Journal of
Applied & Computational Mathematics

Devika and Vani, J Appl Computat Math 2018, 7:3
DOI: 10.4172/2168-9679.1000413

Research Article Open Access

On πg*β-Closed Sets in Topological Spaces

Devika A* and Vani R

Department of Arts and Science, Faculty of Arts and Science, PSG College of Arts and Science, Avinashi Road, Civil Aerodrome Post, Coimbatore, Tamil Nadu, India

Abstract

In this paper, we have introduce a new class of sets in topological spaces called πg*β- closed set and also we have introduce and study the properties of a πg*β-neighbourhood, πg*β-interior and πg*β-closure in topological spaces.

Keywords: πg*β-Open set; πg*β-Closed set; g-Neighbourhoods; πg*β-Interior; πg*β-Closure

2010 AMS Mathematics subject classification: 54A05

Introduction

The study of g-closed sets in a topological space was initiated by Andrijevi [1]. Arya and Nour [2] introduced g*-closed sets. Aslim [3] introduced the concepts ofπ-closed sets. Dontchev [4] and Dontchev and Noiri [5] introduced πg-closed sets. Gnanambal [6] and Janaki [7] introduce and study the πgβ-closed sets. The aim of this paper, is to introduce and study the concepts of πg*β-closed sets [8-10], πg*β-open sets in topological spaces and obtain some of their properties [11-15]. Also, we introduce πg*β-neighbourhood (briefly πg*β-nbhd) in topological spaces by using the notion of πg*β-open sets. Further we have prove that every nbhd of x in X is g*β-nbhd of x but not conversely [16-20].

Preliminaries

Let us recall the following definitions which we shall require in sequal.

Definition

A subset A of a topological space (X, τ) is called

1. A pre-open set [16] if A⊆int(cl(A)) and a pre-closed set if cl(int(A)) ⊆A.
2. A semi-open set [9] if A⊆ cl(int(A))and a semi-closed set if int(cl(A)) ⊆A.
3. An-open set [11] if A⊆ int(cl(int(A))) and an-closed set if cl(int(cl(A))) ⊆A.
4. A semi-pre open set(β-open) [1] if A⊆cl(int(cl(A))) and a semi-pre closed set(=β-closed) if int(cl(int(A))) ⊆A.
5. A regular open set [17] if A=int(cl(A)) and a regular closed set if A=cl(int(A)).
6. π-closed [20] if A is the union of regular closed sets.

The intersection of all semi-closed (resp.pre-closed, semi-preclosed, regular-closed and-closed) sets containing a subset A of (X,τ) is called the semi-closure (resp.pre-closure, semi-pre-closure, regular-closure and α-closure) of A and is denoted by scl(A) (resp. pcl(A),spcl(A), rcl(A) and cl(A)).

Definition

A subset A of a topological space(X,τ) is called

1. A regular generalized closed set (briefly rg-closed) [13] if cl(A)⊆U whenever A⊆U and U is regular open in (X,τ).
2. Aπ generalized closed set (briefly πg-closed) [5] if cl(A) ⊆U whenever A⊆U and U is-open in (X,τ).
3. A π generalized α closed set (briefly πgα-closed) [7] if αcl(A) ⊆U whenever A⊆U and U is π-open in (X,τ).
4. A π generalized regular closed set (briefly πgr-closed) [8] if rcl(A)⊆U whenever A⊆U and U is π-open in (X,τ).
5. A generalized preclosed set (briefly πgp-closed) [14] if pcl(A)⊆U whenever A⊆U and U is-open in (X,τ).
6. A π generalized semi-closed set(briefly πgs-closed) [3] if scl(A)⊆U whenever A⊆U and U is π-open in (X,τ).
7. A π generalized β closed set(briefly πgβ-closed) [15] if β cl(A)⊆U whenever A⊆U and U is π-open in (X,τ).
8. A generalized preregular closed set(briefly gpr-closed) [6] if pcl(A)⊆U whenever A⊆U and U is regular open in (X,τ).
9. A αgeneralized regular closed set(briefly αgr-closed) [19] if αcl(A)⊆U whenever A⊆U and U is regular open in (X,τ).
10. A regular generalized β closed set(briefly rgβ-closed)[15] if βcl(A)⊆U whenever A⊆U and U is regular open in (X,τ).
11. A regular w generalized closed set(briefly rwg-closed)[11] if cl(int(A))⊆U whenever A⊆U and U is regular open in (X,τ).

