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Editorial

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On Abstract Gronwall Lemmas

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In this paper, we present some problems related to Abstract Gronwall Lemmas. We begin our considerations with some notions from operatorial inequality [1-6].

Let $(X,\to,\le)$ be an ordered L-space [2-6], $A:X\to X$ an operator. We denote F_A as the fixed points set of A. We consider that the equation x=A(x) has a unique solution x_A^*. The operatorial inequality problem is the following [1-4]:

Find conditions under which

(i) $x\le A(x)\Rightarrow x\le x_A^*$;

(ii) $x\ge A(x)\Rightarrow x\ge x_A^*$.

To have a concrete result for this problem it is necessary to determine the x_A^*, or to find y: $z\in X$ such that $x_A^*\in[y,z]$.

Abstract Gronwall Lemma (AGL) [2,3,5,6].

Let $(X,\to,\le)$ be an ordered L-space and $A:X\to X$ an operator.

We suppose that

(i) A is PO;

(ii) A is increasing.

Then,

(i) $x\le A(x)\Rightarrow x\le x_A^*$;

(ii) $x\ge A(x)\Rightarrow x\ge x_A^*$.

Abstract Gronwall-Comparison Lemma (AGCL) [2,3,5,6]. Let $(X,\to,\le)$ be an ordered L-space and $A,B:X\to X$ two operators. We suppose that:

(i) A and B are POs;

(ii) A is increasing;

(iii) $A\le B$.

Then,

$$x\le A(x)\Rightarrow x\le x_B^*. \tag{1}$$

We present two problems as follows. First is the simplest and most useful integral inequality.

Lemma Gronwall [7- 9]. Let $y,g\in C([a,b,\mathbb{R}_+)$

Suppose, for $c\ge 0$, we have

$$y(x)|\le c+\int_a^x y(s)g(s)ds, x\ge a\ge 0. \tag{2}$$

Then,

$$y\le c\exp(\int_a^x g(s)ds), x\ge a\ge 0. \tag{3}$$

In this case $(X,\xrightarrow{\|\cdot\|},\le)=C([a,b],\xrightarrow{\|\cdot\|}\le)$, *where* $\|.\|$ is the Bielecki norm on $C([a,b]), \|y\|:=\max_{x\in[a,b]}(|y(x)|e^{-T(x-a)}), T\in\mathbb{R}_+^*$ and $A:X\to X$ is defined by

$$A(y)(x)=c+\int_a^x y(s)g(s)ds. \tag{4}$$

The fixed point of the operator A is $x_A^*=c\exp(\int_a^x g(s)ds)$ and the inequality (3) derived from AGL.

The second problem is analogous to Gronwall lemma. Thus, we have:

Lemma Wendorff [1,7-9]. We consider that

(i) $u,v\in C([0,a]\times[0,b],\mathbb{R}_+),c\in\mathbb{R}_+$;

(ii) v is increasing

If u is a solution of inequality

$$u(x,y)\le c+\int_0^x\int_0^y \upsilon(s,t)u(s,t)dsdt, x[0,a], y\in[0,b] \tag{5}$$

then

$$u(x,y)\le c\exp(\int_0^x\int_0^y \upsilon(s,t)dsdt). \tag{6}$$

In this case, the operator A is the second part of (5) inequality, but the function $u(x,y)\le c\exp(\int_0^x\int_0^y \upsilon(s,t)dsdt)$ is not the fixed point of the operator A. This inequality is derived from AGCL.

For other applications see Lungu and Popa [10], Lungu and Rus [11], and Popa and Lungu [12].

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