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One Step, Three Hybrid Block Predictor-Corrector Method for the Solution of $y''' = f(x, y, y', y'')$

Adesanya A Olaide*, Fasansi M Kolawole and Odekunle M Remilekun

Department of Mathematics, Modibbo Adama University of Technology, Yola, Adamawa State, Nigeria

Abstract

We developed a one step-three hybrid point constant order predictor corrector method for the solution of general third order initial value problems. The method was developed using method of interpolation and collocation of power series approximate solution to generate a continuous linear multistep method which was evaluated at some selected grid point to give the discrete linear multistep method. The predictors are implemented in block method while the corrector gave the solution at an overlapping interval. The basic properties of both the corrector and the predictors were investigated and found to be zero stable, consistent and convergent. The region of absolute stability was also investigated. The efficiency of the derived method was tested on some numerical examples and found to compete favourably with the existing methods.

Keywords: Hybrid points; Collocation; Interpolation; Approximate solution; Block method; Zero stable; Consistent; Convergent

Introduction

This paper considers the numerical solution to the general third order initial value problems of the form

$$y''' = f(x, y, y', y''), y^{(k)}(x_0) = y_0^{(k)}, k = 0,1,2 \tag{1}$$

Where $f : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^m, y^{(k)}(x_0), y_0^{(k)} \in \mathbb{R}^m$, f is continuous and satisfies the uniqueness theorem given by Awoyemi et al. [1].

Direct method for the solution of higher order ordinary differential equations has been established in literature to be better than the method of reduction to system of first order ordinary differential equations [1-5]. Hybrid method has equally been established to have circumvented the Dahlquist barrier theorem, gave better error estimation than the K-step method especially when the problem are stiff and oscillatory [6].

Scholars have proposed different numerical schemes which include the predictor corrector method and block method. It has been reported that predictor corrector method are not efficient because the predictors are in reducing order of accuracy, moreover the cost of developing the separate predictors, human and computer time involved in the execution are too costly [7,8]. Block method was later proposed to cater for some of the setbacks of the predictor corrector method. It should be reminded that Milne in 1953 first developed block method to serve as a predictor to a predictor-corrector algorithm before it was later adopted as a full method [5,7-11] revisited the Milne approach and they concluded that though the method is expensive than the block method but gives better result than the block method. They tagged Milne's approach as constant order predictor-corrector method.

In this paper, we combine the unique properties of hybrid method and the constant order predictor-corrector method to develop a new numerical scheme for the solution of third order initial value problems of ordinary differential equations. This paper is organised as follows; section two discussed the algorithms in developing both the predictor and the corrector. Section three considers the analysis of the basic properties of both the predictor and the corrector. In section four, we test the efficiency of the derived method on some of numerical examples, and discussion of result and finally we concluded in section five.

Methods and Materials

Derivation of the block predictor

We consider the approximate solution of the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{2}$$

Where r and s are the number of interpolation and collocation points respectively. a_j's are the unknown coefficients to be determined. x is the polynomial basis function of degree j.

The third derivation of (2) gives

$$y'''(x) = \sum_{j=3}^{r+s-1} j(j-1)(j-2)a_j x^{j-3} \tag{3}$$

Substituting (3) into (1) gives

$$f(x, y, y', y'') = \sum_{j=3}^{r+s-1} j(j-1)(j-2)a_j x^{j-3} \tag{4}$$

Interpolating (2) at x_{n+r}, $r = 0, \frac{1}{4}, \frac{1}{2}$ and collocating (4) at x_{n+s}, $s = 0\left(\frac{1}{4}\right)1$ gives a non linear system of equation in the form

$$AX = U \tag{5}$$

where

$$A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7]^T$$

***Corresponding author:** Adesanya A Olaide, Department of Mathematics, Modibbo Adama University of Technology, Yola, Adamawa State, Nigeria, E-mail: torlar10@yahoo.com

