Applied & Computational Mathematics
Journal of Applied & Computational Mathematics ISSN: 2168-9679
Ganie and Syed, J Appl Computat Math 2015, 4:6
DOI: 10.4172/2168-9679.1000273
Research Article Open Access

On Some New Generalized Difference Sequence Space of Fuzzy Numbers and Statistical Convergence

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Abstract

The main aim of the present paper is to introduce the concept of lacunary almost statistical convergence and strongly almost convergence of the generalized difference sequences of Fuzzy numbers. We also investigate their some basic topological properties.

Keywords: Almost convergence; Fuzzy set; Lacunary sequence; Generalized difference sequence spaces

2010 Mathematics subject classification: 40A05; 26A03; 11B05

Introduction

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper N, R and C denotes the set of non-negative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex) and let l_∞ and c be Banach spaces of bounded and convergent sequences $x=\{x_n\}_{n=0}^{\infty}$ with supremum norm $\|x\|=\sup|x_n|$. Let T denote the shift operator on ω, that is, $Tx=\{x_n\}_{n=1}^{\infty}$, $T^2x=\{x_n\}_{n=2}^{\infty}$ and so on. A Banach limit L is defined on l_∞ as a non-negative linear functional such that L is invariant i.e., $L(Sx) = L(x)$ and L(e) = 1,e = (1,1,1,…) [1].

Lorentz called a sequence $\{x_n\}$ almost convergent if all Banach limits of x, $L(x)$, are same and this unique Banach limit is called F - limit of x. In his paper, Lorentz proved the following criterian for almost convergent sequences.

A sequence x = $\{x_n\} \in l_\infty$ is almost convergent with F - limit of $L(x)$ if and only if

$$\lim_{m\to\infty} t_{mn}(x) = L(x)$$

where, $t_{mn}(x)=\frac{1}{m}\sum_{j=0}^{m-1}T^j x_n, (T^0=0)$ uniformly in n ≥ 0.

We denote the set of almost convergent sequences by f. This was further studied by Ganie [2], Nanda [3] and many others.

A complex number sequence x is said to be statistically convergent to the number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n}|\{k \le n : |x_k - L| \ge \varepsilon\}| = 0, \tag{1}$$

where the vertical bars indicate the number of elements in the enclosed set. In this case we write S - limit $x = L$ or $x_k \to L(S)$. We shall also use S to denote the set of all statistically convergent sequences. The idea of statistical convergence was introduced by Fast [4] and was further studied by several authors [5-9].

By a lacunary sequence we mean an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. Throughout this paper, the intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio k_r / k_{r-1} will be abbreviated by q_r.

Let θ be a lacunary sequence; the number sequence x is S_θ - convergent to L provided that for every $\varepsilon > 0$,

$$\lim_r \frac{1}{h_r}|\{k \in I_r : |x_k - L| \ge \varepsilon\}| = 0. \tag{2}$$

In this case we write S_θ - limit $x = L$ or $x_k \to L(S_\theta)$, and we define

$$S_\theta = \{x : for\ some\ L, S_\theta - \lim x = L\}.$$

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [10] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [11] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties and has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by many others [4,7-9,12-18].

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on the real line R. For $X, Y \in D$ we define

$$d(X:Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

Where $X = [a_1, a_2]$, $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space.

Let I = [0,1]. A fuzzy real number X is a fuzzy set on R and is a mapping X : R → I associating each real number t with its grade membership X (t). A fuzzy real number X is called convex if

$$X(t) \ge X(s) \wedge X(r) = \min(X(s), X(r)), \text{ where } s < t < r.$$

A fuzzy real number X is called if normal if there exists $t_0 \in$ R such that $X(t_0) = 1$.

A fuzzy real number X is called upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$ for all $a \in I$ and given $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$ is open in

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Received November 13, 2015; **Accepted** December 16, 2015; **Published** December 21, 2015

Citation: Ganie AH, Syed MA (2015) On Some New Generalized Difference Sequence Space of Fuzzy Numbers and Statistical Convergence. J Appl Computat Math 4: 273. doi:10.4172/2168-9679.1000273

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the usual topology of R. The set of all upper semi-continuous, normal, convex fuzzy numbers is denoted by *R(I)*. The α - level set of a fuzzy real number X for $0 < \alpha \le 1$ denoted by X^α is defined by $X^\alpha = \{t \in R : X(t) \ge \alpha\}$. The 0 - level set is the closure of strong 0 - cut.

For each $r \in \mathbb{R}, \overline{r} \in \mathbb{R}(I)$ is defined by $\overline{r} = \begin{cases} \overline{r}, & \text{if } t = r, \\ 0, & \text{if } t \ne r. \end{cases}$

The absolute value of $|X|$ of $X \in R(I)$ is defined by [19].

