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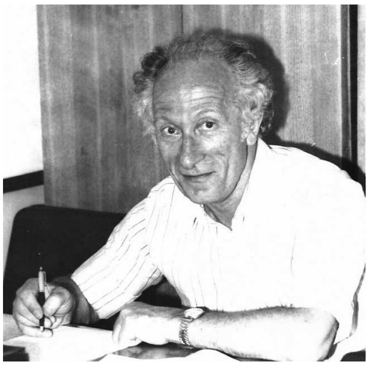
# Maks A. Akivis

*On the occasion of his 85th birthday and 65 years of scientific activity*



**Abstract**

The biography and bibliography of one of the great contemporary geometers Maks A. Akivis and the description of his scientific activities are presented.

**2000 MSC:** 01A70

One of the great contemporary geometers, Professor, Dr. Maks Aizikovich Akivis celebrated his 85th birthday and 65 years of scientific activity on January 5, 2008. On this occasion we want to honor the continuing scholarly productivity of Akivis whose scientific activity prior to 1993 and through 1998 was recognized in articles *Maks Aizikovich Akivis* published in Uspekhi Mat. Nauk **48** (1993), no. 3 (291), 213–216; in Webs and Quasigroups, 1993, pp. 4–8; and in Webs and Quasigroups, 1998/1999, pp. 7–11. In the last publication, a complete list of Akivis' publications prior to 1999 and the list of Ph. D. theses written under his supervision were published.

Maks Aizikovich Akivis was born on January 5, 1923 in Novosibirsk, USSR. While he was in high school, he demonstrated outstanding mathematical talent and ability. In 1940 he entered the Faculty of Mechanics and Mathematics of Moscow State University. During the Great Patriotic War (WWII) his studies were interrupted from 1942 to 1945 while he served in the Soviet Army and participated in the liberation of Prague and the capture of Berlin. For his patriotic service he was awarded many orders and medals.

After WWII he resumed his studies at Moscow State University. However, he was not able to graduate normally: he was expelled when he was a fifth year student for "ideological reasons". These "ideological reasons" also kept Akivis, one of the best students of the Faculty of Mechanics and Mathematics, from becoming a graduate student of Moscow State University. Not until 1958 was he able to defend his Ph. D. thesis which summarized the results of his undergraduate work. In 1964 Akivis defended a second (Doctor of Science) dissertation, and one year later, in 1965 he became a full professor.

Professor I. M. Gelfand, who taught at Moscow State University in the 1940s, recalls: "M. A. Akivis and E. B. Dynkin were my best students at Moscow State University in the 1940s. Akivis choose Differential Geometry as his field of research. Unfortunately, the end of his career as a student was darkened by the conditions in the Soviet Union at that time, and he could not pursue the normal graduate studies. However, he was able to overcome all difficulties and became one of the best scientists of his generation in the area of classical differential geometry."

From 1956 to 1960 Akivis taught at the Tula Mechanical Institute, and from 1960 to 1994 he was a Professor in the Department of Mathematics at Moscow Institute of Steel and Alloys.

Since 1948 Akivis published more than 150 scientific books and papers. His results in multidimensional projective and conformal differential geometry, in web theory and in the theory of differential geometric structures are fundamental. Many of these results are classical and are cited in numerous papers.

The first Akivis papers were devoted to the $T$-pairs of complexes (three-parameter families) of straight lines. His advisor S. P. Finikov suggested that he applies the notion of harmonic intersection of ruled surfaces which had been introduced by É. Cartan to the theory of congruences (two-parameter families) of straight lines in a three-dimensional projective space. Akivis solved this problem brilliantly: he found a new geometric property of the $T$-pairs of congruences introduced by Finikov and extended his results to the pairs of complexes of straight lines. As he showed, the $T$-pairs of complexes of straight lines he discovered are transferred by the Plücker mapping into a configuration of a five-dimensional projective space consisting of a tangentially degenerate two-dimensional submanifold and a three-dimensional submanifold carrying a net of conjugate lines. In the 1950s and 1960s Akivis devoted a series of papers to the projective theory of submanifolds of the type indicated above. As a result, he created a new area of projective differential geometry which continues to be developed successfully to this day.

The results on the $T$-pairs of complexes comprised Akivis' Ph. D. thesis (the Candidate of Science dissertation), and many of his results on multidimensional projective differential geometry became a part of the monograph *Projective differential geometry of submanifolds* (North-Holland, 1993) he wrote jointly with V. V. Goldberg.