πg*β-Closed Sets

In this section, we introduce a new class of sets called πg*β-open sets, πg*β-closed sets and study some of its properties.

Definition

A subset A of a topological space (X,τ) is called πg*β-closed set if βcl(A)⊆U whenever A⊆U and U is πg-open in (X,τ).

***Corresponding author:** Vani R, Department of Arts and Science, Faculty of Arts and Science, PSG College of Arts and Science, Avinashi Road, Civil Aerodrome Post, Coimbatore, India, Tel: 0422 430 3300; E-mail: vanisrivatsan@gmail.com

Received May 30, 2017; **Accepted** July 18, 2018; **Published** July 23, 2018

Citation: Devika A, Vani R (2018) On πg*β-Closed Sets in Topological Spaces. J Appl Computat Math 7: 413. doi: 10.4172/2168-9679.1000413

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Theorem

Every r-closed set is $\pi g^*\beta$-closed.

Proof: Let A be r-closed set in X. Let U be a πg-open set such that A⊆U. Since A is r-closed, we have rcl(A)=A⊆U. But, βcl(A)⊆rcl(A)⊆U. Therefore βcl(A)⊆U. Hence A is a $\pi g^*\beta$-closed set in X.

Remark: The converse of the above theorem is not true as seen from the following example.

Example: Let X={a; b; c} and τ ={θ,{a}},{a,b},{{a,c},X}.Let $\pi g^*\beta$-closed set={θ,{b},{c},{{b,c},X} and γ-closed set={θ,X}.Let A={b}. Then the subset A is$\pi g^*\beta$-closed but not a γ-closed set.

Remark: The following diagram shows the relationship of πg^*s-closed set with other known existing sets (Figure 1).

Example: Let X={a,b,c} with τ={θ,{b},{c},{b,c},X}. Then πg*β-closed set={θ,{a},{b},{c},{a,c},{a,b},X},πg-closed, πgα-closed and πgγ-closed={θ,γ-closed, γg-closed, πgp-closed, πgs-closed and πgs-closed and πgβ-closed set={θ,{a},{b},{c},{a,b},{b,c},{a,c},X}. Let A={a}. Then the subset A is gs-closed, sg-closed, gp-closed, gsp-closed, gr-closed, gp-closed, gs-closed and πgβ-closed set but not πg^*s-closed set.

Example: Let X={a,b,c} with τ={θ,{a},{b},{a,b}X}.Then πg^*s-closed set={θ,{a},{b},{c},{a,c},{b,c},X}, π-closed, γg-closed, αg-closed, πg-closed, πgα-closed={θ,{c},{b,c},{a,c}X} and γwg={θ,{a,b},b,c},{a,c} X}. Let A={a}. Then the subset Ais πg^*s-closed but not π-closed, γ-closed,αg-closed,πgα-closed and γwg-closed set.

Theorem

Union of two $\pi g^*\beta$-closed subset is $\pi g^*\beta$ closed.

Proof: Let A and B be any two $\pi g^*\beta$-closed sets in X such that A⊆U and B⊆U where U isπg-open in X and so A∪B⊆U. Since A and B are $\pi g^*\beta$-closed. A⊆βcl(A) and B⊆βcl(A) and hence A∪B⊆βcl(A)∪βcl(B)⊆βcl(A∪B). Thus, A∪B is πg*β-closed set in (X,τ).

Example: Let X={a;b;c} and τ={θ,{c},{a,c}{b,c},X}. Let A={a} and B={b} then A∪B ={a}∪{b}={a;b} is $\pi g^*\beta$ closed set.

Theorem

Intersection of two $\pi g^*\beta$-closed subset is $\pi g^*\beta$ closed.

