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On the Consecutive Integers $n+i-1=(i+1)\ P_i$

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Abstract

By using the Jiang's function $J_2(\omega)$ we prove that there exist infinitely many integers n such that $n=2P_1$, $n+1=3P_2$,, $n+k-1=(k+1)\ P_k$ are all composites for arbitrarily long k, where P_1, P_2,…, P_k are all primes. This result has no prior occurrence in the history of number theory.

Keywords: Consecutive integers; Jiang's function

Introduction

Theorem 1

There exist infinitely many integers n such that the consecutive integers $n=2P_1$, $n+1=3P_2$,…, $n+k-1=(k+1)\ P_k$ are all composites for arbitrarily long k, where P_1, P_2,…, P_k are all primes.

Proof: Suppose that $P_i=\frac{m}{i+1}x+1$. We define the prime equations:

$$P_i=\frac{m}{i+1}x+1, \tag{1}$$

Where i=1, 2,…, k

The Jiang's function [1] is:

$$J_2(\omega)=\prod_{3\le P}(P-k-1-\chi(P))\neq 0 \tag{2}$$

Where $(P)=-k$ if $P^2\ m$; $\chi(P)=-k+1$ if Pm; $\chi(P)=0$ otherwise, $\omega=\prod_{2\le P}P$.

Since $J_2\ (\omega)\to\infty$ as $\omega\to$, there exist infinitely many integers x such that P_1, P_2,…, P_k are all primes.

We have the asymptotic formula of the number of integers $x\le N$ [1]

$$\pi_{k+1}(N,2)\sim\frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}, \tag{3}$$

Where, $\varphi(\omega)=\prod_{2\le P}(P-2)$.

From (1) we have,

$$n=mx+2=2\left(\frac{mx}{2}+1\right)=2P_1,$$

$$n+1=mx+3=3\left(\frac{m}{3}x+1\right)$$

$$=3P_2,\cdots,n+k-1=mx+k+1=(k+1)\left(\frac{m}{k+1}x+1\right)=(k+1)P_k\cdot$$

Example 1: Let k=5, we have n=2 × 53281, n+1=3 × 35521, n+2=4 × 26641, n+3=5 × 21313, n+4=6 × 17761.

Theorem 2

There exist infinitely many integers n such that the consecutive integers $n=(1+2^b)\ P_1$, $n+1=(2+2^b)\ P_2$,…, $n+k-1=(k+2^b)\ P_k$ are all composites for arbitrarily long k, where P_1, P_2,, P_k are all primes [2].

Proof: Suppose that $m=\prod_{i=1}^{k}(i+2^b)$. We define the prime equations:

$$P_i=\frac{m}{i+2^b}x+1, \tag{4}$$

Where i=1, 2,…, k.

The Jiang's function [1] is:

$$J_2(\omega)=\prod_{3\le P}(P-k-1-\chi(P))\neq 0 \tag{5}$$

Where $\chi(P)=-k$ if P^2m; $\chi(P)=-k+1$ if Pm; $\chi(P)=0$ otherwise.

Since, $J_2\ (\omega)\to\infty$ as$\omega\to\infty$, there exist infinitely many integers x such that P_1, P_2,…, P_k are all primes.

We have the asymptotic formula of the number of integers $x\le N$ [1]

$$\pi_{k+1}(N,2)\sim\frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}, \tag{6}$$

From (4) we have:

$$n=mx+1+2^b=(1+2^b)\left(\frac{m}{1+2^b}x+1\right)=(1+2^b)P_1,$$

$$n+1=mx+2+2^b=(2+2^b)\left(\frac{m}{2+2^b}x+1\right)=(2+2^b)P_2,\cdots,$$

$$n+k-1=mx+k+2^b=(k+2^b)\left(\frac{m}{k+2^b}x+1\right)=(k+2^b)P_k\cdot$$

Example 2: Let b=1 and k=4, we have n=3 × 27361, n+1=4 × 20521, n+2=5 × 16417, n+3=6 × 13681.

Theorem 3

There exist infinitely many integers n such that the consecutive integers $n=3P_1$, $n+2=5P_2$,…, $n+2=5P_2$,…,$n+2\ (k-1)=(2k+1)\ P_k$ are all composites for arbitrarily long k, where P_1, P_2,, P_k are all primes [3].