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$$U=\left[y_n, y_{n+\frac{1}{4}}, y_{n+\frac{1}{2}}, f_n, f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1}\right]^T$$

$$X=\begin{bmatrix} 1 & x_n & x_n^{\ 2} & x_n^{\ 3} & x_n^{\ 4} & x_n^{\ 5} & x_n^{\ 6} & x_n^{\ 7} \\ 1 & x_{n+\frac{1}{4}} & x^2_{n+\frac{1}{4}} & x^3_{n+\frac{1}{4}} & x^4_{n+\frac{1}{4}} & x^5_{n+\frac{1}{4}} & x^6_{n+\frac{1}{4}} & x^7_{n+\frac{1}{4}} \\ 1 & x_{n+\frac{1}{2}} & x^2_{n+\frac{1}{2}} & x^3_{n+\frac{1}{2}} & x^4_{n+\frac{1}{2}} & x^5_{n+\frac{1}{2}} & x^6_{n+\frac{1}{2}} & x^7_{n+\frac{1}{2}} \\ 0 & 0 & 0 & 6 & 24x_n & 60x^2_{\ n} & 120x^3_{\ n} & 210x^4_{\ n} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{4}} & 60x^2_{n+\frac{1}{4}} & 120x^3_{n+\frac{1}{4}} & 210x^4_{n+\frac{1}{4}} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{2}} & 60x^2_{n+\frac{1}{2}} & 120x^3_{n+\frac{1}{2}} & 210x^4_{n+\frac{1}{2}} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{3}{4}} & 60x^2_{n+\frac{3}{4}} & 120x^2_{n+\frac{3}{4}} & 210x^4_{n+\frac{3}{4}} \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{\ n+1} & 120x^3_{\ n+1} & 210x^4_{\ n+1} \end{bmatrix}$$

Solving (5) for the unknown constants a_j's and substituting into (2) gives a continuous hybrid linear multistep method in the form.

$$y(x)=\alpha_0 y_0+\alpha_{\frac{1}{4}} y_{n+\frac{1}{4}}+\alpha_{\frac{1}{2}} y_{n+\frac{1}{2}}+h^3\left[\sum_{j=0}^{1}\beta_j f_{n+j}+\beta_v f_v\right], v=\frac{1}{4},\frac{1}{2},\frac{3}{4} \quad (6)$$

where

$$\alpha_0=8t^2-6t+1$$

$$\alpha_{\frac{1}{4}}=-16t^2+8t$$

$$\alpha_{\frac{1}{2}}=8t^2-2t$$

$$\beta_0=\frac{1}{80640}(4096t^7-17920t^6+31360t^5-28000t^4+13440t^3-3262t^2+307t)$$

$$\beta_{\frac{1}{4}}=\frac{1}{40320}(8192t^7-32256t^6+46592t^5-26880t^4-4242t^2-793t)$$

$$\beta_{\frac{1}{2}}=\frac{1}{13440}(4096t^7-14336t^6+17024t^5-6720t^4+434t^2-57t)$$

$$\beta_{\frac{3}{4}}=\frac{1}{40320}(8192t^7-25088t^6+25088t^5-8960t^4+574t^2-79t)$$

$$\beta_1=\frac{1}{80640}(4096t^7-10752t^6+9856t^5-3360t^4+210t^2-29t)$$

$$t=\frac{x-x_n}{h}$$

Solving (6) for the independent solution gives a continuous block method of the form

$$y^m_{n+k}=\sum_{m=0}^{2}\frac{(kh)^m}{m!}y_n^{(m)}+h^3\left[\sum_{j=0}^{1}\sigma_j f_{n+j}+\sigma_v f_{n+v}\right] \quad v=\frac{1}{4}(\frac{1}{4})\frac{3}{4}. \quad (7)$$

$$\sigma_0=\frac{1}{2520}(128t^7-560t^6+980t^5-875t^4+420t^3)$$

$$\sigma_{\frac{1}{4}}=\frac{-1}{315}(64t^7-252t^6+364t^5-210t^4)$$

$$\sigma_{\frac{1}{2}}=\frac{1}{210}(64t^7-224t^6+266t^5-105t^4)$$

$$\sigma_{\frac{3}{4}}=\frac{-1}{315}(64t^7-196t^6+196t^5-70t^4)$$

Evaluating (7) at $\frac{1}{4}(\frac{1}{4})1$ gives a discrete block method of the form

$$A^0 Y_m^{(i)}=\sum_{i}^{2}h^{(i)}e_i y_n^{(i)}+h^{(3-i)}\left[d_i f(y_n)+b_i f(y_m)\right] \quad (8)$$

where

$$Y_m^{(i)}=\left[y^{(i)}_{n+\frac{1}{4}}, y^{(i)}_{n+\frac{1}{2}}, y^{(i)}_{n+\frac{3}{4}}, y^{(i)}_{n+1}\right]^T$$

$$F(Y_m)=\left[f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1}\right]^T$$

$$y_n^{(i)}=\left[y^{(i)}_{n-1}, y^{(i)}_{n-2}, y^{(i)}_{n-3}, y^{(i)}_n\right]^T$$

$A^0=4X4$ Identity matrix.