Proof: Let A and B be any two $\pi g^*\beta$-closed sets in X such that A⊆U and B⊆U where U is πg-open in X and so A∩B⊆U. Since A and B are $\pi g^*\beta$-closed. A⊆cl(A) and B⊆cl(A) and hence A∩B⊆βcl(A)∩βcl(B)⊆βcl(A∩B). Thus, A∩B is $\pi g^*\beta$-closed set in (X,τ).

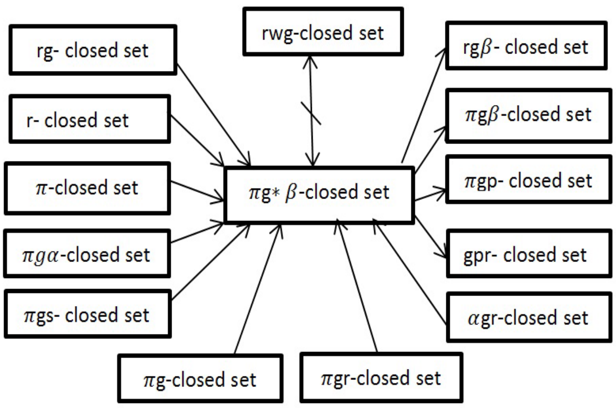

Figure 1: A→B represents A implies B but not conversely.

Example: Let X={a;b;c} and τ={θ,{a},{c},{a,c},X} . Let A={a;b}and B={b;c} then A∩B ={a;b}∩{b} is a $\pi g^*\beta$ closed set.

Theorem

A subset A of X is $\pi g^*\beta$-closed if f βcl(A)-A contains no non-empty closed set in X.

Proof: Let A be a $\pi g^*\beta$-closed set. Suppose F is a non-empty closed set such that F⊆βcl(A)-A. Then F ⊆βcl(A)∩A^c, since βcl(A)-A= βcl(A)∩A^c. Therefore F⊆βcl(A) and F⊆A^c. Since F⊆A^c is open, it is πg-open. Now, by the definition πg^*-closed set, βcl(A)⊆ F^c, That is F⊆$[\beta cl(A)]^c$. Hence F⊆cl(A)∩$[\beta cl(A)]^c$=θ. That is F=θ, which is a contradiction. Thus, βcl(A)-A contains no non-empty closed set in X.

Conversely, assume that βcl(A)-A contains no non-empty closed set. Let A⊆U, where U is πg-open. Suppose that βcl(A) is not contained in U, then βcl(A)∩U^c is a non-empty closed subset of βcl(A)-A, which is a contradiction. Therefore βcl(A)⊆U and hence A is πg*β-closed.

Theorem

For any element x∈X. The set X is $\pi g^*\beta$ closed set or πg-open.

Proof: Suppose X{x} is not πg-open, then X is the only πg-open set containing X{x}.This implies βclX{x}⊂X. Hence X{x} is $\pi g^*\beta$ closed or πg-open set in X.

Theorem

If A is an $\pi g^*\beta$ closed subset of X such that A⊂ B⊂ βcl(A) then B is an $\pi g^*\beta$ closed set in X.

Proof: Let A be an $\pi g^*\beta$ closed set of X such that A⊂ B⊂ βcl(A). Let U be a πg-open set of X such that B⊂U, then A⊂U. Since A is $\pi g^*\beta$-closed, we have βcl(A)⊂U. Now, βcl(B)⊂βcl(β cl(A))⊂U, therefore B is an $\pi g^*\beta$closed set in X.

Definition

A subset A of a topological space (X,τ) is called $\pi g^*\beta$-open set if and only if $A^c$is $\pi g^*\beta$-closed in (X,τ).

Theorem

Let A⊂X is $\pi g^*\beta$-open if and only if F⊂int(A),where F is πg-open and F⊆A.

Proof: Let A be a $\pi g^*\beta$-open set in X. Let F be πg-closed set and F⊂A. Then X-A⊂X-F, where X-F is πg-open, since X-A is $\pi g^*\beta$ closed, βcl(X-A)⊂X-F. Therefore βcl(X-F)=X-int(A)⊂X-int(A)⊂X-F, i.e.,) F⊂ int(A). Conversely, suppose F is πg-closed and F⊂A implies F⊂int(A). Let X-A⊂U, where U is πg-open. Then X-U⊂A, where X-U is πg-closed, By hypothesis, X-U ⊂int(A), i.e.,) X-int(A)⊂U since βcl(X-A)=X-int(A), βcl(A) U, where U is πg-open this implies X-A is $\pi g^*\beta$-closed and hence A is $\pi g^*\beta$-open.

