Journal of Generalized Lie Theory and Applications
ISSN: 1736-4337
Azzam et al., J Generalized Lie Theory Appl 2018, 12:1
DOI: 10.4172/1736-4337.1000291
Research Article Open Access

On Finite and Infinite Principal Door and Connected-Topologies

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Abstract

A space X which carries topology τ is a door space if each subset of X is either open or closed. In this paper a characterization of the principle door and a formula for the number of the door topologies on a set X_n of n points are given. Some properties of the principal connected topologies on non-empty set X are discussed and the minimal τ_0-topologies on X are also characterized. Finally a few results about the number of the chain topologies on X_n are proved.

Keywords: Prfincfipal topologfical spaces; Door; Connected; Mfinfimal τ_0; Chain topologies

AMS mathematics subject classification: 54D10, 54D15, 54C08 and 54C10.

Introduction

Frohlich [1] defined the principal ultratopology on a set X to be the topology on X which is strictly weaker than the discrete topology D on X and which is of the form $D_{yz}=E_z\cup U(y)$, where $U(y)$ is the principal ultra filter generated by $\{y\}$ and E_z is the excluding point topology on X with the excluding point z. In fact $D_{yz}=E_z\cup P_y$, where P_y is the particular point topology on X with the particular point y. Then, D_{yz} is the principal ultratopology on X in which each open set containing z contains y. Steiner [2] defined the minimal open set at a point $x\in X$ in a space X which carries topology τ to be the open set $U_x\in\tau$ such that $x\in U_x$ and is contained in each open set containing x. Steiner also defined the principal topology τ on X to be the topology with the minimal basis consists only of minimal open sets at the points of X, proved that τ is the principal if and only if arbitrary intersections of open sets are open and characterized the door topologies on X. In Farrag and Sewisy [3,4] and Farag and Abbas [5] described algorithms for construction and enumeration all strictly weaker topologies than a given topology on a set X_n of n points, all topologies and all hyperconnected, all door, connected and regular topologies on X_n.

Door Principal Topologies

Let X be a space which carries topology τ, Q and S be two any properties of topologies on X. Then, τ, is:

(1) An E-topology on X if $\cup\{G\in:GX\}\neq X$ [4].

(2) A P-topology on X if $\cap\{G\in:G\phi\}\neq\phi$ [4].

(3) An h-topology on X if $G\cap H\phi$ for any $G,H\in\tau\backslash\{\phi\}$. This is the irreducible [6] as well as is the hyperconnected [2]. If τ is finite then, h and P are equivalent but in general this is not true. For let, $\tau=\{N,\phi,N\setminus\{1,2,3,...,n\}:n\in N\}$ where N is the set of the positive integers then, (N,τ) is h but not P.

(4) An E^*-topology on X if there is a point $p\in X$ such that $E_p\tau$ where, E_p is the excluding point topology in X.

(5) An P^*-topology on X if there is a point $p\in X$ such that $P_p\tau$, where P_p is the particular point topology on X with the particular point $p\in X$.

(6) An $S(k)$-topology on X if there are k singleton members of τ.

(7) A $Q\vee S$-topology if it is Q or S.

(8) A $Q\wedge S$ -topology if it is both Q and S.

(9) A $Q\backslash S$ -topology if it is Q and not S.

Throughout this paper $|U|$ denote the cardinality of the set U. A finite set of n points is denoted by X_n and N_n, $N_n(Q)$ denote the number of all topologies and the number of the Q -topologies on X_n. Then:

(1) $N_n(E)=N_n(P)=\sum_{k=0}^{n-1}{}^{n}c_kN_k$ where N_k is the number of all topologies on X_k [4].

(2) $N_n(E^*\wedge E)=N_n(P^*\wedge P)=n$ and $N_n(E^*\wedge P^*)=n(n-1)+1$.

