Journal of
Applied & Computational Mathematics
Duan, J Applied Computat Mathemat 2012, 1:3
DOI: 10.4172/2168-9679.1000e109
Editorial Open Access

On the Power Series Expansion of a Nonlinear Function of a Power Series

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Nonlinear Differential Equation

Introduction

The power series method (PSM) is classical in resolution of differential equations. For a nonlinear differential equation, such as

du/dt=f(u), u(t0)=C, (1)

where f(u) is an analytical nonlinearity, the PSM requires to expand the nonlinear function of a power series into a power series. Adomian and Rach [1,2] gave the formula we require

$$f\left(\sum_{n=0}^{\infty} a_n (t-t_0)^n\right) = \sum_{n=0}^{\infty} A_n (t-t_0)^n, \quad (2)$$

where A_n, depending on a0, a1, . . . , an, are called the Adomian polynomials, which were defined as [3]

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left(\sum_{n=0}^{\infty} a_k \lambda^k\right)\Big|_{\lambda=0} \quad (3)$$

We note that the Adomian polynomials were initially used in the Adomian decomposition method [3,4], and they are expressed in the components uj of the Adomian decomposition series. The PSM combined with the Adomian polynomials is called the modified decomposition method [5]. For practical calculation and programming, the Adomian polynomials can be expressed as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left(\sum_{k=0}^{M} a_k \lambda^k\right)\Big|_{\lambda=0}, 0 \le n \le M. \quad (4)$$

The first five Adomian polynomials are

$A_0=f(a_0)$,

$A_1=f'(a_0)a_1$,

$A_2=f'(a_0)a_2 + f''(a_0)\frac{a_1^2}{2!}$,

$A_3=f'(a_0)a_3 + f''(a_0)a_1a_2 + f'''(a_0)\frac{a_1^3}{3!}$,

$A_4=f'(a_0)a_4 + f''(a_0)\left(\frac{a_2^2}{2!}+a_1a_3\right)+f'''(a_0)\frac{a_1^2a_2}{2!}+f^{(4)}(a_0)\frac{a_1^4}{4!}$.

We observe that $A_n = \sum_{k=1}^{n} f^{(k)}(a_0)C_n^k, n \ge 1$, (5)

where C_n^k are the sums of all possible products of k components from a1, a2, • • • , a_{n-k+1}, whose subscripts sum to n, divided by the factorial of the number of repeated subscripts [6], which is called Rach's Rule [7,8].

Other different algorithms for the Adomian polynomials have been developed by Rach [9], Wazwaz [10], Abdelwahid [11], and several others [12-17].

New Fast Algorithms and Applications

We review the new fast algorithms for the Adomian polynomials. In [15-17] recursion relations for C_n^k in (5) have been presented.

Algorithm 1 [15].

For $n \ge 1, C_n^1 = a_n$,

for $n \ge 2$ and $\left[\frac{n}{2}\right] < k \le n, C_n^k = C_{n-1}^{k-1}\Big|_{p1\to p1+1,}$

for $n \ge 4 and 2 \le k \le \left[\frac{n}{2}\right], C_n^k = C_{n-1}^{k-1}\Big|_{p1\to p1+1,} + C_{n-k}^k\Big|_{a_j \to a_{j+1}}$,

where p1 → p1 + 1 stands for replacing $\frac{a_1^{p1}}{p1!} by \frac{a_1^{p1+1}}{(p1+1)!}, p1 \ge 0.$

Algorithm 2 [17].

For $n \ge 1, C_n^1 = a_n$,

for $2 \le k \le n, C_n^k = \frac{1}{n}\sum_{j=0}^{n-k}(j+1)a_{j+1}C_{n-1-j}^{k-1}$.

In the two algorithms the recursion operation does not involve the differentiation, but only requires the operations of addition and multiplication, which greatly facilitates calculation and programming. In most practical cases, the exact solution of a nonlinear differential equation is unknown. We obtain the m-term approximation for the solution

$$u(t), \phi_m(t) = \sum_{k=0}^{m-1} a_k (t-t_0)^k. \quad (6)$$

With the fast algorithms for the Adomian polynomials we can efficiently calculate the $\phi_m(t)$ for large m. Further we can use the acceleration convergence techniques, such as the Pad´e approximants and the iterated Shanks transforms, to extend the effective region of convergence and increase the accuracy for the approximate solution.

Another important application is to derive the high-order numeric scheme for nonlinear differential equations more efficiently. For each subinterval [ti, t_{i+1}] we apply the m-term approximation $\phi_m(t)$ (t; t_i, C_i), where i=0, 1, . . . , and C_0 is the initial value while C_i, i>0, is the value at t=t_i of the last approximation $\phi_m(t)$ (t; t_i−1,C_{i-1}).

For the MATHEMATICA subroutine for generating the Adomian polynomials and further readings we suggest readers to refer to [16-18].

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Received June 28, 2012; **Accepted** June 30, 2012; **Published** July 04, 2012

Citation: Duan JS (2012) On the Power Series Expansion of a Nonlinear Function of a Power Series. J Applied Computat Mathemat 1:e109. doi:10.4172/2168-9679.1000e109

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