Theorem

If int(A)⊂B⊂A and A is $\pi g^*\beta$-open then B is also $\pi g^*\beta$ open.

Proof: We know that if A is $\pi g^*\beta$-closed and A⊂B⊂βcl(A) then B is also $\pi g^*\beta$-closed. Here X-A is $\pi g^*\beta$-closed, then X-B is also $\pi g^*\beta$-closed. Hence B is $\pi g^*\beta$-open.

Theorem

If A⊂X is $\pi g^*\beta$-closed then βcl(A)-A is πg-open.

Proof: Let A be a $\pi g^*\beta$-closed set in X. Let F be a πg-closed set such that F⊂βcl(A)-A. Then βcl(A)-A does not contain any non-empty πg-closed set. Therefore F=θ, so F⊂int(βcl(A)-A). This shows βcl(A)-A is πg-open. Hence A is $\pi g^*\beta$-closed in (X,τ).

Theorem

If int(B)⊆B⊆A and if A is $\pi g^*\beta$-open in X, then B is $\pi g^*\beta$-open in X.

Proof: Suppose that int(B)⊆B⊆A and A is $\pi g^*\beta$-open in X then $A^c \subseteq B^c \subseteq cl(A^c)$. Since A^c is $\pi g^*\beta$-closed in X. we have B is $\pi g^*\beta$-open in X.

Theorem

If A is γwg-open and $\pi g^*\beta$-closed then A is πg-closed.

Proof: Let A be a γwg-open and πg-closed set in X. Let A⊂A where A is wg-open. Since A is $\pi g^*\beta$ closed; βcl(A)⊂A whenever A⊂A and A is wg-open. the implies βcl(A)=γwg. Hence A is πg-closed.

Theorem

If A is wg-open and $\pi g^*\beta$-closed then A is πg-closed.

Proof: Let A be a wg-open and $\pi g^*\beta$-closed set in X. Let A⊂A where A is wg-open. Since A is $\pi g^*\beta$-closed; βcl(A)⊂A whenever A⊂A and A is wg-open. the implies βcl(A)=γwg. Hence A is πg-closed.

g-Neighbourhoods

Definition

Let X,τ) be a topological space and let x∈X, A subset N of X is said be $\pi g^*\beta$-neighbourhood of x if there exists an $\pi g^*\beta$-open set G such that x ∈G⊆N. The collection of all $\pi g^*\beta$-neighbourhood of x∈X is called $\pi g^*\beta$ neighbourhood system at x shall be denoted by $\pi g^*\beta$-N(X).

Theorem

Every neighbourhood N of x∈X is $\pi g^*\beta$-neighbourhood of X.

Proof: Let N be a neighbourhood of point x ∈X, To prove that N is a $\pi g^*\beta$-neighbourhood of x by definition of neighbourhood, there exists an open set G, such that x ∈G⊆ N. Hence N is $\pi g^*\beta$-neighbourhood of X.

Remark: In general, a $\pi g^*\beta$-neighbourhood N of x∈X need not be a nbhd of x in X as seen from the following example.

Example: Let X={a;b;c} with topology τ={θ ;X; {a};{a;c}}. Then $\pi g^*\beta$-o(X)={θ;X;{b}; {c}; {b; c}}. The set {a;b} is $\pi g^*\beta$-nbhd of point b, since the $\pi g^*\beta$-open set {b} is such that b∈{b}⊂{a;b}. However the set {a; b} is not a nbhd of the point b, since no open set G exists such that b∈G ⊂{a;b}.

Theorem

If a subset N of a space X is $\pi g^*\beta$-open, then N is $\pi g^*\beta$-nbhd of each of its points.

Proof: Suppose N is $\pi g^*\beta$-open. Let x∈N. We claim that N is $\pi g^*\beta$-nbhd of x. For N is a $\pi g^*\beta$-open set such that x∈N ⊆N. Since x is an arbitrary point of N, it follows that N is a $\pi g^*\beta$-nbhd of each of its points.

