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Numerical Solution of Vibrating Double and Triple-Panel Stepped Thickness Plates

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Abstract

The main objective of the present paper is to achieve a modified numerical method for investigating the vibration characteristics of the stepped thickness plate with many types of boundary conditions surrounding certain number of panels. The presented technique relies on dividing the entire plate into several regions of uniform thickness separated by sudden steps. Each region is divided to number of strips which are assembled and solved numerically by the Finite Strip-Transition Matrix method FSTM. A convenient basic function is applied to reduce the partial differential equation of motion of plate inside a single region into an ordinary differential one. Step continuity conditions are applied to achieve the final solution of plate. Regional rigidities of plates and mass per unit area are changed due to the change of plate thickness from a region to another. Consequently, new straining actions are occurred and then compatibility conditions become necessary to modify the nodal vector at each step. Various types of restrained boundary conditions against rotation are included in the present paper. The validity of present method is checked and the accuracy of the results is compared with those available in literature showing a good agreement.

Keywords: Numerical; Transition; Stepped; Panel; Vibration; Plate

Nomenclatures

$W_i = W_i(z,h,t)$ Regional dimension-less plate displacement in the domain of region R_i; $i = 1,2,3$.

a,b	Plate dimension in ζ,η directions respectively
x,y	Plate coordinates
t	Time
ζ,η	Plate dimension-less coordinates; $\zeta = \frac{x}{a}, \eta = \frac{y}{b}$
$\beta = \frac{a}{b}$	Aspect ratio
h_i	Regional thickness of plate
ρ	Plate density
M	Number of panel
k	Parameter of homogenous sub-grade
$(\bar{m})_i = \frac{\rho}{g} h_i$	Plate mass per unit area
$(D)_i = \frac{Eh_i^3}{(12-\nu^2)}$	Regional flexural rigidity of plate
$K_G = \frac{ka^4}{D_1}$	Normalized parameter of homogenous sub-grade
k	Parameter of homogenous sub-grade
ω	Natural frequency
$\lambda_m^2 = \omega^2 a^4 (\frac{\bar{m}}{D})_1$	Natural frequency parameter
c_m, d_m	Integral values
$\alpha_i = \frac{h_1}{h_i}$	Panel thickness ratio
$\{V_m\}_i$	Regional nodal vector
$[Y_K]_i$	Regional transition matrix
$\{V_m\}_{K_j}, \{V_m\}_{K_{j-1}}$	Nodal vectors of strip K_j
Φ	Restrained cofficient of rotation and translation at $\xi = 0,1$
φ	Restrained cofficient of rotation and translation at $\eta = 0,1$
$\gamma_i = (\frac{h_i}{h_{i+1}})^3; i = 1,2$	Step thickness ratio
ν_x	Poisson's ratio

Introduction

A certain structural optimization of a panel may be achieved by possessing a suitable variation of thickness of plate structure. The importance of types of structures increases when the aero-space and underground structures are considered. Survey of the literature on the flexural vibration of thin plate reveals that the work on this topic has been mainly confined to plates with uniform thicknesses. A relatively few studies have been published on the free vibration of isotropic plates of stepped thicknesses. During the past few decades, many researchers were devoted to mathematical modeling and numerical solution for static elastic multi-structure problems. Since vibration analysis of elastic structures plays important roles in engineering applications, this paper is concerned with Finite Strip-Transition Matrix method (FSTM) for vibration analysis of paneled stepped thickness plate.

Although, Chopra [1] has attempted an exact solution for a simply supported stepped thickness plate with two panels, Warburton [2] pointed out that continuity conditions used by Chopra [1] were incorrect. He presented a modified analytical technique for two paneled stepped plate with different properties of orthotropic. Sakata [3] proposed an approximate formula for estimating the fundamental

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nature frequency of an isotropic plate with stepped thickness from the natural frequencies of the isotropic plate reduced from the orthotropic one. Recently, Farag [4] applied a closed form solution for vibrating surfaces of partially restrained and clamped double-panel plates via a power matrix exponential method. Xiang and Wang [5] also