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \ge 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\overline{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\overline{d}(X,Y) = \sup_{0 \le \alpha \le 1} d(X^\alpha, Y^\alpha).$$

Then $\overline{d}$ defines a metric on R(I) [19]. The additive identity and multiplicative identity in R(I) are denoted by $\overline{0}$ and $\overline{1}$ respectively.

A Fuzzy number is a function X from R^n to [0,1], which is normal, fuzzy convex, upper-semi continuous and the closure of $\{x \in R^n : X(x) > 0\}$ is compact. These properties imply that for each $0 < \alpha \le 1$, the α - level set $X^\alpha = \{t \in R^n : X(t) \ge \alpha\}$ is non-void compact convex subset of R^n, with support $X^0 = \{t \in R^n : X(t) > 0\}$

We denote by L (R^n) the set of all Fuzzy number. The linear structure of L (R^n) induces the addition $X + Y$ and the scalar multiplication λX, $\lambda \in R$, interms of α level sets,

$$|X + Y|^\alpha = |X|^\alpha + |Y|^\alpha \ \& \ |\lambda X|^\alpha = \lambda |X|^\alpha$$

for each $0 \le \alpha \le 1$. Now, for each $1 \le q < \infty$, we define

$$d_q(X,Y) = \left[\int_0^1 \delta_\infty(X^\alpha, Y^\alpha)^q \, d\alpha\right]^{\frac{1}{q}},$$

and

$$d_\infty(X,Y) = \sup_{0 \le \alpha \le 1} \delta(X^\alpha, Y^\alpha),$$

where δ_∞ is Hausdorff metric. It is obvious that $d_\infty(X,Y) = \lim_{q \to \infty} d_q(X,Y)$ and for $q \le r$, we have $d_q \le d_r$. Throughout the text, we will denote d_q by d where $1 \le q < \infty$.

Kizmaz [20] defined the difference sequence spaces Z(Δ) as follows

$$Z(\Delta) = \{x = (x_k) \in \omega : (\Delta x_k) \in Z\}$$

where $Z \in \{l\infty, c, c_0\}$ and $\Delta x_k = x_k - x_{k+1}$. It was further generalized by Tripathy and Esi [21], as follows. Let $m \ge 0$ be an integer then $H(\Delta_m) = \{x = (x_k) : \Delta_m x \in H\}$, for $H = l_\infty$, c and c_0, where $\Delta_m x_k = x_k - x_{k+m}$. The difference sequence space were further studies by Çolak [22], Ganie [15,23] and etc [24-29]. Further, in Tripathy [21] generalized the above notions and unified these as follows:

$$\Delta_m^n x_k = \{x \in \omega : (\Delta_n^m x_k) \in Z\},$$

where

$$\Delta_m^n x_k = \sum_{\mu=0}^{n} (-1)^\mu \binom{n}{r} x_{k+m\mu}$$

and

$$\Delta_0^n x_k = x_k \forall\, k \in \mathbb{N}.$$

Results

In this section, we shall introduce the notion of Fuzzy numbers by using generalized difference operator Δ_m^n and the lacunary sequence k = (k_r) and study their properties.

Definiton 4.1: Let θ = (k_r) be a lacunary sequence; a sequence X = (X_k) of Fuzzy numbers is said to be lacunary almost $\Delta_m^n(\theta)$ - convergent to the Fuzzy number X_0 provided that for every *ε > 0*,

$$\lim_r \frac{1}{h_r} |\{k \in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right) \ge \varepsilon\}| = 0, \tag{3}$$

uniformly in *i*. In this case we write $X_k \to X_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$ or $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim X_k = X_0$. The set of all lacunary almost $\Delta_m^n(\theta)$ - statistically convergent sequences of Fuzzy numbers is denoted by $\hat{S}\left(\Delta_m^n(\theta)\right)$.

Definiton 4.2: Let θ = (k_r) be a lacunary sequence; p = (p_k) be a sequence of strictly positive real numbers and X = (X_k) be a sequence of Fuzzy numbers. Then, the sequence X = (X_k) is said to be a lacunary strongly ( ) -convergent if there is a Fuzzy number X_0 such that

$$\lim_r \frac{1}{h_r} \sum_{k \in I_r} \left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k} = 0, \tag{4}$$

uniformly in *i*. In this case, we write $X_k \to X_0\left(\left[M_\theta^p, \Delta_m^n(\theta)\right]\right)$. By $\left[M_\theta^p, \Delta_m^n(\theta)\right]$, we shall denote the set of all lacunary strongly almost $\Delta_m^n(\theta)$ convergent sequence of Fuzzy numbers.

