

On H-Open Sets and H-Continuous Functions

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Introduction

The author introduced a new topology depending on the original topology on a non-empty set in the paper entitled "On h-open sets and h-continuous functions" by Fadhil Abbas, published in Journal of Applied and Computational Mathematics [1]. However, there are two false theorems and two false examples, namely Theorem 3.5, Theorem 3.8, Example 2.4, and Example 2.5 are unfortunately wrong. 1) Theorem 3.5 is wrong as a counter-example is given in the following.

Counter-example to Theorem 3.5. Let $X = \{a, b, c\} = \{1, 2, 3\}$ and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{1\}\}$. Then $\tau h = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\sigma^h = \{\emptyset, Y, \{1\}, \{2, 3\}\}$. Now define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ as $f(a) = 1, f(b) = 2, f(c) = 3$. Then it is clear that the function f is continuous. Since $\{2, 3\} \in \sigma^h$ and $f^{-1}(\{2, 3\}) = \{b, c\} \in \tau^h$, f is not h-irresolute.

2) Theorem 3.8 is wrong. A counter-example is in the following.

Counter-example to Theorem 3.8 Let $X=Y=Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$,

$\tau^h = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$, $\sigma^h = \{\emptyset, Y, \{a\}, \{b, c\}\}$,

$\eta = \{\emptyset, Z, \{b, c\}\}$, $\eta^h = \{\emptyset, Z, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and let

$f: (X, \tau) \rightarrow (Y, \sigma)$, $g: (Y, \sigma) \rightarrow (Z, \eta)$ be identity functions. It is clear that f is

h-irresolute, g is h-continuous. Since $\{c\} \in \eta$ and $(g \circ f)^{-1}(\{c\}) = \{c\} \in \tau^h$, $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not h-irresolute

3) In Example 2.4. The equality $D(\{a, c\} = \{c\})$ is not correct as well, the correct equality is $D(\{a, c\} = \{b, c\})$

4) But not least, Example 2.5 is not correct, it is false, as b is not an h-limit point of $\{a, b\}$ since $\{b, c\} \cap (A - \{c\}) = \emptyset$. Instead, example may be changed as in the following to get a correct sample for the converse to Theorem 2.5. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $\tau_1 h = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\tau_2 h = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Now that $\tau_1 h \subseteq \tau_2 h$, and b is an h-limit point of $A = \{a, c\}$ with respect to τ_1 while b is not h-limit point of $A = \{a, c\}$ with respect to τ_2 . Note that the author had taken $A = \{a, b\}$, wrongly, which made the example wrong.

5) In Definition 2.1 $U_6 = X$ is to be dropped.

6) In Theorem 3.9, the word "homomorphism" should be "homeomorphisms"

References

1. Fadhil Abbas. "On H-open sets and H-continuous functions." *J Appl Comput Math* 9(2020):1-5.

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