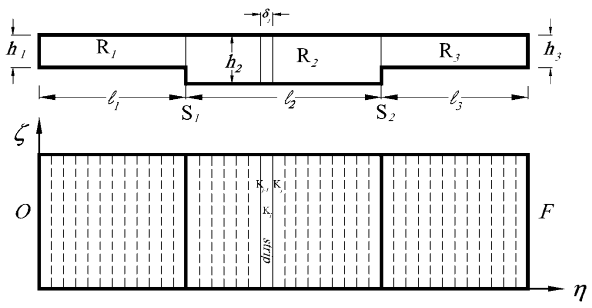

Figure 1: Finite Strips of Triple Panel Stepped Thickness plate.

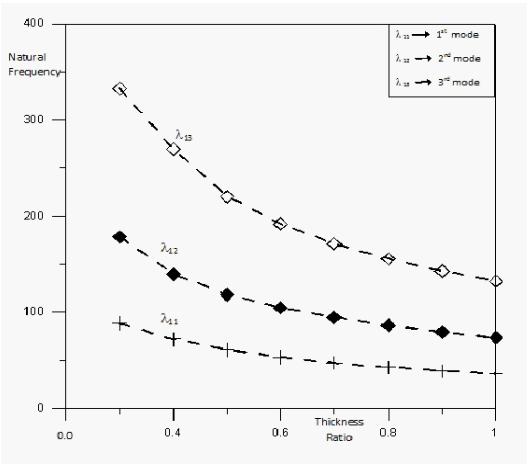

Figure 2-a: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio α_2 for the first three modes where $m=1$ and $n=1,2$ or 3 for CCCC case.

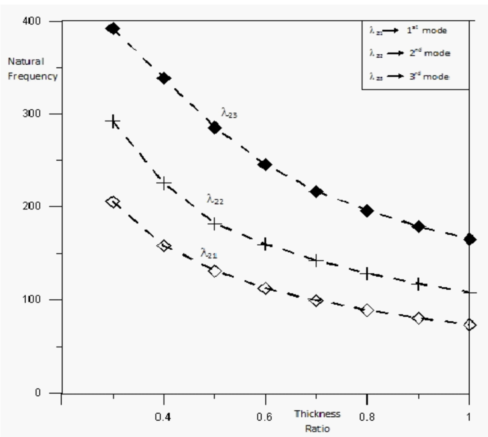

Figure 2-b: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio α_2 or the first three modes where $m=2$ and $n=1,2$ or 3 for CCCC case.

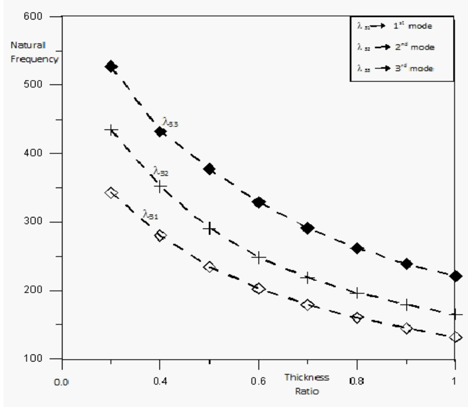

Figure 2-c: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio α_2 for the first three modes where $m=3$ and $n=1,2$ or 3 for CCCC case.

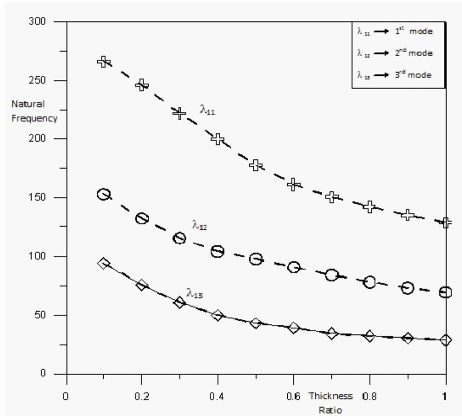

Figure 3-a: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio α_2 for the first three modes where $m=1$ and $n=1,2$ or 3 for CSCS case.