When $i=0$

$$e_0=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}, e_1=\begin{bmatrix}0&0&0&\frac{1}{4}\\0&0&0&\frac{1}{2}\\0&0&0&\frac{3}{4}\\0&0&0&1\end{bmatrix}, e_2=\begin{bmatrix}0&0&0&\frac{1}{3}\\0&0&0&\frac{1}{8}\\0&0&0&\frac{9}{32}\\0&0&0&\frac{1}{2}\end{bmatrix}$$

$$d_0=\begin{bmatrix}0&0&0&\frac{113}{71680}\\0&0&0&\frac{331}{40320}\\0&0&0&\frac{1431}{71680}\\0&0&0&\frac{31}{840}\end{bmatrix}, b_0=\begin{bmatrix}\frac{107}{64512}&\frac{-103}{107520}&\frac{43}{107520}&\frac{-47}{645120}\\\frac{83}{5040}&\frac{-1}{168}&\frac{13}{5040}&\frac{-19}{40320}\\\frac{1863}{35840}&\frac{-243}{35840}&\frac{45}{7168}&\frac{-81}{71680}\\\frac{34}{315}&\frac{1}{210}&\frac{2}{105}&\frac{-1}{504}\end{bmatrix},$$

When i=1

$$e_1=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}, e_2=\begin{bmatrix}0&0&0&\frac{1}{4}\\0&0&0&\frac{1}{2}\\0&0&0&\frac{3}{4}\\0&0&0&1\end{bmatrix}, d_1=\begin{bmatrix}0&0&0&\frac{367}{23040}\\0&0&0&\frac{53}{1440}\\0&0&0&\frac{147}{2560}\\0&0&0&\frac{7}{90}\end{bmatrix},$$

$$b_1=\begin{bmatrix}\frac{3}{128}&\frac{-47}{3840}&\frac{29}{5760}&\frac{-7}{7680}\\\frac{1}{10}&\frac{-1}{48}&\frac{1}{90}&\frac{-1}{480}\\\frac{117}{640}&\frac{27}{1280}&\frac{3}{128}&\frac{-9}{2560}\\\frac{4}{15}&\frac{1}{15}&\frac{4}{45}&0\end{bmatrix}$$

When i=2

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{2880} \\ 0 & 0 & 0 & \frac{29}{360} \\ 0 & 0 & 0 & \frac{27}{320} \\ 0 & 0 & 0 & \frac{7}{90} \end{bmatrix}, \quad b_2 = \begin{bmatrix} \frac{323}{1440} & \frac{-11}{120} & \frac{53}{1440} & \frac{-19}{2880} \\ \frac{31}{90} & \frac{1}{15} & \frac{1}{90} & \frac{-1}{360} \\ \frac{51}{160} & \frac{9}{40} & \frac{21}{160} & \frac{-3}{320} \\ \frac{16}{45} & \frac{2}{15} & \frac{16}{45} & \frac{7}{90} \end{bmatrix}$$

Development of corrector

Interpolate (2) at $x_{n+r}, r = 0\left(\frac{1}{4}\right)\frac{3}{4}$ and collocate (4) at $x_{n+s}, s = 0(\frac{1}{4})1$ gives a non linear systems of equation of the form (4)