Proposition 2.1

Let X_n be a set of n points then,

$$N_n(E\wedge P)=\sum_{k=0}^{n-1}{}^{n}c_kN_k(P)=\sum_{k=0}^{n-1}{}^{n}c_kN_k(E)\text{ where }N_0(E)=N_0(P)=1.$$

Proof. Let AX_n be such that $|A|=k,1\leq kn$. If τ is an E-topology on $X_n\backslash A$ then $\tau(A)=\{G\cup A:G\in\tau\}\cup\{\phi\}$ is a nondiscrete EP -topology on X_n. If $A=X_n$ then $\{\phi\}$ is not a topology on X_n while $\tau(A)=\{X_n\cup\phi\}\cup\{\phi\}=\{X_n,\phi\}$. Clearly there are ${}^{n}c_k$ nonempty subsets of X_n with the cardinality k and so,

$$N_n(E\wedge P)=\sum_{k=1}^{n}{}^{n}c_kN_{n-k}(E)=\sum_{k=0}^{n-1}{}^{n}c_kN_k(E).$$

Secondly; if τ is a p-topology on $X_n\backslash A$ then $\tau(A)=\{X_n,G:G\in\tau\}$ is an $E\wedge P$-topology on X_n. If $A=X_n$ then $\{\phi\}$ is not a topology on X_n while $\tau(A)=\{X_n,\phi\}$. Similarly,

$$N_n(E\wedge P)=\sum_{k=1}^{n}{}^{n}c_kN_{n-k}(P)=\sum_{k=0}^{n-1}{}^{n}c_kN_k(P)$$

Example 2.2: By using Example 3 [4] then,

$$N_5(E\wedge P)-N_5(E\setminus P)=2111-1190=921=\sum_{k=0}^{4}{}^{5}c_kN_k(E).$$

Remark 2.3: An E^*-nondiscrete topology τ on a nonempty set X may be principal or nonprincipal. For, if X is an infinite set then, $\tau=\{G\subset X:p\notin G$ or $X\backslash G$ is finite$\}$ is a nonprincipal E^*-topology on X.

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Received May 21, 2018; **Accepted** May 30, 2018; **Published** June 11, 2018

Citation: Azzam AA, Farrag AS, EL-Sanousy E (2018) On Finite and Infinite Principal Door and Connected-Topologies. J Generalized Lie Theory Appl 12: 291. doi: 10.4172/1736-4337.1000291

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Remark 2.4: A principal $E^*\backslash E$-topological space is not connected.

Remark 2.5: The P^*-topologies on a set X are only principal since there is a point $p \in X$ such that $\{x\}^\wedge = \{p,x\} \in \tau$ and either $\{x\}^\wedge = \{p,x\} \in \tau$ or $\{x\}^\wedge = \{x\} \in \tau$.

Remark 2.6: A principal topology τ on a set X is E^* if and only if τ_c is P^* where, $\tau_c = \{X \setminus G : G \in \tau\}$.

Remark 2.7: If β is the minimal basis for a P^*-topology on a non-empty set X. Then, $|U|$ is 1 or 2 for each $U \in \beta$ and there is at least a point $p \in X$ such that $\cap\{U \in \beta : |U| = 2\} = \{p\} \in \tau$. So, a $P^*\backslash P$-topological space (X,τ) is not connected because $G = \cup\{U \in \beta : |U| = 2\}$ and $X \setminus G = \cup\{U \in \beta : P \notin U\}$ are two nonempty members of τ.

Theorem 2.8: A principal topological space (X,τ) is door if and only if it is E^*P^*.

Proof. Clearly if τ is an E^*P^*-topology on a set X then it is door. Conversely; let (X,τ) be a door principal nonultra topological space and U_x be the minimal open set at the point x for each $x \in X$. Then, $y \in U_x\backslash\{x\}$ and $X\backslash\{y\} \in$ implies that $U_x \subset X \setminus \{y\}$ which implies that $y \in X \setminus \{y\}$ which is impossible. Hence, $X \setminus \{y\} \notin \tau$ which implies that $\{y\} \in \tau$ because (X,τ) is a door space. Let $p \in X$ be such that $|U_p| \geq 3$ then, $x \in X\backslash\{p\}$ and $|U_x| \geq 2$ implies that there is a point $t \in U_x \setminus \{x\}$. Hence $\{p,t\} \notin \tau$ because $U_p \not\subseteq \{p,t\}$ because $|U_p| \geq 3$. If $X \setminus \{p,t\} \in \tau$ then $U_x \subset X \setminus \{p,t\}$ which implies that $t \in X \setminus \{p,t\}$ which is impossible. Hence $X \setminus \{p,t\} \notin \tau$ which contradicts the assumption that (X,τ) is door. Therefore, $U_x=\{x\}$ for each $x \in X \setminus \{p\}$ which implies that (X,τ) is E^*. If $x \in X$ such that $|U_x|=2$, then, there is a point $p \in U_x \setminus \{x\}$ such that $\{p\} \in \tau$. If $x,y \in X$ are such that $U_x \neq U_y$ and $|U_x| = |U_y| = 2$. Then, there are two points $q \in U_x \setminus \{x\}$ and $U_x \not\subseteq \{x,r\}$. If $q \neq r$ then $\{x,r\} \notin \tau$ because $U_x \not\subseteq \{x,r\}$ and $X \setminus \{x,r\} \in \tau$ implies that $U_y \subset X \setminus \{x,r\}$ implies that $r \in X \setminus \{x,r\}$ which is impossible. Hence, $X \setminus \{x,r\} \notin \tau$ which contradicts the assumption that (X,τ) is door. This contradiction implies that $q=r=p$. So, $|U_x|$ is either 1 or 2 for each $x \in X$ and such that $|U_x| = |U_y| = 2$ and $U_x \neq U_y$ implies that $U_x \cap U_y = \{p\}$ this is if and only if $\cap\{U_x \in \beta : |U_x| = 2\} = \{p\}$. So, (X,τ) is P^*.