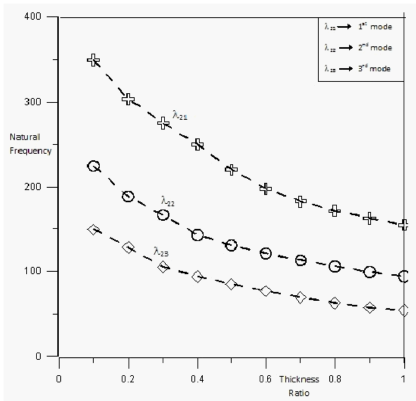

Figure 3-b: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio for the first three modes where $m=2$ and $n=1,2$ or 3 for CSCS case.

studied the exact vibration solutions of stepped rectangular plates. Xiang et al. [6,7] extended these studies in cooperation with others to include the case of stepped rectangular mindlin plate and the case of stepped circular mindlin plate. Gorman and Singha [8] outlined the vibration analysis of stepped cantilever plate using a supper position method. A numerical approach for rectangular stepped plate with sides restrained against rotation has been proposed by Laura and Filipich [9]. They ensured that vibration of plate with stepped thicknesses was studied in a very few number of literature. Filipich et al. [10] used a simple polynomial coordinate function which identically satisfied the restrained boundary conditions of plate with discontinuous thickness. Modal study via a discrete numerical approach, such as finite element method by Mukherjee and Mukhopadhyay [11], or finite strip method by Cheung and Li [12,13] introduces solution accuracy depending upon the suggested element size. Vibrations of plates with variable thickness were studied by Zanzi and Laura [14], Gutierrez et al. [15], Laura and Gutierrez [16] via different methods. Kaabi and Aksu [17,18] applied a modified method to examine the dynamic behavior of rectangular plates with bilinear variation of thickness. Cheung and Li [19] achieved a simple finite strip method to analyze the hunched continuous bridges. Vibration of orthotropic rectangular plate with free edges in the case of discontinuously varying thickness was discussed by Laura et al. [20]. They extended their studies to include two other boundary conditions different from Laura et al. [21]. Farag [22] applied Finite Strip Transition Matrix method to solve the free and forced vibration problems of uniform thickness plate. In extended work, Farag [23] analyzed the stepped thickness plate analytically by means of the matrix exponential method. Recently, a semi and fully discrete finite element methods for investigating vibration analysis of elastic plate-plate structures are proposed by Junjiam et al. [24]. Semie [25] explained the numerical modeling of thin plates using the finite element method. Sanches et al. [26] studied the dynamic stationary response of reinforced plate by the boundary element method.

The present paper offers a modified numerical method relying on reducing the partial differential equation of motion into an ordinary differential one. A convenient basic function is used to obtain the later equation which is solved by Finite Strip Transition Matrix Method (FSTM) inside each panel of plate. Due to the sudden change of thickness between two adjacent panels, continuity conditions must be satisfied to generate the final solution of the entire plate. The known basic function is expressed in a variable parallel to the steps. The reduced differential equation of rectangular stepped plate is still carrying the other unknown variable in the other plate direction. The restrained boundary conditions are included and employed to derive the solution of the particular cases of clamped and simply supported edges.

Equation of Motion

The isotropic rectangular plates with elastically restrained boundary conditions in the two directions ζ and η of plates are considered here. The plate thickness shown in figure 1 is a uniform in ζ-direction, while it changes suddenly in η-direction at a step S_i from h_i to h_{i+1}.

Referring to the domain of the isotropic region R_i, the regional dimensionless partial differential equation of motion of plate vibration is:

$$\bar{\nabla}^4 W_i + \left(\frac{\bar{m}a^4}{D}\right)_i \frac{\partial^2 W_i}{\partial t^2} + \left(\frac{k\, a^4}{D}\right)_i W_i = 0; \qquad i = 1,2,3 \tag{1}$$

Where

$$\bar{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \beta^2 \frac{\partial^2}{\partial y^2}$$

The different symbols are denoted in the nomenclatures part where the suffix *i* means that the mentioned magnitudes are calculated inside the region R_i.