where

$$A = \left[a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\right]$$

$$U = \left[y_n, y_{n+\frac{1}{4}}, y_{n+\frac{1}{2}}, y_{n+\frac{1}{4}}, f_n, f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1}\right]$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+\frac{1}{4}} & x^2_{n+\frac{1}{4}} & x^3_{n+\frac{1}{4}} & x^4_{n+\frac{1}{4}} & x^5_{n+\frac{1}{4}} & x^6_{n+\frac{1}{4}} & x^7_{n+\frac{1}{4}} & x^8_{n+\frac{1}{4}} \\ 1 & x_{n+\frac{1}{2}} & x^2_{n+\frac{1}{2}} & x^3_{n+\frac{1}{2}} & x^4_{n+\frac{1}{2}} & x^5_{n+\frac{1}{2}} & x^6_{n+\frac{1}{2}} & x^7_{n+\frac{1}{2}} & x^8_{n+\frac{1}{2}} \\ 1 & x_{n+\frac{3}{4}} & x^2_{n+\frac{3}{4}} & x^3_{n+\frac{3}{4}} & x^4_{n+\frac{3}{4}} & x^5_{n+\frac{3}{4}} & x^6_{n+\frac{3}{4}} & x^7_{n+\frac{3}{4}} & x^8_{n+\frac{3}{4}} \\ 0 & 0 & 0 & 6 & 24x_n & 60x_n^2 & 120x_n^3 & 210x_n^4 & 336x_n^5 \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{4}} & 60x^2_{n+\frac{1}{4}} & 120x^3_{n+\frac{1}{4}} & 210x^4_{n+\frac{1}{4}} & 336x^5_{n+\frac{1}{4}} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{2}} & 60x^2_{n+\frac{1}{2}} & 120x^3_{n+\frac{1}{2}} & 210x^4_{n+\frac{1}{2}} & 336x^5_{n+\frac{1}{2}} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{3}{4}} & 60x^2_{n+\frac{3}{4}} & 120x^3_{n+\frac{3}{4}} & 210x^4_{n+\frac{3}{4}} & 336x^5_{n+\frac{3}{4}} \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{n+1} & 120x^3_{n+1} & 210x^4_{n+1} & 336x^5_{n+1} \end{bmatrix}$$

Solving (4) for the unknown constant a_j's and substituting into (2) gives a continuous hybrid linear multistep method in the form

$$y(x) = \alpha_0 y_0 + \alpha_{\frac{1}{4}} y_{n+\frac{1}{4}} + \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + \alpha_{\frac{3}{4}} y_{n+\frac{3}{4}} + h^3 \left[\sum_{j=0}^{1} \beta_j f_{n+j} + \beta_v f_v\right], v = \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \quad (9)$$

where

$$\alpha_0 = \frac{1}{21}(16384t^8 - 65536t^7 + 100352t^6 - 71680t^5 + 21504t^4 + 44t + 21)$$

$$\alpha_{\frac{1}{4}} = \frac{-1}{7}(16384t^8 - 65536t^7 + 100352t^6 - 71680t^5 + 21504t^4 - 1124t^2 + 114)$$

$$\alpha_{\frac{1}{2}} = \frac{1}{7}(16384t^8 - 65536t^7 + 100352t^6 - 71680t^5 + 21504t^4 - 1180t^2 + 156t)$$

$$\alpha_{\frac{3}{4}} = \frac{-1}{21}(16384t^8 - 65536t^7 + 100352t^6 - 71680t^5 + 21504t^4 - 1236t^2 + 170t)$$

$$\beta_0 = \frac{1}{161280}(8192t^8 - 24576t^7 + 14336t^6 + 266880t^5 - 45248t^4 + 26880t^3 - 7142t^2 + 679t)$$

$$\beta_{\frac{1}{4}} = \frac{1}{20160}(118784t^8 - 479232t^7 + 743680t^6 - 542976t^5 - 169344t^4 - 11082t^2 + 1629t)$$

$$\beta_{\frac{1}{2}} = \frac{1}{26880}(172032t^8 - 679936t^7 + 1025024t^6 - 718592t^5 + 212352t^4 - 12110t^2 + 1671t)$$

$$\beta_{\frac{3}{4}} = \frac{-1}{20160}(4096t^8 - 12288t^7 + 12544t^6 - 5376t^5 + 896t^4 - 22t^2 + 3t)$$

$$\beta_1 = \frac{1}{161280}(8192t^8 - 24576t^6 - 16128t^5 + 4032t^4 - 198t^2 + 27t)$$

Evaluating (9) at t=1 gives our corrector as

(10)

Analysis of the Basic Properties of the Method

Order of the method

Let the linear operator associated with the block method be defined as

$$\ell\{y(x):h\} = A^0 Y_m^{(i)} - \sum_{i}^{2} h^{(i)} e_i y_n^{(i)} - h^{(3-i)}\left[d_i f(y_n + b_i F(Y_m)\right] \quad (11)$$

Expanding (11) in Taylor series and comparing the coefficient of h gives

$$\ell\{y(x):h\} = c_0 y(x) + c_1 h y(x) + \ldots + c_p h^p y^p(x) + c_{p+2} h^{p+2} y^{p+2}(x) + \ldots$$

Definition: The linear operator Δ and associated block method are said to be of order p if $c_0 = c_1 = c_2 = \ldots = c_{p+1} = 0$, $c_{p+2} \neq 0$. c_{p+2} is called the error constant

