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Research Article

On the Tuning Parameter for the Adaptive Bonferroni Procedure under Positive Dependence

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Abstract

Guo introduced an adaptive Bonferroni procedure and he proved that his adaptive Bonferroni procedure controls the familywise error rate under a conditional dependence model. However, how to choose the tuning parameter λ to control the familywise error rate in the procedure under positive dependence is not clear in his paper. In this paper, we suggest that $\lambda = \alpha$. Simulation studies are provided.

Keywords: Adaptive Bonferroni procedure; Familywise error rate; Holm procedure

Introduction

Simultaneously testing a family of m null hypotheses $H_i(i = 1,...,m)$ can arise from many circumstances such as comparing several treatments with a control. A main concern in multiple testing is the multiplicity problem, namely, that the probability of committing at least one Type I error sharply increases with the number of the hypotheses tested at a prespecified level. The probability of at least one false rejection is referred to as the familywise error rate (FWER). Several procedures have been proposed for controlling the familywise error rate, including proposals by Holm [1] and Hochberg [2]. When some null hypotheses are false, these procedures are often conservative by a factor given by the proportion of the true null hypotheses among all null hypotheses.By exploiting knowledge of this proportion Hochberg & Benjamini [3] introduced adaptive Bonferroni, Holm and Hochberg procedures for controlling the familywise error rate. These adaptive procedures estimate, the proportion and then use it to derive more powerful testing procedures. However, whether or not the adaptive procedure ultimately control the FWER has not yet been mathematically established. Recently, Guo [4] offered a partial answer to the open problem. He considered the aforementioned adaptive Bonferroni procedure, modified it slightly by replacing the estimate of the number of true null hypothesis by the estimate that Storey et al. [5] considered in the context of false discovery rate, and proved that, when the p-values are independent or exhibit certain types of dependence, his version of adaptive Bonferroni procedure controls the FWER. Guo [4] conducted a simulation study for positive correlated p-values with the tuning parameter $\lambda = 0.2$ to show his procedure controlling FWER. However, Finner and Gontscharuk [6] reported that the adaptive Bonferroni procedure does not control FWER when $\lambda = 0.5$ for positive highly-correlated p-values. Guo [4] did not explain why he chose the tuning parameter $\lambda = 0.2$. These motivated us to do a further simulation study for the adaptive Bonferroni procedure. In this paper we propose to use $\lambda = \alpha$. Then Guo's adaptive procedure controls FWER for positive correlated p-values. This observation has not been reported in the literature.

Guo's Adaptive Bonferroni Procedure

Given m null hypotheses $H_1,...,H_m$, consider testing if $H_i = 0$, true, or $H_i = 1$, false, simultaneously for $i = 1,...,m$, based on their respective p–values $P_1,..,P_m$. Assume that $H_i(i=1,...,m)$, are Bernoulli random variables with $\text{pr}(H=0)=\pi_0 = 1-\text{pr}(H=1)$, and the corresponding p–values P_i can be expressed as

$$P_i=(1-H_i)U_i+H_iG^{-1}(U_i), \qquad (2.1)$$

where $U_i(i=1,...,m)$ are independent and identically distributed uniform (0,1) random variables that are independent of all H_i;G_i is some cumulative distribution function on (0,1) and $G_i^{-1}(u)$ is the inverse of G_i. The P_is are conditionally independent given $H_i(i=1,...,m)$, but H_is may be dependent. If the His are independent, then (2.1) reduces to the conventional random effect model [7-9].