While studying the $T$-pairs of complexes, Akivis noticed that the Plücker hyperquadric, on which the set of straight lines of a three-dimensional projective space is realized, is endowed with conformal geometry. This very simple observation led to some interesting and important generalizations. In the 1960s Akivis constructed the invariant theory of submanifolds of the conformal space. In particular, for a submanifold of an arbitrary dimension, he found a system of tensors defining a submanifold up to a conformal transformation. These results led to a second (Doctor of Science) dissertation of Akivis and were developed further in his work on the theory of conformal and pseudoconformal structures.

In his investigations, Akivis combined the method of exterior forms and moving frames of É. Cartan and the method of extensions and inclusions of G. F. Laptev with the classical tensorial methods. A broad mathematical outlook combined with a unusual geometric intuition allowed Akivis to obtain the important and profound results in complete and final form and to achieve extraordinary clarity in the exposition of his results.

In 1959 Akivis translated from German into Russian the book *Einführung in die Geometrie der Waben* written by W. Blaschke. In this period many papers on the algebraic theory of webs and quasigroups appeared. All these impressed Akivis very much and influenced his scientific interests. In particular, he noticed that some important results of the algebraic theory of webs do not have an adequate interpretation in the framework of the classical geometric theory of webs. Akivis concluded from this that it would be necessary to develop the theory of multidimensional webs which were studied in the 1930s in only three papers of G. Bol and S. S. Chern.

In 1969 Akivis published two papers: *Three-webs of multidimensional surfaces* (Trudy Geometr. Sem., vol. 2) and *On canonical expansions of equations of a local analytic quasigroup* (Dokl. Akad. Nauk SSSR **188** (1969), no. 5).

In the first paper, Akivis studied in detail the structure equations of a multidimensional three-web which were obtained by Chern as far back as 1936. Akivis not only studied the structure equations of a multidimensional three-web but also introduced an important notion of transversality and defined two new classes of multidimensional three-webs: transversally geodesic and paratactical. In addition, he found the tensorial characterizations of parallelizable, group, hexagonal, transversally geodesic and paratactical three-webs and studied in detail an affine connection (which is called the Chern connection now) canonically associated with a three-web.

Akivis became completely immersed into the geometry of webs, and because of his unusual geometric insight, he was able to obtain simple, elegant and effective results in the theory of multidimensional three-webs.

In the second paper of 1969, applying isotopic transformations, Akivis reduced the Taylor expansions of equations of a local analytic quasigroup to a canonical form and found under what conditions this quasigroup possesses one-parameter subquasigroups. Ideologically, this paper was close to the paper *Analytic loops* written in 1955 by A. I. Mal'cev. Namely this Akivis paper gave rise to that theory of local analytic loops in which the equivalence relation is the isotopy (not the isomorphism). The canonical expansion of a local analytic loop found by Akivis is the widest generalization of the Campbell–Hausdorff series for Lie groups.

Both Akivis' papers of 1969 opened a new era in the development of the theory of webs and the theory of local analytic loops (related to the theory of webs) and became the foundation of these theories. The ideas contained in these two papers are still far from being exhausted, and they continue to be the source of new and interesting results. Akivis himself as well as his students are also continuing to further develop and generalize on these ideas.

Thus, on the one hand, we have the structure equations of a three-web (or a local analytic loop) generalizing the Maurer–Cartan equations of a Lie group. On the other hand, we have the canonical expansion of a local analytic loop generalizing the Campbell–Hausdorff series for Lie groups. So, the theory of three-webs and smooth loops is similar to the theory of Lie groups and is a generalization of the latter. Therefore, it is natural to classify these objects according to how close they are to Lie groups. Developing these considerations, Akivis arrived at the notion of a closed $G$-structure of order $k$ which is important not only in the theory of webs but also in the general theory of $G$-structures: in 1975 Akivis introduced a closed $G$-structure as a $G$-structure defined by a formally completely integrable system of exterior differential equations. Examples of closed $G$-structures are Lie groups, symmetric spaces and some of their generalizations, many known classes of three-webs, etc. The order $k$ of a closed $G$-structure indicates how close this $G$-structure is to Lie groups for which $k = 2$.

During this period Akivis formulated a few problems and conjectures which greatly influenced the further development of the theory of webs. We indicate two of them which were later solved by Akivis and A. M. Shelekhov:

1. Determine whether the $G$-structure associated with a hexagonal three-web of arbitrary dimension is closed.

2. Prove that a three-web possesses a closed $G$-structure of order $k$ if and only if the canonical expansion of the coordinate loops of the three-web is completely determined by the $k$th order jet.