Order of the predictors: Expanding (8) in Taylor series when gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(-\ )}{j!} y_n^{(j)} - y_n - \frac{1}{4}hy_n' - \frac{1}{32}h^2 y_n'' - \frac{113}{71650}h^2 y_n''' - \sum_{j=0}^{\infty}\frac{}{j!} y_n^{(j\ 3)}\left[\frac{107}{64512}\left(\frac{1}{4}\right)^j - \frac{103}{107520}\left(\frac{2}{5}\right)^j + \frac{43}{107520}\left(\frac{3}{4}\right)^j - \frac{47}{645120}(\ )\right] \\ \sum^{\infty}\frac{(-\ )}{!} y_n^{(\ )} - y_n - \frac{1}{2}hy_n' - \frac{1}{8}h^2 y_n'' - \frac{331}{40320}h^2 y_n''' - \sum^{\infty}\frac{}{!}{}^{(\ \ 3)}\left[\frac{83}{5040}\left(\frac{1}{4}\right)^j - \frac{1}{168}\left(\frac{2}{5}\right)^j + \frac{13}{5040}\left(\frac{3}{4}\right)^j - \frac{19}{40320}(\ )\right] \\ \sum_{j=0}^{\infty}\frac{(-\ )}{j!} y_n^{(j)} - y_n - \frac{3}{4}hy_n' - \frac{9}{32}h^2 y_n'' - \frac{1431}{71650}h^2 y_n''' - \sum_{j=0}^{\infty}\frac{}{j!} y_n^{(j\ 3)}\left[\frac{1863}{35840}\left(\frac{1}{4}\right)^j - \frac{243}{35840}\left(\frac{2}{5}\right)^j + \frac{45}{7168}\left(\frac{3}{4}\right) \quad \frac{81}{71680}(\ )\right] \\ \sum_{j=0}^{\infty}\frac{(h)^j}{j!} y_n^{(j)} - y_n - hy_n' - \frac{1}{2}h^2 y_n'' - \frac{31}{840}h^2 y_n''' - \sum_{j=0}^{\infty}\frac{h^j}{j!} y_n^{(j\ 3)}\left[\frac{34}{315}\left(\frac{1}{4}\right)^j - \frac{1}{210}\left(\frac{2}{5}\right)^j + \frac{2}{105}\left(\frac{3}{4}\right)^j - \frac{1}{504}(\ )\right] \end{bmatrix}$$

Comparing the coefficient of h, the order of the method is five with error constants

$$\left[\frac{139}{2642411520}, \frac{1}{2949120}, \frac{243}{293601280}, \frac{1}{645120}\right]^T$$

Expanding (8) in Taylor series when i=1 gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(\frac{1}{4}h)^{j+1}}{j!} y_n^{(j+1)} - hy_n' - \frac{1}{4}h^2 y_n'' - \frac{367}{23040}h^3 y_n''' - \sum_{j=0}^{\infty}\frac{h^{j+3}}{j!} y_n^{(j+3)}\left[\frac{3}{128}\left(\frac{1}{4}\right)^j - \frac{47}{3840}\left(\frac{1}{2}\right)^j + \frac{29}{5760}\left(\frac{3}{4}\right)^j - \frac{7}{7680}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(\frac{1}{2}h)^{j+1}}{j!} y_n^{(j+1)} - hy_n' - \frac{1}{2}h^2 y_n'' - \frac{53}{1440}h^3 y_n''' - \sum_{j=0}^{\infty}\frac{h^{j+3}}{j!} y_n^{(j+3)}\left[\frac{1}{10}\left(\frac{1}{4}\right)^j - \frac{1}{48}\left(\frac{1}{2}\right)^j + \frac{1}{48}\left(\frac{3}{4}\right)^j - \frac{1}{480}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(\frac{3}{4}h)^{j+1}}{j!} y_n^{(j+1)} - hy_n' - \frac{3}{4}h^2 y_n'' - \frac{147}{2560}h^3 y_n''' - \sum_{j=0}^{\infty}\frac{h^{j+3}}{j!} y_n^{(j+3)}\left[\frac{117}{640}\left(\frac{1}{4}\right)^j + \frac{27}{1280}\left(\frac{1}{2}\right)^j + \frac{3}{128}\left(\frac{3}{4}\right)^j - \frac{9}{2560}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(h)^{j+1}}{j!} y_n^{(j+1)} - hy_n' - h^2 y_n'' - \frac{7}{90}h^3 y_n''' - \sum_{j=0}^{\infty}\frac{h^{j+3}}{j!} y_n^{(j+3)}\left[\frac{4}{15}\left(\frac{1}{4}\right)^j - \frac{1}{15}\left(\frac{1}{2}\right)^j + \frac{4}{45}\left(\frac{3}{4}\right)^j\right] \end{bmatrix} = 0$$