If V is the number of true null hypotheses rejected, then the familywise error rate is defined to be the probability of one or more false rejections, i.e. $\text{FWER} = \text{pr}\{V > 0\}$. Let $P_{1:m} \leq \cdots \leq P_{m:m}$ be the ordered values of $P_1,...,P_m$ and $H_{(1)},...,H_{(m)}$ be the corresponding null hypotheses. The Bonferroni procedure controls the familywise error rate at level $\pi_0\alpha$ for test statistics with arbitrary dependence by rejecting H_i whenever $P_i \leq \alpha/m$. Holm [1] proposed a step-down version of the Bonferroni procedure, which controls the familywise error rate at α. Let $\alpha_i = \alpha/(m-i+1)(i = 1,...,m)$ and r be the largest i such that $P_{1:m} \leq \alpha_1,...,P_{i:m} \leq \alpha_i$, then under the Holm procedure, we reject the hypotheses $H_{(1)},...,H_{(r)}$. If r is not defined, then no hypothesis is rejected.

Because the above Bonferroni-type procedures are conservative by the factor π_0, knowledge of π_0 can be useful for improving the performance of Bonferroni and Holm's procedures. Several estimators of π_0 have been introduced; see [5,10], among others. Guo [4] used Storey et al.[4]'s simple estimator:

$$\widehat{\pi}_0(\lambda) = \frac{m - R(\lambda) + 1}{(1-\lambda)m}, \qquad (2.2)$$

where $0 < \lambda < 1$ is a prespecified constant, $R(\lambda) = \sum_{i=1}^{m} I(P_i \leq \lambda)$ is the number of p-values less than or equal to λ, and $I(\ )$ is an indicator function. Storey et al.'s estimator is a simplified version of Schweder

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and Spjotvoll's estimator, which was used in the adaptive procedures of Hochberg & Benjamini [3] and Benjamini & Hochberg [11]. Based on $\hat{\pi}_0(\lambda)$, Guo's adaptive Bonferroni procedure is defined as follows

Definition 2.1 *The level α adaptive Bonferroni procedure.*

1. Given a fixed $\lambda \in (0,1)$, find $R(\lambda) = \sum_{i=1}^{m} I(P_i \le \lambda)$ and then calculate $\hat{\pi}_0$ based on (2.2).

2. Reject $H_{(1)},\dots,H_{(\hat{r})}$, where

$\hat{r} = \max\{i = 1,\dots,R(\lambda) : P_{i:m} \le \frac{\alpha}{\hat{\pi}_0 m}\}$.

If the maximum does not exist, reject no hypothesis.

Guo proved that the adaptive Bonferroni procedure above controls the familywise error rate at level α in the conditional dependence model.

Simulations of Familywise Error Rate and Power for Dependent p-values

It is recognized that the dependence issue is always very complicated in multiple testing. We simulate six different types of dependence structures to compare numerically the FWER control level and the power of Guo's adaptive Bonferroni procedure (denoted by a Bon in tables) with that of the Bonferroni (denoted by Bonintables) and Holm procedures for dependent p-values. We set α = 0.01, 0.05 and λ= α, 0.1, 0.2, 0.5 depending on the type of dependence structure of p-values. The simulated FWER and average power, the expected proportion of false nulls that are rejected, are based on 10000 replications. With 10000 repetition, the standard error of the estimated coverage near α is $\sqrt{\frac{\alpha(1-\alpha)}{10000}}$, and it never exceeds $\sqrt{\frac{0.5(1-0.5)}{10000}}$ =0.005.

Example 1 (positive equicorrelation)