Corollary 2.9: A principal door topological space (X,τ) is connected if and only if it is $E_p \vee P_p$ where $p \in X$.

Remark 2.10: In previous study [7] proved that a door topological space (X,τ) is T_0.

Clearly both the principal E^* and P^*-topological spaces are T_0. It is T_4 if τ is nonprincipal E^* in which there is a point $p \in X$ such that $\{p\}^\wedge \notin \tau$ and $\{p\}^\wedge = \{p\}$.

Theorem 2.11: Let X_n be a set of n points then, $N_n(E^*) = N_n(P^*) = n(2^{n-1} - 1) + 1$,

Proof. Let, β be the minimal basis for a nondiscrete E^*-topology on X_n. Then, τ is $S(n-1)$ and so there is a point $p \in X_n$ such that $\{p\} \notin \beta$ and the member $U \in \beta$ which is the minimal open set at the point p is such that $|U| \geq 2$. If $|U| = k \geq 2$, then U can be the minimal open set at each of its points i.e. p can be any point of U. Accordingly, we may have k distinct minimal bases β's for E^*-topologies on X_n. Since the number of such subsets U's of X_n is ${}^n c_k$ then the number of the corresponding distinct minimal bases for E^*-topologies on X_n is $k\ {}^n c_k$. Therefore, $2 \leq k \leq n$ implies that: $N_n(E^*) = \sum_{k=2}^{n} k\ {}^n c_k + 1 = n(2^{n-1} - 1) + 1$.

Secondly; let β be the minimal basis for a nondiscrete P^*-topology on X_n. Then there is a point $p \in X_n$ such that $\{p\} \in \beta$ and $|U|$ is 1 or 2 for each $U \in \beta$ such that $\cap\{U \in \beta : |U| = 2\} = \{p\}$. If $T = \{U \in \beta : |U| = 2\}$ and $|T| = k - 1 \geq 1$ then $T \cup \{\{p\}\} = \{\{p\},\{p,x_1\},\{p,x_2\},\ldots,\{p,x_{k-1}\}\} \subset \beta$. Clearly any of the points $x_1, x_2, \ldots, x_{k-1}$ can take the position of the point p and we may have k distinct minimal bases for P^*-topology on X_n. Since the number of the subsets $\{p, x_1, x_2, \ldots, x_{k-1}\}$ of X_n is ${}^n c_k$, then the number of the corresponding minimal bases for P^*-topology on X_n is $k\ {}^n c_k$. Therefore, $2 \leq k \leq n$ implies that:

$$N_n(P^*) = \sum_{k=2}^{n} k\ {}^n c_k + 1 = n(2^{n-1} - 1) + 1.$$

Clearly, $N_n(E^*) = N_n(P^*)$ and as a direct consequence of Theorems (2.7) and (2.10) we have Theorem (2.12).

Theorem 2.12: The number of all door topologies on X_n is:

$$N_n(DO) = n(2^n - n - 1) + 1.$$

Proof. $N_n(DO) = N_n(E^* \vee P^*) = N_n(E^*) + N_n(P^*) - N_n(E^* \wedge P^*)$.