It is clear that an infinitesimal analog for closed $G$-structures was needed. Akivis found this analog: this is the so-called $W$-algebras ("$W$" is from the English word "web") generalizing the notion of triple Lie systems. The $W$-algebras and their generalization, the $W_k$-algebras (introduced by Akivis and Shelekhov in 1985), play in the theory of webs with a closed $G$-structure the same role as the Lie algebras in the theory of Lie groups. Later on, in 1986 K. H. Hofmann and K. Strambach developed the theory of abstract $W$-algebras and named them *Akivis algebras*.

In 1973–1974 Akivis introduced two very important classes of three-webs: isoclinic webs and their particular case— Grassmann webs which at the same time are transversally geodesic and which are defined on the Grassmann manifold of straight lines of a projective space by means of three hypersurfaces.

It is difficult to overestimate the importance of the Grassmann webs for the general theory of webs. First, the Grassmann webs are geometrically described in a very simple manner, and by taking the defining hypersurfaces in a certain special way, it is possible to construct three-webs with different *a priori* given properties. Secondly, Akivis proved that any isoclinic transversally geodesic three-web is equivalent to a Grassmann three-web, and any isoclinic hexagonal three-web is equivalent to a Grassmann three-web whose defining hypersurfaces belong to one and the same hypercubic (such a three-web was later called algebraic). When in 1978 Chern and P. A. Griffiths formulated the problems of Grassmannizability and algebraizability of $d$-webs, they did not know that in 1973–1975 these problems were solved by Akivis and Goldberg for $d = 3, 4$ and $d = n + 1$. A complete solution of these problems was given by Akivis in his papers of 1980 and 1983. Note that at the same time the methods of the theory of webs were used to solve the following important problem of algebraic geometry: to find necessary and sufficient conditions for $d$ submanifolds of dimension $r$ to belong to the same algebraic submanifold of dimension $r$ and degree $d$.

In the solution of the problems of Grassmannizability and algebraizability, the notion of an almost Grassmann structure which is naturally connected with a web played an essential role. Akivis defined an almost Grassmann structure as a fibration of Segre cones on a manifold, and this geometric definition has been used by I. M. Gelfand, S. G. Gindikin and other authors in integral geometry.

All of these, as well as some other results of Akivis not mentioned here, were summarized in several survey papers, manuals and finally in the monograph *Geometry and algebra of multidimensional three-webs* (Kluwer Academic Publishers, 1992) written by Akivis jointly with Shelekhov.

In 1993, together with B. A. Rosenfeld, Akivis completed a scientific biography *Élie Cartan* (1869 – 1951) which was published in book form by the American Mathematical Society. In 2007 this book was published in Russian.

In 1994 Akivis moved to Israel to join his younger daughter and grandchildren. There, in Israel, he received the Shapiro stipend which is awarded to elderly scientists actively involved in research. He resumed his research first in Ben-Gurion University of the Negev, Beer Sheva, and since 1997 in Jerusalem College of Technology (JCT). After Akivis moved to Israel, he has collaborated very actively with his lifelong coauthor Goldberg (New Jersey Institute of Technology, U.S.A.).

During his first years in Israel (1994–1996), Akivis jointly with Goldberg worked on the monograph *Conformal differential geometry and its generalizations* (John Wiley & Sons, 1996). In this book, the authors study systematically differential geometry of conformal and pseudoconformal spaces and submanifolds in these spaces; manifolds endowed with conformal and pseudoconformal structures; and Grassmannians and manifolds with Grassmann structures. Since a Grassmann structure occurs on manifolds bearing multidimensional webs, the authors also study webs from this point of a view. This book summarizes the authors' many years of work in the fields of differential geometry indicated above. The book was of great success. A half dozen excellent reviews of this book by well-known mathematicians were published. The first printing was sold out.

In addition to the monograph, in 1993–2008 Akivis published more than 40 papers in different fields of differential geometry. An outline of Akivis' works follows.

First we will present Akivis' work in conformal differential geometry:

- He studies a real geometry of four-dimensional manifolds endowed with conformal structures of different signatures. He considers the isotropic $\alpha$- and $\beta$-fibration on these manifolds and finds the conditions for conformal structures on such manifolds to be integrable or semiintegrable. This research is connected with the Penrose's twistor theory and the theory of Einstein's spaces in general relativity.

- Jointly with Goldberg, Akivis develops and complements some results considered in the above mentioned monograph. In particular, they prove the conformal invariance of isotropic geodesics of pseudoconformal spaces and that the principal lines on a four-dimensional manifold endowed with conformal structure of Lorentzian signature are isotropic geodesics. They connect these results with Petrov's classification of Einstein's spaces.