Proof: Suppose that $m=\prod_{i=1}^{k}(2i+1)$. We define the prime equations:

$$P_i=\frac{m}{2i+1}x+1, \tag{7}$$

Where i=1, 2,…, k.

The Jiang's function [1] is:

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$$J_2(\omega) = \prod_{3 \le P} (P - k - 1 - \chi(P)) \neq 0 \tag{8}$$

Where $\chi(P)=-k$ if $P^2\ m$; $\chi(P)=-k+1$ if $P\ m$; $\chi(P)=0$ otherwise.

Since, $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many integers x such that $P_1, P_2, \ldots, P_k$ are all primes.

We have the asymptotic formula of the number of integers $x \le N$ [1]

$$\pi_{k+1}(N,2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \tag{9}$$

From (7) we have:

$$n = mx + 3 = 3(\frac{m}{3}x + 1) = 3P_1,$$

$$n + 2 = mx + 5 = 5(\frac{m}{5}x + 1) = 5P_2, \cdots,$$

$$n + 2(k-1) = mx + 2k + 1 = (2k+1)\left(\frac{m}{2k+1}x + 1\right) = (2k+1)P_k$$

***Example 3*:** Let k=4, we have $n=3 \times 631$, $n+2=5 \times 379$, $n+4=7 \times 271$, $n+6=9 \times 211$.

Theorem 4

There exist infinitely many integers n such that the consecutive integers $n+2=3P_2, \ldots, n+2=3P_2, \ldots, n+2\ (k-1)=(2k+1)\ P_k$ are all composites for arbitrarily long k, where $P_1, P_2,, P_k$ are all primes [4].

Proof: Suppose that $m = \prod_{i=1}^{k}(2i-1)$. We define the prime equations:

$$P = \frac{\quad}{2 \quad 1} x + \tag{10}$$

Where i=1, 2,…, k.

The Jiang's function [1] is:

$$J_2(\omega) = \prod_{3 \le P} (P - k - 1 - \chi(P)) \neq 0 \tag{11}$$

Where $\chi(P)=-k$ if $P^2\ m$; $\chi(P)=-k+1$ if $P\ m$; $\chi(P)=0$ otherwise.

Since, $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many integers x such that $P_1, P2, \ldots, P_k$ are all primes.

We have the asymptotic formula of the number of integers $x \le N$ [1]

$$\pi_{k+1}(N,2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \tag{12}$$

From (10) we have:

$n=P_1=mx+1$,

$$n + 2 = mx + 3 = 3(\frac{m}{3}x + 1) = 3P_2, \cdots,$$

$$n + 2(k-1) = mx + 2(k-1) = (2k-1)\left(\frac{m}{2k-1}x + 1\right) = (2k-1)P_k \cdot$$

***Example 4*:** Let k=4, we have n=9661, $n+2=3 \times 3221$, $n+4=5 \times 1933$, $n+6=7 \times 1381$.

Theorem 5

There exist infinitely many integers n such that the consecutive integers $n=3P_1, \ldots, n+4=7P_2, \ldots, n+4\ (k-1)=(4k+1)\ P_k$ are all composites for arbitrarily long k, where $P_1, P_2, \ldots, P_k$ are all primes [5].

***Example 5*:** Let k=4, we have $n=3 \times 2311$, $n+4=7 \times 991$, $n+8=11 \times 631$, $n+12=15 \times 463$.

Theorem 6

There exist infinitely many integers n such that the consecutive integers $n=5P_1, \ldots, n+4=9P_2, \ldots, n+4\ (k-1)=(4k+1)\ P_k$ are all composites for arbitrarily long k, where $P_1, P_2, \ldots, P_k$ are all primes [6].

Conclusion

Jiang's function $J_2(\omega)$ prove that there exist infinitely many integers n such that $n=2P_1$, $n+1=3P_2, \ldots, n+k-1=(k+1)\ P_k$ are all composites for arbitrarily long k, where $P_1, P_2,, P_k$ are all primes which result has no prior occurrence in the history of number theory.

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