Connected Principal-Topology

Let (X,τ) be a principle topological space, β be the minimal basis for τ and let $T \subset \beta$ be such that $\cap\{B : B \in T\} \neq \phi$ and such that $H \in \beta\backslash T$ implies that $H \cap [\cap\{B : B \in T\}] = \phi$. If $\cap\{B : B \in T\} = V$ then $V \in \tau$ is a minimal open set at each of its points since $x \in V$ implies that $U_x \in T$ and if $G \in \tau$ such that $x \in G$, then $V \subset U_x \subset G$. The family $\{V_\lambda : \lambda \in \Delta\} \subset \tau$ of such minimal open sets in non-empty and is a pair wise disjoints family of members of τ. Clearly,

(1) if $\cap\{G : G \in T\} = \phi$ for each $T \subset \beta$ then each member of β is minimal at each of its points and by (X,τ) is regular [5].

(2) if $\cap\{B : B \in T\} \neq \phi$ for each $T \subset \beta$ then there exists $\lambda \in \Delta$ such that $\cap\{B \in \beta : B \neq \phi\} = V$ which is the unique minimal open set at each of its points and (X,τ) is P. If $A_\lambda = \cup\{U_x : V_\lambda \subset U_x\}$. Then, $\cap\{U : U \in T\} \neq \phi$ for, if τ is an E-topology on X then $A_\lambda=X$ for each $\lambda \in \Delta$. Otherwise let, $x \in X$, $T\beta$ be such that $U_x \in T$ and $\cap\{U : U \in T\} \neq \phi$ such that $G \in \beta \setminus T$ implies that $G \cap [\cap\{U : U \in T\}] = \phi$.

If $\cap\{U : U \in T\} = U^*$ then $U^* \subset Ux$ and there is a point $\lambda \in \Delta$ such that $U^*=V_\lambda$ which implies that $x \in A$.

Theorem 3.1: Let τ be a principal topology on a set X, $\{V_\lambda : \lambda \in \Delta\} \subset \tau$ be the family of all open sets each of which is minimal at each of its points and $A_\lambda = \cup\{U_x : V_\lambda \subset U_x\}$ for each $\lambda \in \Delta$. If:

(a) $V \neq X$, then (X,τ) is connected implies that $A \neq V_\lambda$ for any $\lambda \in \Delta$.

(b) (X,τ) is connected then for each $\lambda \in \Delta$ there exists $\mu \in \Delta$ such that $A_\lambda \cap A_\mu \neq \phi$.

(c) $A_\lambda \cap A_\mu \neq \phi$ for each two distinct points $\lambda, \mu \in \Delta$ then (X,τ) is connected but not conversely.

Proof. (a): Let $\lambda \in \Delta$ be any point such that $V \neq X$ and $x \in X$ be any point. Then $t \in V_\lambda \cap U_x$ implies that $U_x \cap V_\lambda \neq \phi$ which implies by the definition of A_λ that $x \in A_\lambda$ and if $A_\lambda=V_\lambda$, then $x \in V_\lambda$ in fact $U_x=V_\lambda$. Therefore $U_x \cap V_\lambda \neq \phi$ implies that $x \in V$ and the contrapositive of this result is $x \in X \setminus V_\lambda$ implies that $U_x \cap V_\lambda = \phi$ implies that $X \setminus V_\lambda \in \tau$ implies that (X,τ) is not connected. Therefore (X,τ) is connected implies that A_λ V_λ.

(b) Let, $\lambda \in \Delta$ be any point such that $A_\lambda \cap A_\mu = \phi$ for each point $\mu \in \Delta$. Then A and $X\backslash A_\lambda$ are open sets which implies that $(X,)$ is not connected. The contrapositive of this result, $(X,)$ is connected implies that for each $\lambda \in \Delta$ there is $\mu \in \Delta$ such that $A_\lambda \cap A_\mu \neq \phi$.

(c) Let (X,τ) be disconnected. Then there is a subset G of X such that $G, X\setminus G \in \tau\setminus\{X,\phi\}$. Then there is a point $x \in G$ which implies that $U_x \subset G$ and $\cup\{A_\lambda : \lambda \in \Delta\} = X$ implies that there is a point $\lambda \in \Delta$ such that $x \in A$ which implies by the definition of A that there is a point $t \in X$ such that $V \subset U_t$ and $x \in U_t$ which implies that $U_x \subset U_t$. If $t \in X \setminus G$ then $U_t \subset X\setminus G$ which implies that $V_\lambda \subset U_t \subset G$, a contradiction. Hence $t \in G$ which implies that $V_\lambda \subset U_t \subset G$. If $y \in A_\lambda \cap (X\setminus G)$ then $y \in A_\lambda$ implies that there is a point $p \in X$ such that $y \in U_p, V_\lambda \subset U_p$ and $p \in X\setminus G$ implies that $V_\lambda \subset U_P \subset X\setminus G$ implies that $V_\lambda \subset G \cap (X\setminus G)$ while $p \in G$ implies that $y \in U_p \subset G$ which implies that $y \in G\cap(X\setminus G)$. Therefore $A_\lambda \cap (X\setminus G) = \phi$ which implies that $A \subset G$. Similarly there is a point $\mu \in \Delta$ such that $A_\mu \subset X\setminus G$. Hence (X,τ) is not connected implies that there are two points $\lambda, \mu \in \Delta$ such that $A_\lambda \cap A_\mu = \phi$ and the contrapositive of this result is if $A_\lambda \cap A_\mu \neq \phi$ for each two distinct points $\lambda, \mu \in \Delta$ then (X,τ) is connected.