In this example, our simulation study is similar to Guo's simulation study. The number of tests m = 200 was set for $H_{0i}: \mu_i$=0 against $H_{ai}: \mu_i \neq 0$ with the fraction of the true null hypotheses π_0 = 0.1,0.2,...,0.9. Let $Z_0,Z_1,\dots,Z_m$ be distributed independently and identically as N(0,1) and $Yi = \sqrt{\rho}Z_0 + \sqrt{1-\rho}Z_i + \mu_i$, where $\rho = 0.1,\dots,0.9$ and $\mu_i = 0, i = 1,\dots,m_0 = \pi_0 m$, μ_i = 6, $i = m_0$+1,...,m. We only report ρ = 0.5,0.9 here for space limits. When ρ = 0,Y_i are independent and the p-values are independent, a special case of the conditional dependence model. Guo studied λ=0.2 for α = 0.05 and 1000 replications. Note that $0.01+3\sqrt{\frac{0.01(1-0.01)}{10000}} = 0.013$. When α = 0.01 and λ = 0.2, Tables 1 and 2 indicate that the adaptive Bonferroni procedure does not control FWER for ρ = 0.5, 0.9; when ρ = 0.9, the adaptive Bonferroni procedure does control FWER even for λ = 0.1. Table 3 indicates that for λ = 0.2 the adaptive Bonferroni procedure controls FWER when α = 0.05 and ρ = 0.5, which matches the result in Guo [4]. Table 4 demonstrates that when π_0 = 0:1; 0:2, the adaptive Bonferroni procedure does not control FWER for λ = 0.2 when α = 0.5 and ρ = 0.9 (note that $0.05+3\sqrt{\frac{0.05(1-0.05)}{10000}} = 0.0565$). However, for $\lambda = \alpha$, the adaptive Bonferroni procedure does control FWER for all ρ =0.1,...,0.9 and its FWER level is more closer to α than the Bonferroni procedure and the Holm procedure. The powers of the adaptive Bonferroni procedure are larger than the powers of the Bonferroni procedure and the Holm procedure even for $\lambda = \alpha$ from Tables 1-4.

$\pi_0 = \frac{m_0}{m}$	Bon	Holm	adaptive Bon		
			λ = 0.2	λ = 0.1	$\lambda = \alpha$
0.1	.0006(.9743)	.0059(.9911)	.0167(.9960)	.0127(.9953)	.0081(.9940)
0.2	.0016(.9743)	.0053(.9879)	.0157(.9930)	.0121(.9919)	.0077(.9903)
0.3	.0024(.9742)	.0065(.9854)	.0160(.9905)	.0126(.9892)	.0079(.9875)
0.4	.0034(.9741)	.0067(.9832)	.0154(.9882)	.0128(.9868)	.0085(.9849)
0.5	.0042(.9741)	.0067(.9813)	.0152(.9861)	.0127(.9847)	.0085(.9827)
0.6	.0048(.9741)	.0068(.9796)	.0153(.9844)	.0126(.9828)	.0082(.9808)
0.7	.0054(.9740)	.0070(.9779)	.0155(.9826)	.0119(.9810)	.0084(.9789)
0.8	.0060(.9739)	.0072(.9764)	.0156(.9811)	.0121(.9793)	.0081(.9772)
0.9	.0069(.9740)	.0076(.9752)	.0158(.9796)	.0126(.9777)	.0089(.9757)

value in the parenthesis is the corresponding power.

Table 1: FWER and power when α =0.01, ρ =0.5.

$\pi_0 = \frac{m_0}{m}$	Bon	Holm	adaptive Bon		
			λ = 0.2	λ = 0.1	$\lambda = \alpha$
0.1	.0003(.9735)	.0014(.9853)	.0292(.9997)	.0172(.9996)	.0043(.9948)
0.2	.0004(.9734)	.0014(.9834)	.0327(.9997)	.0208(.9994)	.0047(.9910)
0.3	.0005(.9735)	.0013(.9818)	.0341(.9996)	.0197(.9991)	.0040(.9879)
0.4	.0007(.9734)	.0010(.9802)	.0338(.9996)	.0191(.9985)	.0040(.9850)
0.5	.0008(.9734)	.0011(.9789)	.0335(.9995)	.0191(.9979)	.0036(.9828)
0.6	.0009(.9734)	.0010(.9776)	.0333(.9994)	.0188(.9973)	.0032(.9806)
0.7	.0009(.9734)	.0010(.9765)	.0327(.9993)	.0182(.9965)	.0031(.9787)
0.8	.0009(.9733)	.0010(.9754)	.0323(.9992)	.0175(.9958)	.0032(.9770)
0.9	.0009(.9732)	.0010(.9743)	.0318(.9991)	.0176(.9950)	.0030(.9753)

value in the parenthesis is the corresponding power.