Definiton 4.3: Let θ = (k_r) be a lacunary sequence. Then the sequence X = (X_k) of Fuzzy numbers is said to be Δ_m^n bounded if the set $\left\{\left(\Delta_m^n X_k\right) : k \in \mathbb{N}\right\}$ of Fuzzy numbers is bounded. We shall denote by $l_\infty(\Delta_m^n)$ the set of all Δ_m^n - bounded sequences of Fuzzy numbers.

Theorem 4.1: Let $X = (X_k), Y = (Y_k) \in \hat{S}\left(\Delta_m^n(\theta)\right)$ and α ∈ R. Then,

(i) $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim(\alpha X_k) = \alpha \hat{S}\left(\Delta_m^n(\theta)\right) - \lim X_k$

(ii) $\hat{S}\left(\Delta_m^n(\theta)\right) - \lim(X_k + Y_k) = \hat{S}\left(\Delta_m^n(\theta)\right) - \lim X_k + \hat{S}\left(\Delta_m^n(\theta)\right) - \lim Y_k$

Proof: We left it as an easy exercise for the reader.

Theorem 4.2: Let θ = (k_r) be a lacunary sequence; p = (p_k) be a sequence of strictly positive real numbers with $0 < h = \inf p_k \le p_k \le \sup p_k = H < \infty$, then

(i) $X_k \to X_0\left(\left[M_\theta^p, \Delta_m^n(\theta)\right]\right) \Rightarrow X_k \to X_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$,

(ii) $X_k \in l_\infty(\Delta_m^n) \ \& \ X_k \in X_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right) \Rightarrow X_k \to X_0\left(\left[M_\theta^p, \Delta_m^n(\theta)\right]\right)$.

Proof: Let $X_k \to X_0\left(\left[M_\theta^p, \Delta_m^n(\theta)\right]\right)$. Then, for *ε > 0*, we have

$$\frac{1}{h_r} \sum_{k \in I_r} \left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k}$$

$$\ge \frac{1}{h_r} \sum_{\substack{k \in I_r \\ d\left(\Delta_m^n X_{k+i}, X_0\right) \ge \varepsilon}} \left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k}$$

$$\ge \frac{1}{h_r} \sum_{\substack{k \in I_r \\ d\left(\Delta_m^n X_{k+i}, X_0\right) \ge \varepsilon}} \varepsilon^{p_k}$$

$$\ge \frac{1}{h_r} \sum_{\substack{k \in I_r \\ d\left(\Delta_m^n X_{k+i}, X_0\right) \ge \varepsilon}} \min\left(\varepsilon^h, \varepsilon^H\right)$$

$$\ge \frac{1}{h_r} \left|\left\{k \in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right) \ge \varepsilon\right\}\right| \min\left(\varepsilon^h, \varepsilon^H\right)$$

uniformly in *i*, there by proving part (i).

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(ii) Now to prove part (ii) We suppose that $X_k \in l_\infty(\Delta_m^n)$ and $X_k \in X_0\left(\hat{S}\left(\Delta_m^n(\theta)\right)\right)$. Then, there is a constant $Q > 0$ such that

$$d\left(\Delta_m^n X_{k+i}, X_0\right) \le Q.$$

Then, for $\varepsilon > 0$, we have

$$\frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k}$$

$$\ge \frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon}}\left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k}$$

$$+\frac{1}{h_r}\sum_{\substack{k\in I_r\\ d\left(\Delta_m^n X_{k+i}, X_0\right)<\varepsilon}}\left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{p_k}$$

$$\le \max\left(Q^h, Q^H\right)\frac{1}{h_r}\left|\left\{k\in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right| + \max\left(\varepsilon^h, \varepsilon^H\right).$$

Therefore, we conclude that $X_0 \in \left(\left[M_\theta^p, \Delta_m^n(\theta)\right]\right)$.◊

From Theorem 2.2 above, we have the following result:

Corollary 4.3: $\hat{S}\left(\Delta_m^n(\theta)\right)\cap l_\infty(\Delta_m^n) = \left[M_\theta^p, \Delta_m^n(\theta)\right]\cap l_\infty(\Delta_m^n)$.

Theorem 4.4: Let θ = (k_r) be a lacunary sequence. If the sequence X = (X_k) is almost Δ_m^n - statistically convergent to the Fuzzy number X_0 and $\lim\inf_{(r)}\left(\frac{k_r}{r}\right) > 0$, then it is almost $\Delta_m^n(\theta)$ -statistically convergent to X_0.

Proof: For $\varepsilon > 0$, we have

$$\left|\left\{k \le r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right| \supset \left|\left\{k\in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right|.$$

This gives,

$$\frac{1}{r}\left|\left\{k \le r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right|$$

$$\ge \frac{1}{r}\left|\left\{k\in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right|$$

$$= \frac{h_r}{r}\frac{1}{h_r}\left|\left\{k\in I_r : d\left(\Delta_m^n X_{k+i}, X_0\right)\ge\varepsilon\right\}\right|.$$

On taking limits as r → ∞ and using the fact that $\lim\inf_{(r)}\left(\frac{k_r}{r}\right) > 0$, we conclude that X is $\Delta_m^n(\theta)$ -statistically convergent to X_0.