Method of Solution

The partial differential equation (1) is solved numerically under the proposed boundary conditions at $(\zeta = 0,1)$ and $(\eta = 0,1)$ by using the Technique of Finite Strip-Transition Matrix FSTM. In this Technique, the investigated plate is divided into three stepped panels which are separated by a number M-1 of steps. Each individual panel is divided to a number N of finite strips separated by a number N+1 of nodal

$\alpha_2 = h_1/h_2$	$\sigma_{mn} = \frac{\lambda_{mn}}{\pi^2}$	SSSS;			CSCS;			CCCC;
		Present	Xiang et al. [5]	Farag [4]	Present	Xiang et al. [5]	Farag [4]	Present
	σ_{11}	02.9015	0 2.9015	02.9015	04.1711	04.1711	04.1711	05.3028
1/2	σ_{12}	07.1157	07.1156	07.1156	09.9048	0 9.9047	09.9047	10.5159
	σ_{13}	13.7849	13.7850	13. 7848	18. 0455	18.0450	18. 0453	18.6238
	σ_{11}	02.4470	2.4471	02.4470	03.5609	03.5610	03.5609	04.4631
2/3	σ_{12}	06.2230	6.2229	06.2229	08.7200	08.7199	08.7199	09.2528
	σ_{13}	11.9125	11.9480	11.9124	15.6217	15.6210	15.6215	16.0596

Table 1: Comparesons of the natural frequency coefficient σ_{mn} for square Plates SSSS and CSCS.

	Natural frequencey parameters λ_{mn}					
α_2,α_3	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}
1.0, 1.0	35.9195	73.5929	132.3785	211.2511	309.0456	416.1822
0.9, 1.0	37.9563	77.3438	138.6326	222.3554	327.3623	437.8916
0.8, 1.0	40.5830	81.3422	145.6952	236.6180	348.0211	459.2436
0.7, 1.0	43.9586	85.5344	154.3948	236.6180	370.3955	480.2570
0.6, 1.0	46.3017	71.6753	102.2939	255.4175	332.9891	385.8950
0.5, 1.0	54.0737	95.5105	184.6845	267.9179	410.2973	539.8316
0.4, 1.0	62.4249	104.1121	215.7646	343.2175	425.5430	616.2196
0.3, 1.0	76.9445	122.0380	268.1892	366.2887	447.2614	819.1406
0.2, 1.0	109.3091	168.1862	333.4251	376.4200	543.4720	1265.0045
0.1, 1.0	215.0652	311.9320	359.3840	410.0752	1080.9598	2590.7068

Table 2: The natural frequency parameter λ_{mn} for 3-panels square clamped plate CCCC.

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lines as in figure 1. The basic function of strip [23], is applied to reduce the partial differential equation of motion into an ordinary differential one. The reduced differential equation is solved numerically by the FSTM as an initial value problem under the proposed initial boundary conditions at $(\eta = 0)$. The initial nodal conditions of each strip are applied to derive the strip end nodal conditions which are used as initial nodal conditions to the next strip. All nodal straining actions of each strip are obtained until the end of the current panel is reached. Because the intermediate end of a current panel is sudden step, the initial nodal conditions of the next panel must be modified by satisfying the compatibility conditions at this step. The FSTM is applied for all strips of all panels until the final boundary end of plate is reached. Satisfying the boundary conditions at $(\eta = 1)$, one can obtain the final solution of the equation motion of plate.

Reduction technique

The regional displacement of the stepped plate is:

$$W_i(\zeta,\eta,t) = \sum_{m=1}^{M} (U_m(\zeta))_i (V_m(\eta))_i \sin\omega t \qquad (2)$$

Where $(U_m(\zeta))_i$ is known regional shape function satisfying the plate boundary conditions [23] at $(\zeta = 0,1)$ and $(V_m(\eta))_i$ is unknown regional longitudinal function to be determined at $(\eta = 0,1)$.

