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On the Existence Problem of Joint Probability Distributions in Quantum Logic

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Introduction

The concept of joint probability distributions lies at the heart of classical probability theory, forming a mathematical backbone for statistical inference, stochastic modeling, and our understanding of simultaneous observables. However, in the quantum mechanical realm, this classical notion faces profound challenges due to the fundamentally non-classical nature of quantum logic. In contrast to the Boolean structure of classical logic, quantum propositions are represented by closed subspaces (or equivalently, projection operators) of a Hilbert space, forming a non-distributive orthomodular lattice. This inherent non-distributivity means that not all sets of observables in quantum mechanics can be jointly measurable or simultaneously assigned definite values a direct consequence of the uncertainty principle and non-commutativity of observables. As a result, the question of whether a joint probability distribution exists for a given set of quantum observables becomes non-trivial and deeply tied to the structure of quantum logic itself. This issue is not just a technical quirk but cuts to the core of foundational problems in quantum theory, including contextuality, non-locality, and the interpretation of quantum measurements. The existence problem for joint distributions in quantum logic involves examining under what conditions, if any, one can assign a global probability distribution that is consistent with all the marginal (measurable) distributions obtained from quantum experiments [1].

Description

At the core of this inquiry is the structure of quantum logic as formalized by Birkhoff and von Neumann in their foundational work, which introduced the idea that quantum mechanical propositions correspond to elements of an orthomodular lattice rather than a Boolean algebra. In classical systems, propositions about physical quantities (e.g., "position is x", "momentum is p") obey classical logic, where operations such as conjunction, disjunction, and negation satisfy distributivity. However, in quantum systems, observables are not simultaneously measurable unless they commute, and this non-commutativity gives rise to the failure of distributivity. Thus, a collection of propositions in quantum mechanics does not necessarily admit a classical joint probability assignment. Instead, quantum probability is defined in terms of the Born rule, which gives the probability of measurement outcomes for individual observables or compatible (i.e., commuting) sets of observables. This gives rise to the central question: Can we define a global joint distribution over all observables that reproduces these quantum marginals? In many cases, the answer is no, and this impossibility is at the heart of quantum contextuality a phenomenon proven through the Kochen-Specker theorem [2].

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To analyze the existence problem mathematically, one typically considers a set of projection operators representing propositions in quantum logic and asks whether there exists a single classical probability measure defined over a Boolean algebra that includes these projections, such that the measure agrees with quantum mechanical predictions for all measurable subsets. This question becomes particularly relevant in finite-dimensional Hilbert spaces (such as spin systems) where one can construct explicit examples of sets of observables that violate classical constraints. In this context, the Kochen-Specker theorem demonstrates that no global truth assignment (and hence no global joint distribution) is consistent with the predictions of quantum mechanics, highlighting the inherent contextuality of the theory. Similarly, Bell-type inequalities, which assume the existence of joint distributions for all variables, are violated in quantum experiments, underscoring the failure of this classical notion. Despite this, researchers have developed partial solutions or workarounds by identifying specific conditions under which joint distributions can exist. For instance, if all observables under consideration commute, then there exists a joint spectral measure, and a joint probability distribution can be derived in a well-defined way. Furthermore, in certain cases, it is possible to construct quasi-probability distributions, such as the Wigner function or the Husimi Q function, which represent quantum states in phase space but may take on negative or non-classical values. While these distributions cannot be interpreted as true probabilities [3].

they retain useful information about the system's behavior and allow for an approximate notion of joint distribution. Another avenue of exploration involves convex sets of compatible observables, where contextuality does not manifest, and classical distributions can be assigned locally. In this vein, quantum logicians and probabilists have worked to define measures on orthomodular lattices, attempting to extend Kolmogorov-style probability theory into the quantum domain, with varying degrees of success.Philosophically, the absence of joint distributions raises deep questions about the nature of reality and measurement in quantum theory. If one cannot assign pre-existing values to observables prior to measurement, and no joint distribution exists to reflect hidden variables, then we are forced to abandon classical realism in favor of an operational or relational interpretation of quantum mechanics. This has fueled various interpretations such as the Copenhagen interpretation, quantum Bayesianism (QBism), and the many-worlds interpretation, each offering different resolutions to the tension. In particular, contextuality the impossibility of explaining measurement outcomes without reference to the measurement context emerges as a hallmark of quantum behavior, now considered a resource for quantum computation and quantum cryptography. Modern advances in quantum information theory have reframed contextuality not as a limitation but as a powerful non-classical resource that can be harnessed in quantum protocols, such as magic-state distillation and randomness generation [4].

Experimentally, the existence problem has been investigated through tests of contextuality using trapped ions, photons, and superconducting qubits. These experiments attempt to verify whether the statistical predictions of quantum mechanics can be embedded in any classical joint distribution. Time and again, the results have affirmed the absence of such a distribution, consistent with theoretical predictions. Moreover, quantum contextuality inequalities, such as those developed by Klyachko, Cabello, and others, provide rigorous statistical tests for the (non)existence of joint distributions. These tests serve not only as proofs of principle but also as benchmarks for quantum devices, potentially defining standards for future quantum technologies. In parallel, the development of non-commutative probability theory and quantum measure theory offers a mathematically rigorous platform to formalize the probabilistic structure of quantum theory without relying on classical assumptions. These frameworks shift the emphasis away from joint distributions toward operationally defined measurement statistics, better suited to describe inherently non-classical systems. Ultimately, the problem of the existence of joint distributions on quantum logic reflects the departure of quantum mechanics from the Kolmogorovian paradigm that underpins classical probability theory. This shift necessitates not only new mathematical tools but also a reconceptualization of what it means to describe reality probabilistically. The interplay between logical structure (orthomodularity), algebraic structure (non-commutative operator algebras), and statistical structure (state measures) defines a rich, interdisciplinary domain where physics, mathematics, and philosophy intersect [5].

Conclusion

In conclusion, the problem of the existence of joint probability distributions within the framework of quantum logic highlights a fundamental departure of quantum theory from classical paradigms. The non-distributive nature of quantum logic, the non-commutativity of observables, and the phenomenon of contextuality collectively render the classical notion of joint distributions inapplicable in general. While certain subsets of commuting observables admit consistent joint probabilities, the general case resists such simplification, reflecting the deep structural differences between quantum and classical worlds. The implications of this are not merely philosophical but operational, affecting the foundations of quantum computing, cryptography, and information theory. The absence of global joint distributions underscores the impossibility of assigning pre-measurement properties to quantum systems in a classical sense, and it demands new frameworks such as non-classical probability theories, contextual models, and quasi-probability formalisms to describe quantum phenomena accurately. Moreover, modern perspectives suggest that what was once perceived as a limitation the failure of joint distributions may in fact be strength. Contextuality and non-classical correlations are now being explored as unique resources for quantum technologies

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Conflict of Interest

No conflict of interest.

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