Theorem 4.5: Let $0 < p_k \le q_k < \infty$ and $\left(\frac{q_k}{p_k}\right)$ be bounded. Then, $\left[M_\theta^q, \Delta_m^n(\theta)\right] \subset \left[M_\theta^p, \Delta_m^n(\theta)\right]$.

Proof: Let $X \in \left[M_\theta^q, \Delta_m^n(\theta)\right]$. Let us denote $w_{k,i} = \left[d\left(\Delta_m^n X_{k+i}, X_0\right)\right]^{q_k}$ and $\mu_k = \frac{p_k}{q_k}$, so that $0 < \mu_k \le 1$ for each k. We define the sequences ($u_{k,i}$) and ($v_{k,i}$) as follows: Let $u_{k,I} = w_{k,i}$ and $v_{k,I} = 0$ for $w_{k,I} \ge 1$ and $u_{k,I} = 0$ and $v_{k,I} = w_{k,i}$ for $w_{k,I} < 1$. Then, for all k ∈ N, it is obvious that $w_{k,I} = u_{k,I} + v_{k,i}$ and $w_{k,i}^{\mu_k} = u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}$. Hence, we conclude that $u_{k,i}^{\mu_k} = u_{k,i} \le w_{k,i}$ and $v_{k,i}^{\mu_k} \le v_{k,i}^{\mu_k}$. Consequently, we have

$$\frac{1}{h_r}\sum_{k\in I_r} w_{k,i}^{\mu_k} = \frac{1}{h_r}\sum_{k\in I_r}\left(u_{k,i}^{\mu_k} + v_{k,i}^{\mu_k}\right)$$

$$\le \frac{1}{h_r}\sum_{k\in I_r} w_{k,i} = \frac{1}{h_r}\sum_{k\in I_r} v_{k,i}^{\mu_k}.$$

Since, μ < 1 for each m, we have

$$\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}^{\mu_k} = \left(\frac{1}{h_r}v_{k,i}\right)^{\mu}\left(\frac{1}{h_r}\right)^{1-\mu}$$

$$\le \left(\sum_{k\in I_r}\left[\left(\frac{1}{h_r}v_{k,i}\right)^{\mu}\right]^{\frac{1}{\mu}}\right)^{\mu}\left(\sum_{k\in I_r}\left[\left(\frac{1}{h_r}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}}\right)^{1-\mu}$$

$$= \left(\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}\right)^{\mu}.$$

By Holder's inequality and hence we have

$$\frac{1}{h_r}\sum_{k\in I_r} w_{k,i}^{\mu_k} \le \frac{1}{h_r}\sum_{k\in I_r} w_{k,i} + \left(\frac{1}{h_r}\sum_{k\in I_r} v_{k,i}\right)^{\mu}.$$

This shows that $X \in \left[M_\theta^p, \Delta_m^n(\theta)\right]$.◊

Theorem 4.6: $\left[M_\theta^p, \Delta_m^n(\theta)\right]_\infty \subset l_\infty\left(\Delta_m^n\right)$,

where

$$\left[M_\theta, \Delta_m^n(\theta)\right]_\infty = \left\{X = (X_k) : \frac{1}{h_r}\sum_{k\in I_r}\left[d\left(\Delta_m^n X_{k+i}, 0\right)\right] < \infty\right\}.$$

Proof: We first suppose that $X \in \left[M_\theta^p, \Delta_m^n(\theta)\right]_\infty$. Hence, we can find a constant λ > 0 such that

$$\frac{1}{h_1} d\left[\Delta_m^n X_{1+i}, \bar{0}\right] \le \frac{1}{h_r}\sum_{k\in I_r} d\left[\Delta_m^n X_{k+i}, \bar{0}\right] \le \lambda,$$

for all i and hence we have $X \in l_\infty\left(\Delta_m^n\right)$.

Conversely, we suppose that $X \in l_\infty\left(\Delta_m^n\right)$. Therefore, we can find a constant β such that for all j, we have

$$d\left(\Delta_m^n X_j, 0\right) \le \beta,$$

so that

$$\frac{1}{h_r}\sum_{k\in I_r} d\left[\Delta_m^n X_{k+i}, \bar{0}\right] \le \frac{k_2}{h_r} 1 \sum_{k\in I_r} 1 \le \beta,$$

for all k and i. Consequently, $X \in \left[M_\theta^p, \Delta_m^n(\theta)\right]_\infty$.◊

Acknowledgement

We would like to express our sincere thanks for the refree(s) for the kind remarks that improved the presentation of the paper.

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