Table 2: FWER and power when α = 0.01, ρ = 0.9.

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$\pi_0 = \frac{m_0}{m}$	Bon	Holm	adaptive Bon	
			λ = 0.2	$\lambda = \alpha$
0.1	.0037(.9904)	.0345(.9979)	.0539(.9993)	.0434(.9988)
0.2	.0072(.9903)	.0284(.9967)	.0475(.9985)	.0368(.9978)
0.3	.0108(.9903)	.0279(.9956)	.0465(.9977)	.0343(.9968)
0.4	.0133(.9903)	.0263(.9947)	.0439(.9970)	.0315(.9959)
0.5	.0158(.9903)	.0259(.9938)	.0423(.9963)	.0320(.9951)
0.6	.0178(.9903)	.0257(.9931)	.0429(.9956)	.0312(.9942)
0.7	.0203(.9903)	.0253(.9923)	.0414(.9950)	.0308(.9935)
0.8	.0224(.9902)	.0258(.9916)	.0404(.9944)	.0313(.9928)
0.9	.0241(.9903)	.0262(.9910)	.0408(.9940)	.0311(.9922)

value in the parenthesis is the corresponding power.

Table 3: FWER and power when α = 0.05, ρ = 0.5.

$\pi_0 = \frac{m_0}{m}$	Bon	Holm	adaptive Bon	
			λ = 0.2	$\lambda = \alpha$
0.1	.0017(.9899)	.0107(.9956)	.0636(1.0000)	.0268(.9999)
0.2	.0022(.9899)	.0074(.9948)	.0585(1.0000)	.0228(.9999)
0.3	.0023(.9899)	.0069(.9939)	.0549(.1.0000)	.0211(.9998)
0.4	.0025(.9899)	.0058(.9931)	.0528(1.0000)	.0196(.9997)
0.5	.0027(.9899)	.0054(.9925)	.0504(1.0000)	.0192(.9996)
0.6	.0028(.9899)	.0044(.9919)	.0484(1.0000)	.0182(.9995)
0.7	.0029(.9899)	.0042(.9914)	.0481(1.0000)	.0176(.9993)
0.8	.0033(.9900)	.0042(.9910)	.0471(1.0000)	.0168(.9992)
0.9	.0036(.9900)	.0043(.9905)	.0471(1.0000)	.0165(.9990)

value in the parenthesis is the corresponding power.

Table 4: FWER and power when α = 0.05, ρ = 0.9.

Example 2 (positive block dependence)

This example largely follows the set-up of Example 3 in Finner and Gontscharuk [6]. Let

$$\mu = 1_{25} \otimes \begin{pmatrix} a \\ a \\ 0 \\ 0 \end{pmatrix}$$

and $\Sigma = \sigma^2 J_{25} \otimes \{(1-\rho) J_4 + \rho 1_{4\times4}\}$, $\rho \in (0,1)$, where 1_n denotes a column vector of 1s of length n,$1_{n\times n}$ denotes an $n\times n$ matrix of 1s and J_n is the identity matrix. Let $X_j \sim N_{100}(\mu,\Sigma)$, j = 1,...,n, be independent and identically distributed. We use σ =1 and a =1.0,1.5,2.0 in the simulation. We consider the multiple test problem $H_{ai}: \mu_i = 0$ versus $H_{ai}: \mu_i \neq 0$, i = 1,..,100. We use the test statistic $T_i = \frac{\overline{X_i} - \sqrt{n}}{s_i}$ as Finner and Gontscharuk [6], where $\widehat{X}_i = \frac{\sum_{j=1}^n X_{ij}}{n}$ and $S_i^2 = \frac{\sum_{j=1}^n (X_{ij} - \overline{X_i})^2}{(n-1)}$. Therefore, the test statistics have a $t_{(n-1)}$ distribution. The p-values corresponding to T_i is $P_i = 2F_{t_{n-1}}\left(-\left|T_i\right|\right)$, where F_{tv} denotes the cumulative distribution function of a central t-distribution with v degrees of freedom. For illustration, we simulate this model for n = 10, a = 2.0; n = 16, a = 1.5; n = 25, a = 1.0, and only three values of ρ = 0.1,0.5,0.9. Table 5 indicates that the adaptive Bonferroni procedure controls FWER well for each ρ and λ. Moreover, for the adaptive Bonferroni procedure, FWER decreases slightly but power does not change much when ρ increases and its FWER and power seem to be nearly independent of λ.