Theorem

Let X be a topological space. If F is a $\pi g^*\beta$-closed subset of X, and x∈F^c: P rove that there exists a $\pi g^*\beta$-nbhd N of x such that N∩F=θ .

Proof: Let F be $\pi g^*\beta$-closed subset of X and x∈F^c: T hen F^c is $\pi g^*\beta$-open set of X. So by Theorem 4.5 F^c contains a $\pi g^*\beta$-nbhd of each of its points. Hence there exists a $\pi g^*\beta$-nbhd N of x such that N⊂F^c: That is N∩F=θ.

$\pi g^*\beta$-Interior

Definition

Let A be a subset of X. A point x∈X is said to be $\pi g^*\beta$-interior point of A if A is a $\pi g^*\beta$-nbhd of x. The set of all $\pi g^*\beta$-interior points of A is called the $\pi g^*\beta$-interior of A and is denoted by $\pi g^*\beta$-int(A).

Theorem

If A be a subset of X. Then $\pi g^*\beta$-int(A)=∪{G:G is $\pi g^*\beta$-open,G⊆A}.

Proof : Let A be a subset of X:x∈ $\pi g^*\beta$-int(A)⇔x is a $\pi g^*\beta$-interior point of A.

- A is a $\pi g^*\beta$-nbhd of point x.
- There exists $\pi g^*\beta$-open set G such that x ∈ G⊆A.
- x ∈∪{G:G is $\pi g^*\beta$-open, G⊆A.

Hence $\pi g^*\beta$-int(A)=∪{G: G is $\pi g^*\beta$-open, G⊆A}:

Theorem

Let A and B be subsets of X. Then

1. $\pi g^*\beta$-int(X)=X and $\pi g^*\beta$-int(θ)=θ .
2. $\pi g^*\beta$-int(A)⊆A.
3. If B is any $\pi g^*\beta$-open set contained in A, then B⊆$\pi g^*\beta$-int(A).
4. If A⊆B, then $\pi g^*\beta$-int(A)⊆ $\pi g^*\beta$-int(B).
5. $\pi g^*\beta$-int(π $\pi g^*\beta$-int(A))=π $\pi g^*\beta$-int(A).

Proof: 1. Since X and θ are $\pi g^*\beta$-open sets, by Theorem $\pi g^*\beta$-int(X)= ∪{G:G is $\pi g^*\beta$-open, G⊆X}=X∪{all fall $\pi g^*\beta$-open sets}=X. That is $\pi g^*\beta$-int(A)=X. Since θ is the only $\pi g^*\beta$-open set contained in θ, $\pi g^*\beta$-int(θ)=θ .

2. Let x∈ $\pi g^*\beta$-int(A))⇒x is a $\pi g^*\beta$-interior point of A.

- A is a $\pi g^*\beta$-nbhd of x.
- X∈A. Thus x ∈ $\pi g^*\beta$-int(A))⇒ x ∈A. Hence $\pi g^*\beta$-int(A) A.

3. Let B be any $\pi g^*\beta$-open sets such that B⊂A. Let x ∈B, then since B is a $\pi g^*\beta$-open set contained in A. x is a $\pi g^*\beta$-interior point of A. That is x∈ $\pi g^*\beta$-int(A). Hence B⊆$\pi g^*\beta$-int(A).

4. Let A and B be subsets of X such that A⊆B. Let x∈ $\pi g^*\beta$-int(A). then x is a $\pi g^*\beta$-interior point of A and so A is $\pi g^*\beta$-nbhd of x. Since B⊃A, B is also a $\pi g^*\beta$-nbhd of x. This implies that x ∈ $\pi g^*\beta$-int(B). Thus we have shown that x ∈ $\pi g^*\beta$-int(A)) x∈ $\pi g^*\beta$-int(B). Hence $\pi g^*\beta$-int(A)⊂ $\pi g^*\beta$-int(B).