Conversely let, $X=\{1,2,3,4,5,6,7\}$ and $\beta=\{\{1\}, \{1,2,3\}, \{3\}, \{3,4,5\}, \{5\}, \{5,6,7,\}, \{7\}\}$ be the minimal basis for a topology τ on X. Then, (X,τ) is connected while $A(\{1\}) = \{1,2,3\}$ and $A(\{7\}) = \{5,6,7\}$ are disjoint.

Remark 3.2: If we denote the connected topologies on a set X by CTD-topologies. Then, the number of the connected topologies on a set X_n of n points is:

$$N_n(CTD) = N_n[(P\vee E)\vee CTD\setminus(P\vee E)] = 2N_n(E)\setminus N_n(P\wedge E) + N_n(CTD\setminus(P\vee E)).$$

Theorem 3.3: A principal T_0-topological space (X,τ) is minimal T_0 if and only if the minimal basis β for τ is totally ordered by the inclusion operator.

Proof. Let, (X,τ) be a T_0-topological space such that the minimal basis β is totally ordered by the inclusion operator and τ^* be a strictly weaker topology on X that τ. Then, by Theorem (2.8) [3] there are two distinct points $y,z \in X$ such that $\tau^* = \tau_{yz} = \tau \cap D_{yz}$ such that $y \notin U_z$. Then, $z \notin U_y$ because β is totally ordered by the inclusion operator which implies that $U_y = U_y \cup U_z$ so, U_y is the minimal open set at both the points y and z in τ_{yz} which implies that $(X,_{yz})$ is not T_0. Therefore, (X,τ) is minimal T_0.

Conversely; let (X,τ) be the minimal T_0-topological space, $y,z \in X$ be two distinct points such that $y \notin U_z$ and $y \notin U_y$. If $\tau_{yz} = \tau \cap D_{yz}$ and $G \in \tau_{yz}$ such that $z \in G$ then,

(1) yU_z implies that $\tau \neq \tau_{yz}$,

(2) $y,z \in U_y \cup U_z$ implies that $U_y \cup U_z \in \tau_{yz}$,

(3) $G \in \tau_{yz}$ implies that $G \in \tau$ and

(4) $z \in G$ implies that $y \in G$ because $G \in D_{yz}$ which implies that $U_y \cup U_z \subset G$. Hence, $U_y \cup U_z \in \tau_{yz}$ is the minimal open set at z. If $x \in X\setminus\{y,z\}$ then $U_x = U_y \cup U_z$ implies that either $U_x = U_y$ or $U_x = U_z$ which contradicts the assumption that $y \notin U_z$ and $z \notin U_y$. Hence, $U_x \neq U_y \cup U_z$ for each $x \in X\setminus\{y,z\}$ and zU_y implies that $U_y \neq U_y \cup U_z$. Then, (X,τ_{yz}) is T_0 which contradicts that (X,τ) is minimal T_0. This contradiction because of the incorrect assumption that $y \notin U_z$ and $z \notin U_y$. Therefore, either $y \in U_z$ or $z \in U_y$ for any two distinct points $y,z \in X$. This completes the proof.

Corollary 3.4: Let, (X,τ) be T_0 then (X,τ_{yz}) is T_0 if and only if $y \notin U_z$ and $z \notin U_y$ for any two distinct points $y,z \in X$ where $\tau \neq \tau_{yz} = \tau \cap D_{yz}$.

Proof. As a direct consequence of the proof of Theorem(3.2) $y \notin U_z$ and $z \notin U_y$ implies that $(X,_{yz})$ is T_0 for any two distinct points $y,z \in X$.