Example 3 (pairwise comparisons)

This example is modified from Example 2 in Finner and Gontscharuk [6]. Let X_{ij}, i = 1,..,k,j=1,..,n, be independent normally distributed random variables with unknown mean μ_i and unknown variance σ^2. We consider the pairwsie comparisons problem

$$H_{oij}: \mu_i = \mu_j \text{ versus } H_{aij}: \mu_i \neq \mu_j, 1 \le i < j \le k$$

for various scenarios of means. The test statistics are given by $T_{ij} = \sqrt{\frac{n}{2}} \frac{\overline{X_i} - \overline{X_j}}{s}$ with $\overline{X_i} = \frac{\sum_{j=1}^n X_{ij}}{n}$ and $S^2 = \frac{\sum_{i=1}^k \sum_{j=1}^n (X_{ij} - \overline{X_i})^2}{k(n-1)}$. Therefore, the test statistics have a $t_k(n-1)$ distribution. The p-values corresponding to T_{ij} is $P_{ij} = 2F_{t_{k(n-1)}}\left(-\left|T_{ij}\right|\right)$. Setting t_0=0, a scenario $\{\mu_{t_1}^1, .., \mu_{t_r}^T\}$ means that $\mu_{t_{i-1}+1} = \cdots = \mu t_{i-1} + t_i = \mu^i$ for i = 1,..,r. So the case $\mu_1 = \mu_2 = \mu_3 = 0$, $\mu_4 = \mu_5 = \mu_6 = \mu_7 = 2$ and $\mu_8 = \mu_9 = \mu_{10} = 4$ corresponds to $\{0_3, 2_4, 4_3\}$ with k =10 and $m = \frac{10\times9}{2} = 45$. Table 6 shows that the adaptive Bonferroni procedure apparently controls FWER for all λ and it is more powerful than the Bonferroni procedure and the Holm procedure.

ρ	Procedure	n = 10,	a = 2.0	n = 16,	a = 1.5	n = 25,	a = 1.0
		FWER	power	FWER	power	FWER	power
0.1	Bon	.0257	.7740	.0269	.9015	.0272	.8115
	aBon (λ = 0.5)	.0500	.8637	.0497	.9433	.0505	.8699
	aBon (λ = 0.2)	.0504	.8648	.0494	.9438	.0499	.8708
	aBon (λ = 0.05)	.0498	.8651	.0503	.9440	.0497	.8708
	Holm	.0442	.8491	.0477	.9411	.0451	.8608
0.5	Bon	.0224	.7741	.0261	.9012	.0267	.8119
	aBon (λ = 0.5)	.0492	.8644	.0472	.9431	.0499	.8697
	aBon (λ = 0.2)	.0476	.8659	.0473	.9435	.0499	.8706
	aBon (λ = 0.05)	.0481	.8662	.0471	.9436	.0488	.8706
	Holm	.0418	.8497	.0444	.9408	.0430	.8608
0.9	Bon	.0216	.7745	.0223	.9012	.0208	.8123
	aBon (λ = 0.5)	.0461	.8646	.0422	.9425	.0421	.8708
	aBon (λ = 0.2)	.0467	.8659	.0416	.9437	.0424	.8711
	aBon (λ = 0.05)	.0458	.8663	.0411	.9437	.0418	.8710
	Holm	.0394	.8496	.0388	.9406	.0360	.8610

Table 5: Simulation study for the positive block dependence model in example2 for α = 0.05.