5. From (2) and (4) $\pi g^*\beta$-int($\pi g^*\beta$-int(A)) ⊆$\pi g^*\beta$-int(A). Let x ∈ $\pi g^*\beta$-int(A) this implies A is a neighbourhood of x, so there exists a $\pi g^*\beta$-open set G such that x ∈G⊆A. so every element of G is an $\pi g^*\beta$-interior of A, hence x∈G⊆$\pi g^*\beta$-int(A) which means that x is an $\pi g^*\beta$-interior point of $\pi g^*\beta$-int(A) that is $\pi g^*\beta$-int(A)⊆ $\pi g^*\beta$-int($\pi g^*\beta$-int(A)). That is $\pi g^*\beta$-int($\pi g^*\beta$-int(A))= $\pi g^*\beta$-int(A). Let A be any subset of X. By the definition of $\pi g^*\beta$-interior $\pi g^*\beta$-int(A)⊂A, by $\pi g^*\beta$-int($\pi g^*\beta$-int(A)) ⊂ $\pi g^*\beta$-int(A). Hence $\pi g^*\beta$-int($\pi g^*\beta$-int(A))⊂∩{F:A⊂F∈ $\pi g^*\beta$-C(X)}=$\pi g^*\beta$-cl(A).

Theorem

If a subset A of space X is $\pi g^*\beta$-open, then $\pi g^*\beta$-int(A)=A.

Proof: Let A be $\pi g^*\beta$-open subset of X. $\pi g^*\beta$-int(A)⊂A. Also, A is $\pi g^*\beta$-open set contained in A. From (3) A⊂$\pi g^*\beta$-int(A). Hence $\pi g^*\beta$-int(A)=A.

Remark: The converse of the above theorem need not be true, as seen from the following example.

Example: Let X={a;b;c} with topology τ={θ, {c}; {b;c}; X}. $\pi g^*\beta$-closed set is {θ, {a}, {b},{a;b}, X}. $\pi g^*\beta$-O(X) is $\pi g^*\beta$-open sets in X={θ, {c}; {b;c}; {a;c}; X} $\pi g^*\beta$-int(A)=$\pi g^*\beta$-int({a;b})={a}∪{b}={a;b}; but {a;b} is not g-open set.

Theorem

If A and B are subsets of X, then $\pi g^*\beta$-int(A)∪ $\pi g^*\beta$-int(B)⊂ $\pi g^*\beta$-int(A∪B).

Proof: Theorem $\pi g^*\beta$-int(A)⊂ $\pi g^*\beta$-int(A∪B) and $\pi g^*\beta$-int(B)⊂ $\pi g^*\beta$-int(A∪B). This implies that $\pi g^*\beta$-int(A)∪ $\pi g^*\beta$-(B)⊂ $\pi g^*\beta$-int(A∪B).

g-Closure in a Space X

Definition

Let A be a subset of a space X. The $\pi g^*\beta$-closure of A is de ned as the intersection of all $\pi g^*\beta$-closed sets containing A. $\pi g^*\beta$-cl(A)=∩{F:A⊂F∈ $\pi g^*\beta$C(X)}.

Theorem

If A and B are subsets of a space X. Then

(1) $\pi g^*\beta$-cl(X)=X and $\pi g^*\beta$-cl(θ)=θ .

(2) A⊂$\pi g^*\beta$-cl(A).

(3) If B is any $\pi g^*\beta$-closed set containing A, then $\pi g^*\beta$-cl(A)⊂B.

(4) If A⊂B, then $\pi g^*\beta$-cl(A)⊂ $\pi g^*\beta$-cl(B).

(5) $\pi g^*\beta$-cl(A)= $\pi g^*\beta$-cl($\pi g^*\beta$-cl(A)).

Proof: (1) By the definition of $\pi g^*\beta$-closure, X is the only $\pi g^*\beta$-closed set containing X. Therefore $\pi g^*\beta$-cl(X)=Intersection of all the $\pi g^*\beta$-closed sets containing X=∩{X}=X: That is $\pi g^*\beta$-cl(X)=X. By the definition of $\pi g^*\beta$-closure, $\pi g^*\beta$-cl(θ)=Intersection of all the $\pi g^*\beta$ closed sets containing θ=∩ any $\pi g^*\beta$-closed sets containing θ=θ . That is $\pi g^*\beta$-cl(θ)=θ .