Eq. (2) is applied to reduce the partial differentia Eq. (1) Inside the Region $R_i;\ i = 1,2,3$ into [4]:

$$(V_m'''')_i + \frac{2c_m}{\beta^2}(V_m'')_i + \frac{1}{\beta^4}[K_G D_1(\frac{1}{D})_i - \lambda_m^2 \frac{D_1}{h_1}(\frac{h}{D})_i + d_m](V_m)_i = 0 \qquad (3)$$

Where

$$K_G = \frac{ka^4}{D_1}, \lambda_m^2 = \omega^2 a^4 (\frac{\overline{m}}{D})_1, c_m = \frac{\int_0^1 U_m U_m'' d\zeta}{\int_0^1 U_m U_m d\zeta}, c_m = \frac{\int_0^1 U_m U_m'' d\zeta}{\int_0^1 U_m U_m d\zeta},$$

$$d_m = \frac{\int_0^1 U_m U_m'''' d\zeta}{\int_0^1 U_m U_m d\zeta}$$

Consequently, the equation of motion for any panel becomes:

$$(V_m'''')_i = \left[\frac{-1}{\beta^4}(\alpha_i^3 K_G - \alpha_i^2 \lambda_m^2 + d_m) \quad 0 \quad (-2\frac{c_m}{\beta^2}) \quad 0 \right]_i \begin{Bmatrix} V_m \\ V_m' \\ V_m'' \\ V_m''' \end{Bmatrix}_i \qquad (4)$$

		Restraint coefficients against rotation $\varphi_o = \varphi_F$					
α_2,α_3	λ_{mn}	0.0	0.02	0.2	2.0	200	∞
1.0, 1.0	λ_{11}	35.9195	34.7141	31.1493	29.1644	28.8413	28.8379
	λ_{12}	73.5929	69.7890	50.1225	55.6106	54.9253	54.9181
	λ_{13}	132.3787	125.0569	109.4910	103.5695	102.7365	102.7279
0.75, 1.0	λ_{11}	42.1654	40.8311	37.1138	35.1818	34.8760	34.7828
	λ_{12}	83.4136	78.8536	67.9529	63.1631	62.4507	62.4432
	λ_{13}	149.7636	141.3653	124.3771	118.1902	117.3307	117.3219
0. 5, 1.0	λ_{11}	54.0736	52.5033	48.5825	46.7700	46.4956	46.4927
	λ_{12}	95.5105	90.5687	79.5519	57.0258	74.3679	74.3610
	λ_{13}	184.8631	175.7379	157.7919	151.4550	150.5831	150.7542
0.25, 1.0	λ_{11}	84.6291	88.3274	58.8198	84.7820	84.6307	84.6291
	λ_{12}	123.5494	135.5536	127.3214	124.0274	123.5543	123.5494
	λ_{13}	233.6159	282.3131	246.3862	235.1273	233.6312	233.6164

Table 3: The natural frequency parameter λ_{mn} for 3-panels square clamped restrained plate CE_RCE_R.

		Restraint coefficients against rotation $\varphi_o = \varphi_F$				
α_2,α_3	λ_{mn}	0.0	0.1	1.0	100	∞
0.75, 1.0	λ_{11}	26.7406	24.9157	23.1746	22.7619	22.7573
	λ_{12}	66.0149	60.3497	56.4543	55.6976	55.6894
	λ_{13}	128.6246	118.5003	113.3414	112.4694	112.4602
0. 5, 1.0	λ_{11}	31.4907	29.3873	27.6339	27.2489	27.2447
	λ_{12}	75.0484	68.5108	64.6539	63.9483	63.9407
	λ_{13}	160.2106	149.9871	144.8592	143.9884	143.9792
0.75, 1.0	λ_{11}	47.4033	43.0552	38.3594	37.1537	37.1398
	λ_{12}	135.0159	121.8370	112.2586	110.3528	110.3321
	λ_{13}	275.0068	251.6332	239.4639	237.3944	237.3724
0. 5, 1.0	λ_{11}	53.3631	48.4637	43.7366	42.6063	42.5936
	λ_{12}	147.6858	132.0205	122.2170	120.3904	120.3904
	λ_{13}	338.6052	315.2836	303.4671	301.4525	301.4311
0.75, 1.0	λ_{11}	77.0914	69.2508	60.3105	57.9252	57.8977
	λ_{12}	231.7011	207.9731	190.3487	186.8081	186.7695
	λ_{13}	479.9336	437.9678	415.9456	412.1925	412.1526
0. 5, 1.0	λ_{11}	85.5963	76.9994	68.1073	65.8863	65.8610
	λ_{12}	249.6733	220.5666	202.1801	198.7270	198.7270
	λ_{13}	588.0605	546.3231	525.1012	521.4784	521.4784

Table 4: The natural frequency parameter λ_{mn} for 3-panels square simply supported- restrained plate SE_RSE_R.