Conversely; if (X,τ_{yz}) is T_0 then, $U_y \neq U_y \cup U_z$ which implies that zU_y and $y \in U_z$ implies that $\tau =_{yz}$ which implies that $y \notin U_z$.

Corollary 3.5: Let, (X_n,τ) be a minimal T_0-topological space. then, there is a point $p \in X$ such that $\tau = \{\phi, \{p\}, \{p,x\}, \{p,x,t\}, ..., X_n\}$. So, the number of the minimal T_0-topologies on X_n is N_n (min. T_0)=n!.

In the chain topology on a set X_n is the topology whose members are completely ordered by the inclusion operator. Clearly the minimal T_0-topologies on X_n are chain topologies and the chain topologies on X_n are connected [8]. Stephen [8] proved that the number of all chain topologies on a set X_n is : $N_n(CH(k)) = \sum_{k=0}^{n-1} {}^n c_k \; N_k(CH)$. Where CH-topology on X_n is a chain topology on X_n and $N_0(CH) = 1$.

The members of a chain topology τ on X_n are such that: $\phi \subset G_1 \subset G_2 \subset ... \subset G_r \subset X_n$ in which G_1 is nonempty and either singleton or nonsingleton. Accordingly τ is either $S(1)$ or $S(0)$ and so $N_n(CH) = N_n(S(0)CH) + N_n(S(1)CH)$. If $|G_1| = k$ and $N_n(CH)$ then τ is said to be $CH(k)$-topology on X_n and the number of the chain topologies in such case is denoted by $N_n(CH)$. So, $N_n(CH(1)) = N_n(S(1)CH)$, $N_n(S(0)CH) = \sum_{k=2}^{n} N_n(CH(k))$ and $N_n(CH) = \sum_{k=1}^{n} N_n(CH(k))$.

Theorem 3.6: Let X_n be a set of n points then:

(1) $N_n(CH(k)) = {}^n c_k N_{n-k}(CH)$,

(2) $N_n(CH) = \sum_{k=0}^{n-1} {}^n c_k N_k(CH)$,

(3) $N_n(CH(k)) = \sum_{r=k}^{n-1} {}^n c_r N_r(CH(k))$ and

(4) $N_n(S(0)CH) = \sum_{k=0}^{n-2} {}^n c_k N_k(CH)$ where $N_0(CH) = 1$.

Proof: Let $A \subset X_n$ be such that $|A| = k, 1 \leq k \leq n$. If τ is a chain topology on $X_n \setminus A$ then, $\tau(A) = \{G \cup A : G \in \tau\} \cup \{\phi\}$ is an $CH(k)$-topology on X_n. Clearly there are ${}^n c_k$ distinct nonempty subset of X_n with cardinality k and therefore:

(1) $N_n(CH(k)) = {}^n c_k N_{n-k}(CH)$,

(2) $N_n(CH) = \sum_{k=1}^{n} {}^n c_k N_{n-k}(CH) = \sum_{k=0}^{n-1} {}^n c_k N_k(CH)$

Let $k \in N$ and $A \subset X_n$ be such that $1 \leq k \leq n-1$ and $|A| = r \geq k$. If τ is a $CH(k)$-topology on A then $\tau(A) = \{X_n, G : G \in \tau\}$ is a $CH(k)$-topology on X_n. If $r=n$ then $A \subset X_n$ and if τ is a $CH(k)$-topology on A then $\tau(A)=\tau$,

(3) $N_n(CH(k)) = \Sigma_{r=k}^{n-1} {}^n c_r N_r(CH(k))$

Clearly if $k > n \geq 0$ then, $N_n(CH(k)) = 0$ and so $N_0(CH(1)) = 0$. Also, if $k=n$ then $A=X_n$ which implies that $\tau=\{X_n,\phi\}$ which implies that $\tau(A) = \{X_n, \phi\}$ which implies that $N_n(CH(n)) = 1$. Therefore, using (1)

(4) $N_n(S(0)CH) = \sum_{k=2}^{n} N_n(CH(k)) = \sum_{k=2}^{n} {}^n c_k N_{n-k}(CH) = \sum_{k=0}^{n-2} {}^n c_k N_k(CH)$.

Conclusion

It is show that we are interested in finding the characterization of the principle door and a formula for the number of the door topologies on a set X_n of n points are given. Some properties of the principal connected topologies on a nonempty set X are discussed and the minimal T_0-topologies on X are also characterized. Also, a few results about the number of the chain topologies on X_n are given.

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