2. By the definition of $\pi g^*\beta$-closure of A, it is obvious that A⊂$\pi g^*\beta$-cl(A).

3. Let B be any $\pi g^*\beta$-closed set containing A. Since $\pi g^*\beta$-cl(A) is the intersection of all g-closed sets containing A, $\pi g^*\beta$-cl(A) is contained in every $\pi g^*\beta$-closed set containing A. Hence in particular $\pi g^*\beta$-cl(A)⊂B.

4. Let A and B be subsets of X such that A⊆B. By the definition of $\pi g^*\beta$-closure, $\pi g^*\beta$-cl(B)=∩{F: B ⊂∈$\pi g^*\beta$C(X)g. If B ⊂F ∈ $\pi g^*\beta$ C(X), then $\pi g^*\beta$-cl(B) F. Since A⊂B, A⊂B ⊂F∈ $\pi g^*\beta$ C(X), $\pi g^*\beta$-cl(A)⊂F. Therefore $\pi g^*\beta$-cl(A) ⊂∩{F: B⊂F∈ $\pi g^*\beta$ C(X)}= $\pi g^*\beta$-cl(B). That is $\pi g^*\beta$-cl(A)⊂ $\pi g^*\beta$-cl(B).

5. Let A be any subset of X. By the definition of $\pi g^*\beta$-closure, $\pi g^*\beta$-cl(A)= ∩{F: A⊂ F∈ $\pi g^*\beta$ C(X)g, If A ⊂F ∈ $\pi g^*\beta$ C(X), then $\pi g^*\beta$-cl(A) F. Since F is $\pi g^*\beta$-closed set containing $\pi g^*\beta$-cl(A), by (3) $\pi g^*\beta$-cl($\pi g^*\beta$-cl(A))⊂F. Hence $\pi g^*\beta$-cl($\pi g^*\beta$-cl(A)) ⊂∩{F: A⊂ F∈ $\pi g^*\beta$ C(X)}= $\pi g^*\beta$-cl(A). that is $\pi g^*\beta$-cl($\pi g^*\beta$-cl(A))=(A).

Theorem

If A⊂X is $\pi g^*\beta$-closed, then $\pi g^*\beta$-cl(A)=A.

Proof: Let A be $\pi g^*\beta$-closed subset of X. By the definition of $\pi g^*\beta$-cl(A), A⊂$\pi g^*\beta$-cl(A). Also A⊂A and A is $\pi g^*\beta$-closed. By Theorem $\pi g^*\beta$-cl(A)⊂A. Hence $\pi g^*\beta$-cl(A)=A.

Remark: The converse of the above theorem need not be true as seen from the following example.

Example: Let X={a;b;c} with topology τ={θ, {a}, {a;b}, {a;c}, X}. $\pi g^*\beta$-closed set is {θ, {b}, {c}, {b; c}, X} and $\pi g^*\beta$-O(X)= $\pi g^*\beta$-open sets in X={θ, {a} ; {a; b} ; {a; c} ; X}. $\pi g^*\beta$-cl(A)=$\pi g^*\beta$-cl({a}={a; b}∩{a; c}={a} but {a} is not $\pi g^*\beta$-closed set.

Theorem

If A and B are subsets of a space X, Then $\pi g^*\beta$-cl(A∩B)⊂ $\pi g^*\beta$-cl(A)∩ $\pi g^*\beta$-cl(B).

Proof: Let A and B be subsets of X. clearly A∩B⊂C and A∩B⊂B. by Theorem, $\pi g^*\beta$-cl(A∩B)⊂$\pi g^*\beta$-cl(A) and $\pi g^*\beta$-cl(A∩B)⊂$\pi g^*\beta$-cl(B). Hence $\pi g^*\beta$-cl(A∩B)⊂ $\pi g^*\beta$-cl(A)∩ $\pi g^*\beta$-cl(B).

Conclusion

This paper is to introduced and study the concepts of $\pi g^*\beta$-closed sets and $\pi g^*\beta$-neighbour hood in topological spaces. We had proved that the defined set was properly contains πgβ-closed and contained in πg-closed set. Further the defined set satisfies the union and intersection property. Hence we conclude that the defined set forms a topology which results this work may be extend widely.

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