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On Discrete Integrable Systems

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It is well known that the theory of continuous integrable system has been extensively developed since the famous work by Zabusky et al [1] and Gardner et al [2,3] on the Korteweg-de Vries (KdV) equation. A most important topic in the continuous integrable systems is detecting integrability for nonlinear partial differential equations. Integrability is closely related to the Lax pairs, the Hamiltonian structure, infinitely many conservation laws, the backlund transformation, arbitrary number of soliton solutions, symmetry, Painleve property and so on [4-8]. A number of continuous integrable systems, such as the KdV, the modified KdV, the nonlinear SchrÄodinger (NLS), the KP and so on, have been extensively studied.

The study of integrable systems has important roles in mathematics and physics. The theory of integrable systems connects with various fields of mathematics such as partial differential equation, ordinary differential equation, Lie group, Lie algebra, differential geometry, computational mathematics. Integrable systems possess many physical applications in fluid mechanics, condensed matter physics, nonlinear optics, field theory, and so on.

On the other hand, there has been an explosion of interest in the study of discrete in-tegrable systems including nonlinear differential-difference equations and partial difference equations. As many authors pointed [7-11], the reasons why the theory of discrete equations attracted much attention are due to the following facts: (1) It is well known that the soliton was first discovered by numerical computation for the cauchy problem of the KdV equation. The motivation of the remarkable work of Zabusky and Kruskal comes from an attempt to understand the phenomenon found by Fermi, Pasta and Ulam (FPU). The model FPU considered was just a differential-difference equation. Thus, the investigation for the semi-discrete equation leads to the discovery of the soliton. (2) It is important to yield a proper discrete version for a continuous integrable system. It is desirable in the numerical simulation of soliton equations that the discrete versions inherit the integrability property of original continuous soliton equations. Ablowitz's work (discretization of the NLS equation) shows the significance of the discrete version keeping integrability [9]. (3) There exist a lot of discrete phenomena in various physical realms such as nonlinear optics, statistical physics, plasma physics, quantum gravity, quantum field theory. The models described these discrete phenomena are nonlinear differential-difference equations and difference equations. Discrete integrable systems have also applications in many other fields such as numerical analysis, discrete geometry, mathematical biology, and economics.

Today, all the topics for continuous integrable systems have been generalized to their discrete versions. Discrete integrable systems also yield some new phenomena. For example, the shorter discrete solitary waves can travel faster than the taller ones. Some of the major research topics and open problems in the area of discrete integrable systems have been addressed by Hietarinta [11], and Halburd et al [12].

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