- Akivis and Goldberg introduce a new conformal differential invariant of a hypersurface of a (pseudo)conformal space. They proved that two hypersurfaces in 1-to-1 point correspondence are conformally equivalent if and only if this invariant is preserved under the correspondence.
- In series of papers, they study the geometry of lightlike hypersurfaces in the de Sitter space and the Darboux map of canal hypersurfaces of a conformal space.
- Akivis and Goldberg also study conformal and almost Grassmann structures and their integrability and semiintegrability in more detail than in their book mentioned above.

Second we outline Akivis' work in web theory and the theory of local differentiable quasigroups:

- In his joint paper with Shelekhov, Akivis applied to web theory the apparatus of the theory of vector fields and differential operators which is dual to the method of exterior forms. Since in recent years the theory of quasigroups is used widely in theoretical physics, this study by the dual method could be useful for research in physics.
- It is well-known that the Bol loops are closely related to the theory of symmetric spaces. Geometrically this connection is most visualizable for Riemannian metrics of constant curvature. Akivis explored this in his papers on Bol's three-webs and non-Euclidean geometry.
- Akivis and Goldberg wrote the 133-page survey paper for *Handbook of Differential Geometry* (Elsevier Science B. V., 1999). In this paper the authors present a very detailed description of numerous works in the field of webs and local differentiable quasigroups. The paper is organized in such a way that it can be used both as an introductory text and an encyclopaedia on this subject.

  The authors give an introduction to the theory of webs with the history of research in this subject, indicate the most interesting directions of this theory and the results obtained, and formulate some open problems.

  In their presentation of the differential geometry of webs, the authors describe those topics of the general theory that in their opinion are the most important. In addition, certain new notions and thoughts were introduced:

  - Instead of the notion of an abstract web, the authors define the complete web which is of opposite sense to the local web and which is more precise.
  - In their presentation of algebraizable webs, the authors distinguish algebraizability in the narrow sense (an ordinary notion of algebraizability in the sense of Chern and Griffiths) and algebraizability in the wide sense. This notion was suggested by examples of webs which are determined by algebraic manifolds but not Grassmannizable, and thus not algebraizable in the sense of Chern and Griffiths.
  - In connection with web theory the authors present the theory of closed $G$-structures (such structures are completely determined by elements of a finite order differential neighborhood) and emphasize the relationship between algebraizable webs and webs associated with the almost Grassmann structures.

- In another survey paper of 2000, Akivis and Goldberg discuss the relationship between local properties of equivalence classes of webs formed by suitable foliations of a manifold and properties of isotopy classes of corresponding coordinate differentiable local loops. Here they discuss the most important interactions of web geometry and algebra, namely famous closure conditions and loop identities; differentiable local loops, their special classes and corresponding Akivis binary-ternary algebras; $(n+1)$-webs and local $n$-quasigroups; special classes of four-webs and related algebraic conditions.

- In 2001, in a joint paper with Goldberg, Akivis constructed a series of examples of irreducible $n$-quasigroups for any $n \geq 3$. This solved the famous Belousov problem of constructing examples of irreducible $n$-quasigroups.

- In 2004, Akivis, Goldberg, and Lychagin found the linearization conditions for planar $d$-webs, $d \geq 4$. This solved the problem posed by Blaschke in the book *Einführung in die Geometrie der Waben* which Akivis translated.

- In 2006, *Bulletin of the American Mathematical Society* (one of the most prestigious mathematical journals) published the paper *Local algebras of a differentiable quasigroup* by Akivis and Goldberg. In this paper, the authors summarize investigations in the theory of differentiable quasigroups and their local algebras, indicating the relation of these investigations with some recent work on this subject.

- Most recently, Akivis jointly with Goldberg published two papers on differential geometry of Veronese-like and Lagrange-like webs. The authors proved that nonparallelizable Veronese-like webs $VLW_t(n, r)$ exist only in the cases $n = 3, r = 1$ and $n = 2, r$ arbitrary. They also prove that nonparallelizable Lagrange-like webs $LLW_t(n, r)$ exist only in the case $n = 2, r$ arbitrary. The class of webs $VLW_t(2, r)$ coincides with the class of webs $LLW_t(2, r)$, and the webs of both classes are formed by isoclinic submanifolds of an isoclinic three-web $W(3, 2, r)$.

Finally, we note that in 2004 Akivis and Goldberg continued their work on projective differential geometry. They wrote a series of papers on varieties with degenerate Gauss maps (vwdGm). These studies were summarized in the monograph *Differential geometry of varieties with degenerate Gauss maps* (Springer-Verlag, 2004).