μ - scenario	Procedure	Results for n = 4		Results for n = 6		Results for n = 8	
		FWER	power	FWER	power	FWER	power
$\{0_3, 2_4, 4_3\}$,	Bon	.0107	.4442	.0120	.6420	.0118	.7967
m = 45, m_0 = 12	aBon (λ = 0.5)	.0450	.5471	.0505	.7488	.0530	.8788
	aBon (λ = 0.2)	.0381	.5428	.0445	.7505	.0504	.8804
	aBon (λ = 0.05)	.0298	.5261	.0369	.7442	.0434	.8792
	Holm	.0211	.4825	.0263	.7070	.0328	.8628
$\{0_4, 1_1, 2_4, 3_1\}$,	Bon	.0125	.2250	.0131	.3963	.0145	.5250
m = 45, m_0 = 12	aBon (λ = 0.5)	.0350	.2949	.0382	.4776	.0444	.6016
	aBon (λ = 0.2)	.0313	.2859	.0329	.4706	.0379	.5966
	aBon (λ = 0.05)	.0239	.2702	.0263	.4565	.0306	.5853
	Holm	.0173	.2409	.0192	.4261	.0219	.5609
$\{0_4, 1_4, 2_4, 3_4, 4_4\}$,	Bon	.0071	.2760	.0077	.4078	.0071	.4954
m = 190, m_0 = 30	aBon (λ = 0.5)	.0265	.3366	.0300	.4748	.0327	.5643
	aBon (λ = 0.2)	.0221	3275	.0259	.4660	.0287	.5564
	aBon(λ = 0.05)	.0161	.3143	.0198	.4532	.0214	.5439
	Holm	.0108	.2899	.0113	.4288	.0131	.5206

Table 6: Simulation study for the pairwise comparisons problem in example 3 for α=0.05.

$\pi_0 = \frac{m_0}{m}$	Procedure	Results for	ρ = 0.1	Results for	ρ = 0.5	Results for	ρ = 0.9
		FWER	power	FWER	power	FWER	power
0.2	Bon	.0118	.9873	.0106	.9874	.0097	.9873
	aBon (λ = 0.5)	.0492	.9960	.0485	.9960	.0411	.9961
	aBon (λ = 0.2)	.0486	.9960	.0487	.9960	.0412	.9961
	aBon (λ = 0.05)	.0489	.9960	.0482	.9960	.0406	.9961
	Holm	.0483	.9960	.0477	.9960	.0380	.9959
0.8	Bon	.0383	.9872	.0366	.9873	.0224	.9874
	aBon (λ = 0.5)	.0480	.9890	.0466	.9892	.0288	.9894
	aBon (λ = 0.2)	.0483	.9890	.0470	.9892	.0279	.9894
	aBon (λ = 0.05)	.0486	.9890	.0460	.9891	.0277	.9893
	Holm	.0480	.9890	.0455	.9891	.0270	.9891

Table 7: Simulation study for the block dependence model in example 4 for α=0.05.

Procedure	Results for −0.1		Results for −0.5		Results for −0.9	
	FWER	power	FWER	power	FWER	power
Bon	.0381	.9845	.0369	.9846	.0322	.9846
aBon (λ = 0.5)	.0485	.9866	.0454	.9866	.0383	.9866
aBon (λ = 0.2)	.0481	.9866	.0454	.9866	.0384	.9866
aBon (λ = 0.05)	.0480	.9866	.0457	.9866	.0385	.9866
Holm	.0482	.9865	.0456	.9866	.0387	.9866

Table 8: Simulation study for the negative block dependence model in example 5 for α = 0.05.