Comparing the coefficients of h, the order of the method is five with the error constant

$$\left[\frac{107}{165150720}, \frac{1}{645120}, \frac{9}{3670016}, \frac{1}{322560}\right]^T$$

Expanding (8) in Taylor series when i=2 gives

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$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(\frac{1}{4}h)^{j+2}}{j!}y_n^{(j+2)}-h^2y_n''-\frac{251}{2880}h^3y_n'''-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{(j+3)}\left[\frac{323}{1440}\left(\frac{1}{4}\right)^j-\frac{11}{120}\left(\frac{1}{2}\right)^j+\frac{53}{1440}\left(\frac{3}{4}\right)^j-\frac{19}{2880}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(\frac{1}{2}h)^{j+2}}{j!}y_n^{(j+2)}-h^2y_n''-\frac{29}{360}h^3y_n'''-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{(j+3)}\left[\frac{31}{90}\left(\frac{1}{4}\right)^j+\frac{1}{15}\left(\frac{1}{2}\right)^j+\frac{1}{90}\left(\frac{3}{4}\right)^j-\frac{1}{360}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(\frac{3}{4}h)^{j+2}}{j!}y_n^{(j+2)}-h^2y_n''-\frac{27}{320}h^3y_n'''-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{(j+3)}\left[\frac{51}{160}\left(\frac{1}{4}\right)^j+\frac{2}{40}\left(\frac{1}{2}\right)^j+\frac{21}{160}\left(\frac{3}{4}\right)^j-\frac{3}{320}(1)^j\right] \\ \sum_{j=0}^{\infty}\frac{(h)^{j+2}}{j!}y_n^{(j+2)}-h^2y_n''-\frac{7}{90}h^3y_n'''-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{(j+3)}\left[\frac{16}{45}\left(\frac{1}{4}\right)^j+\frac{2}{15}\left(\frac{1}{2}\right)^j+\frac{16}{45}\left(\frac{3}{4}\right)^j+\frac{7}{90}(1)^j\right] \end{bmatrix}=0$$

comparing the coefficient of h, the order of the method is five with the error constant

$$\left[\frac{3}{655360},\frac{1}{368640},\frac{3}{655360},\frac{-1}{1935360}\right]^T$$

Order of the corrector: Expanding (10) in Taylor series gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y_n^{(j)}-y_n+2\sum_{j=0}^{\infty}\frac{(\frac{1}{4}h)^j}{j!}y_n^{(j)}-2\sum_{j=0}^{\infty}\frac{(\frac{3}{4}h)^j}{j!}y_n^{(j)}-\frac{1}{7680}h^3y_n'''- \\ \sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[\frac{7}{960}\left(\frac{1}{4}\right)^j+\frac{21}{1280}\left(\frac{1}{2}\right)^j+\frac{7}{960}\left(\frac{3}{4}\right)^j+\frac{1}{7680}(1)^j\right] \end{bmatrix}=0$$

Comparing the coefficient of h gives the order as Seven and the error constant as $\frac{1}{7927234560}$.

Consistency of the method

Consistency of the predictors: A method is said to be consistent, if it has order greater than one. From the above analysis, it is obvious that all our predictors are consistent.

Consistency of the corrector: A linear multistep method is said to be consistent, if it has order ρ ≥ 1 and if

$$\rho(1)=\rho'(1)=...=\rho^{(n-1)}(1)=0,\rho^{(n)}(1)=n!\sigma(1)$$

where $\rho(r)$ and $\sigma(r)$ are the first and second characteristics polynomial respectively, n is the order of the differential equation.

For our method

$$\rho(r)=r+2r^{\frac{1}{4}}-2r^{\frac{3}{4}}-1$$

$$\sigma(r)=\frac{1}{7680}(r+56r^{\frac{3}{4}}+126r^{\frac{2}{4}}+56r^{\frac{1}{4}}+1)$$

$$\rho(1)=3!\sigma(1)=\frac{3}{16}$$

Hence our method is consistent.