We outline below the most significant results of these studies.

- Akivis and Goldberg show that the Griffiths–Harris conjecture on the structure of vwdGm is not complete. As was proved in their papers and in their book, the basic types of vwdGm include not only cones and torses but also hypersurfaces with degenerate Gauss maps. Note that such hypersurfaces form a very wide class of varieties with degenerate Gauss maps.

- They *systematically* use the focal images (the focal hypersurfaces and the focal hypercones) associated with vwdGm. They allow the authors to describe the geometry of vwdGm and give their classification.

- They study singular points and singular varieties of vwdGm.

- They found some new properties of vwdGm such as

  - A new classification of vwdGm.
  - A proof that the hypersurfaces of Sacksteder and Bourgain coincide.
  - Finding an affine analogue of the Hartman–Nirenberg cylinder theorem.

- Establishing the relation between smooth lines on projective planes over two-dimensional algebras and vwdGm.
- A description of a new class varieties with degenerate Gauss maps (twisted cones).

- In the papers and the book, they consider a very large number of examples. Some of these examples (such as the twisted cones and some algebraic hypersurfaces in $\mathbb{P}^4$) are considered there for the first time, and other examples (such as the cubic symmetroid in $\mathbb{P}^5$ and its projection onto $\mathbb{P}^4$) were known earlier but were considered by the authors from a new point of view.

- The following applications of vwdGm were considered:

  - Application of vwdGm to the theory of lightlike hypersurfaces in the de Sitter space.
  - Application of a relation of the theory of vwdGm in projective spaces with the theory of congruences and pseudocongruences of subspaces to the construction of induced connections on submanifolds of projective spaces and other spaces endowed with a projective structure.
  - Establishing a relation between the theory of vwdGm with the theory of smooth lines on projective planes over the complete matrix algebra $\mathbb{M}$ of order two, the algebra $\mathbb{C}$ of complex numbers, the algebra $\mathbb{C}^1$ of double numbers, and the algebra $\mathbb{C}^0$ of dual numbers.

In 1993–2007 Akivis jointly with Goldberg presented the above mentioned papers at numerous conferences: 7th International Conference on Geometry (Israel; 1995), Joint meeting of the American Mathematical Society and Israel Mathematical Union (IMU) (Israel; 1995), 6th, 7th and 10th International Conference on Differential Geometry and Its Applications (Czech Republic; 1995, 1998 and 2007), International Geometric Conference (Hungary; 1996), Annual Meeting of IMU (Israel; 1997), 3rd International Conference on Geometry (Bulgaria; 1997), International Congress on Geometry in honor of Pasquale Calapso (Italy; 1998), Mile High Conference on Quasigroups, Loops and Nonassociative Systems (Denver University, Denver, Colorado, USA; 2005), 7th and 9th International Conference on Geometry (Nahsholim, Israel, 1999; 2007), and International Conference Geometry in Odessa–2007 (Odessa, Ukraine; 2007).

During his long tenure at the Moscow Institute of Steel and Alloys Akivis taught many different special courses; for a long period of time he led the scientific seminar in the theory of webs in which the most important results in web theory and related fields were presented by the mathematicians of the USSR and foreign countries. While working in JCT in 1997–2007, Akivis taught a few special courses and conducted seminars for students and teachers of JCT.

The textbook *Tensor calculus*, written by Akivis jointly with Goldberg in 1969, was republished in the USSR in 1972 and in 2003. The translation of the first four chapters of this book was published in the USA in 1972 by Prentice Hall and was republished in 1977 by Dover Publications. In 2003 the entire book was translated by Goldberg into English and published by World Scientific. This book became a handbook and manual for many engineers, won worldwide recognition and is cited in many papers.

Maks Aizikovich Akivis is a man of extraordinary generosity. His great erudition, attention and kindness to his students and colleagues and his readiness to discuss any mathematical problems make contact and collaboration with him pleasant and fruitful; people of different ages are attracted to him. Maks Aizikovich Akivis is a brilliant teacher who educated many undergraduate and graduate students. He was the mentor and advisor of 28 Ph.D. (the Candidate of Sciences) theses and four Doctor of Sciences dissertations. All Akivis' graduate students are aware of his fatherly care of them. As a role model and with great dignity, integrity and tenacity he transmitted the love of research to all of them. It is with emotion, excitement and admiration we have witnessed how under his guidance, a young person becomes a mature specialist who selflessly loves a beautiful science — geometry.

We, friends, students and colleagues of Maks Aizikovich Akivis whole-heartedly wish him the best of health, happiness and an abundance of joy in his new scientific discoveries.

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