Example 4 (Storey et al. [5]'s block dependence)

This example follows Storey, Taylor and Siegmund [5]'s dependence example. The null statistics have $N(0,1)$ marginal distribution with m_0 = 60,240 and the alternative distributions have marginal distribution $N(6,1)$ with $m_1 = m - m_0$ = 240,60 respectively. The statistics have correlation $\pm\rho$ = 0.1,..,0.9 in group size of 10 as the following.

$$\sum_{ij} = \begin{cases} 1 & i = j, \\ \rho & i < j \le 5, \\ -\rho & i \le 5, j \le 5. \end{cases}$$

See Storey et al. [5] for details. FWER is well controlled for all the procedures and λ choices. For brevity, Table 7 lists the results for ρ = 0.1,0.5,0.9 only.

Example 5 (negative block dependence)

The set-up is similar to Example 4 above but the statistic correlation is negative. The null statistics have N(0,1) marginal distribution with m_0 = 320 and the alternative distributions have marginal distribution N(0,6) with $m_1 = m - m_0$ = 80. The statistics have correlation $-\rho$ in group size of 2 as the following.

$$\sum_{ij} = \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}.$$

FWER is well controlled for all the procedures and λ choices. For brevity, Table 8 lists the results for correlation -0.1,-0.5,-0.9 only.

Example 6 (multivariate equicorrelated *t*-distribution)

We consider the situation that was described in Example 1 and Example 5 in Finner and Gontscharuk [6]. Let Let $X_i \sim N(\mu_i,\sigma^2)$, i = 1,...,m be independent normal random variables and let $\frac{vs^2}{\sigma^2} \in X_v^2$ be independent of the X_is. The multiple testing problem is H_{0i} : μ_i = 0 versus H_{ai} : $\mu_i > 0, i$ = 1,...,m with test statistic $T_i = \frac{X_i}{s}$. Then T =

Procedure	Results for m_0 =80		Results for m_0 =160		Results for m_0 =190	
	FWER	power	FWER	power	FWER	Power
Bon	.0158	.8981	.0279	.8984	.0320	.8984
aBon (λ = 0.5)	.0375	.9524	.0341	.9137	.0331	.9016
aBon (λ = 0.2)	.0413	.9502	.0352	.9103	.0367	.8980
aBon (λ = 0.05)	.0407	.9515	.0346	.9119	.0349	.8998
Holm	.0375	.9423	.0332	.9104	.0336	.9011

Table 9: Simulation study for the multivariate equicorrelated t-distribution model in example6 for α = 0.05.

$(T_1,...,T_m)$ has a multivariate equicorrelated t-distribution. The p-values are $P_i = 1 - F_{t_v}\left(\frac{x_i}{s}\right)$. In the simulation, we have m = 200, v = 15, σ^2 = 1, μ_i = 0, i = 1,..., m_0, and μ_i = 6, $i = m_0$ +1,..., m, m_0 = 80, 160, 190. Table 9 demonstrates that FWER is obviously controlled for all values of m_0 and λ that are considered in the simulation. Moreover, the differences between the three procedures in FWER and power are virtually negligible and independent of the choice of λ when $\frac{m_0}{m}$ is large.

Discussion

Guo [4] mathematically proved that the adaptive Bonferroni procedure controls FWER under a conditional dependence model. A critical point for the adaptive Bonfer-roni procedure is the choice of the tuning parameter λ. Finner and Gontscharuk [6] suggested that λ around ½ may be a good compromise and they further commented that \Anyhow, it seems not easy to give precise guidelines here" (page 1046 of their paper). It is a challenging problem for the proof of FWER control for the adaptive Bonferroni procedure under dependent p-values. In this paper, we suggest that $\lambda = \alpha$ as a guideline, it seems that the adaptive Bonferroni procedure controls FWER for the positve equicorrelated normal distributions in our simulations. This simple choice for the tuning parameter λ will help applications of Guo's adapative Bonferroni procedure.

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