Zero stability

Zero stability of the predictor: A block method is said to be zero stable as $h\to 0$ the root r, j $1(1)k$ of the first characteristics polynomial $\rho(r)=0$ that is $\left|\left[\sum A^0\Re^{K-1}\right]\right|$ satisfying $|\Re|\le 1,$ for those root with $|\Re|=1,$ must be simple.

For our method

$$\rho(r)=\left|r\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}\right|=0$$

$r=0,0,0,1$

Hence our method is Zero stable

Zero stability of the corrector: A linear multistep method is said to be zero stable if the zero's of the first characteristics polynomial

$\rho(r)$ Satisfies $|r|\le 1$ and is simple for $|r|=1$ hence our corrector are zero stable.

Region of absolute stability

Definition: A method is said to be absolutely stable if for a given value of h, all the roots z_s of the characteristics polynomial $\pi(z,\hbar)=\rho(z)+\hbar\sigma(z)=0$, satisfies $|z_s|<s, s=1,2,...,n$, where $\hbar=\lambda h$, $\lambda=\frac{df}{dy}$

Substituting the test equation $y'=\lambda y$ into (8), solving for $\hbar=\lambda h$, and writing $r=e^{i\theta}$, gives the stability region of the corrector and the predictor as shown in Figures 1 and 2 respectively.

Numerical Experiments

Numerical examples

In this section, we test the efficiency of our method on some numerical examples

Problem 1: We consider special third order initial value problem

$$y'''=3\sin x \quad y(0)=1, y'(0)=0, y''(0)=-2 \quad 0\le x\le 1$$

Exact solution $y(x)=3\cos x+\frac{x^2}{2}-2$

Source: [12]

The results are shown in Table 1

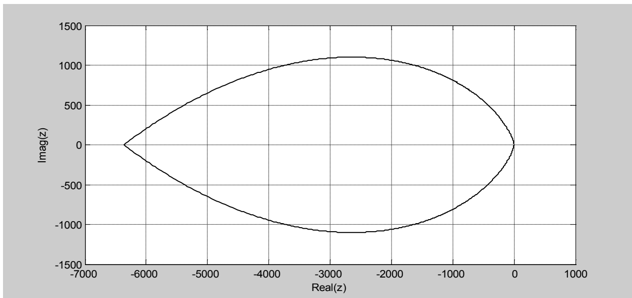

Figure 1: Showing region of absolute stability of the corrector.

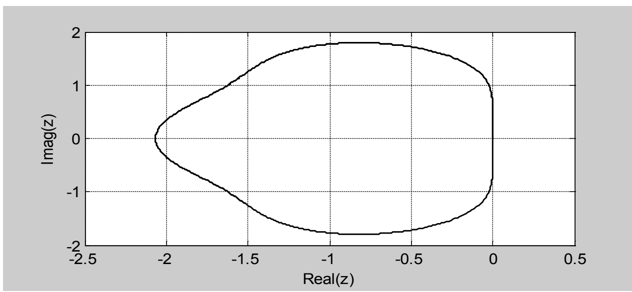

Figure 2: Showing region of absolute stability of the predictor.

Problem II: We consider a linear third order initial value problem

$$y'''(x)+y'(x)=0, y'(0)=1, y''(0)=2 \quad x\in[0,1]$$

Exact solution: $y(x)=2(1-\cos x)+\sin x$

Source: [12]

The result is shown in Table 2

Problem III: We consider a linear third order initial value problem

$$y'''-2y''-9y'-18y=-18x^2-18x+22$$

$$y(0)=-2, y'(0)=-8, y''(0)=-12$$

Exact solution: y(x)= $-2e^{3x}+e^{-2x}+x^2-1$

Source: [13]

The result is shown in Table 3

Note:

ERBM→ Error in block method

ECPCM→ Error in our new method

ERABM→ Error in Adesanya et al. [12]

EROBM→ Error in Olabode and Yusuf [14]

ERAPCM→ Error in Awoyemi and Idowu [13]

Discussion of Result

We have considered three numerical examples in this paper. Problem I considered a special problem which was solved by Adesanya et al. [12] and Oladode and Yusuf [14] using a k-step block method of order six. It was shown that our new method gave good approximation than the existing methods as shown in Table 1. Problem II is a linear problem, solved by Adesanya *et al.* [12] and Olabode and Yusuf [14]. Table 2 shows clearly that our method gave good result than the existing methods. Problem III is also a stiff problem solved by Awoyemi and Idowu [13], where a hybrid method of order seven implemented in predictor corrector method was proposed. Our new method gave good result as shown in Table 3.

