitness values of the best particles found by nine algorithms over 30 runs, are listed in Tables 11-19 [20]. Two different dimensions are tested (10 and 30 dimensions) for the first eight functions. The boldface numbers indicate the best results. The times of reaching global optima out of all runs are presented in the brackets. The successful rates of ten PSOs are listed in Tables 17a, 17b, 18a and 18b which is defined as follows:

$$\text{The successful rate} = \frac{\text{the number of successful runs} \times 100}{\text{the number of all runs (30)}} \quad (18)$$

From Tables 11-19, it can be seen that the results of the proposed method are superior to other nine algorithms, for example, the results of 10-D Sphere and 30-D Griewank and Rastragin and 10-D and 30-D Rosenbrock. On 10-D and 30-D Quadric, the CPSO-outer method is slightly better than PPSO; however, PPSO can get better result of

the best solution. On 30-D Quadric, FDR performs best with high optimization robustness; however, PPSO could find global optimum with high successful rate. For 10-D Sphere and Quadric; however, UPSO and FDR perform robust and show good accuracy, PPSO and CPSO-outer can converge to global optimum with the same robustness. Only PPSO and CPSO-outer methods achieved the best results of sphere, 10-D Quadric, and Griewank. However, 30-D functions are more difficult than 10-D functions; results show that PPSO can perform well when dimensions are increased. On 30-D Griewank, Rastragin and Non Continuous Rastragin, only PPSO could achieve global optimum. Although, other algorithms find global optimums on 10-D functions, in 30-D cases their performances sharply plummet.

### Conclusion

In this paper, PPSO with simple techniques of partitioning search space into blocks and use of improved acceleration coefficients is able to escape from local minima and converge to the global minima robustly. The proposed algorithm contains three stages. Particles are divided into smaller groups and search small regions at the first stage. Then, particles search all of the search space as the transition stage. Finally, based on the results of these two stages, particles continue searching. Moreover, PPSO performs well even if dimensions of search space are increased; however, the performance of some methods considerably drops. In short, PPSO is generally simple, and yields good results in comparison with other variants.

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