x	ERBM	ECPCM	ERABM	EROBM
0.1	3.330669 (-16)	1.110223 (-16)	0.0000+00	9.992007 (-16)
0.2	3.330669 (-16)	2.220446 (-16)	9.99200 (-16)	7.660538 (-15)
0.3	3.330669 (-16)	9.992007 (-16)	1.55431 (-15)	2.287059 (-14)
0.4	1.110223 (-16)	3.330667 (-16)	3.10862 (-15)	5.906386 (-14)
0.5	1.110223 (-16)	2.220446 (-16)	4.66293 (-15)	1.153521 (-13)
0.6	4.440892 (-16)	5.551115 (-16)	6.88338 (-15)	1.982855 (-13)
0.7	5.551115 (-16)	6.661338 (-16)	9.10883 (-15)	3.127498 (-13)
0.8	5.551115 (-16)	3.330669 (-16)	1.14908 (-14)	4.635573 (-13)
0.9	7.216450 (-16)	1.110223 (-16)	1.42108 (-14)	6.542544 (-13)
1.0	1.054712 (-15)	3.608225 (-16)	1.74582 (-14)	8.885253 (-13)

Table 1: showing result of problem I, h=0.01.

x	ERBM	ECPCM	ERABM	EROBM
0.1	3.018419 (-14)	3.01907 (-14)	1.54055 (-09)	1.189947 (-11)
0.2	6.272760 (-15)	6.245005 (-15)	9.84550 (-09)	3.042207 (-09)
0.3	3.398948 (-13)	3.398948 (-13)	2.36528 (-08)	7.779556 (-08)
0.4	1.235012 (-12)	1.234790 (-12)	4.32732 (-08)	7.74955 (-07)
0.5	2.985279 (-12)	2.985168 (-12)	3.90181 (-08)	3.398961 (-06)
0.6	5.907719 (-12)	5..907497 (-12)	6.97008 (-08)	9.501398 (-06)
0.7	1.033573 (-11)	1.033551 (-11)	5.20329 (-08)	1.75558 (-06)
0.8	1.661249 (-11)	1.661205 (-11)	1.35224 (-07)	2.745889 (-05)
0.9	2.508349 (-11)	2.508327 (-11)	4.74034 (-07)	3.888082 (-05)
1.0	3.6088915 (-11)	3.608935 (-11)	1.06936 (-06)	5.137153 (-05)

Table 2: showing result of problem II, h=0.1.

x	ERBM	ECPCM	ERABM	EROBM
0.1	3.018419 (-14)	3.01907 (-14)	1.54055 (-09)	1.189947 (-11)
0.2	6.272760 (-15)	6.245005 (-15)	9.84550 (-09)	3.042207 (-09)
0.3	3.398948 (-13)	3.398948 (-13)	2.36528 (-08)	7.779556 (-08)
0.4	1.235012 (-12)	1.234790 (-12)	4.32732 (-08)	7.74955 (-07)
0.5	2.985279 (-12)	2.985168 (-12)	3.90181 (-08)	3.398961 (-06)
0.6	5.907719 (-12)	5..907497 (-12)	6.97008 (-08)	9.501398 (-06)
0.7	1.033573 (-11)	1.033551 (-11)	5.20329 (-08)	1.75558 (-06)
0.8	1.661249 (-11)	1.661205 (-11)	1.35224 (-07)	2.745889 (-05)
0.9	2.508349 (-11)	2.508327 (-11)	4.74034 (-07)	3.888082 (-05)
1.0	3.6088915 (-11)	3.608935 (-11)	1.06936 (-06)	5.137153 (-05)

Table 3: showing result of problem III, h=0.05.

Conclusion

We have developed a one step, three hybrid points method implemented in constant order block predictor corrector method in this paper. The results re-affirmed that hybrid method gave better approximation especially when the step is low than the k-step method as discussed in section one. We have equally established that the new method gave better approximation than the block method and the convectional predictor corrector method which is in reducing order of accuracy. It should be noted that the new method was able to exhaust all interpolation points hence we developed higher order methods without increasing the grid point as discussed by Adesanya et al. [10]. In our future correspondence, we shall investigate the implication when more interpolation points are considered using the same predictor.

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