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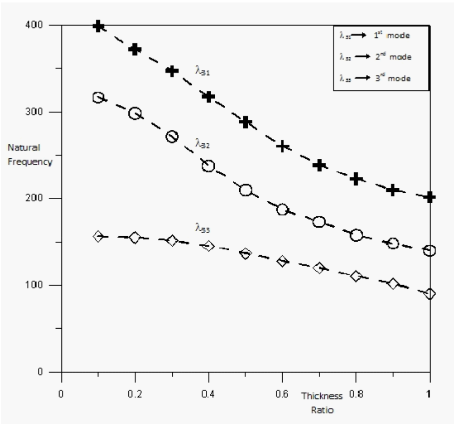

Figure 3-c: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of thickness ratio α_2 for the first three modes where $m=3$ and $n=1,2$ or 3 for CSCS case.

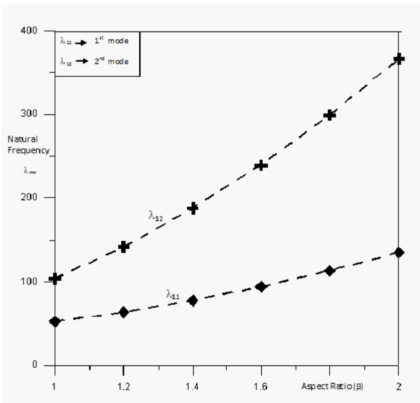

Figure 4-a: Relationship of the variation of natural frequency parameters λ_{mn} due to the change of aspect ratio β for the first two modes where $m=1$ and $n=1,2$ or 3 for CCCC case

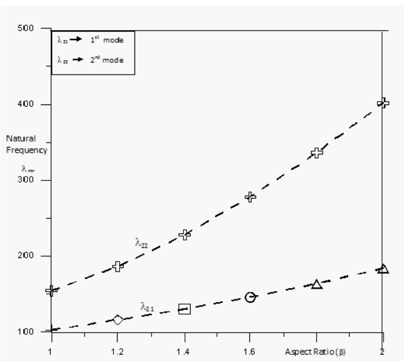

Figure 4-b: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of aspect ratio β for the first two modes where $m=2$ and $n=1,2$ or 3 for CCCC case.

where:

$\alpha_i = \dfrac{h_1}{h_i}$ is the panel thickness ratio at region R_i; $i=1,2,3$

The ordinary differential equation (4) is transformed within a region R_i to a linear system of differential equations expressed as [22]:

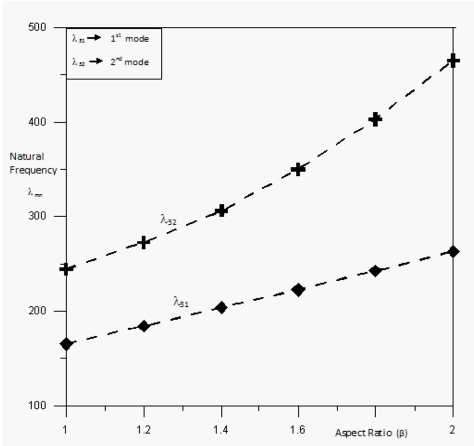

Figure 4-c: Relationships of the variation of natural frequency parameters λ_{mn} due to the change of aspect ratio β for the first two modes where $m=3$ and $n=1,2$ or 3 for CCCC case.

$$\frac{d}{d\eta}\{V_m\}_i = [A_m]_i \{V_m\}_i \tag{5}$$

Where

$$\{V_m\}_i = \begin{Bmatrix} V_m \\ V'_m \\ V''_m \\ V'''_m \end{Bmatrix}_i \tag{6}$$

And

$$[A_m]_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{-1}{\beta^4}(\alpha_i^3 K_G - \alpha_i^2 \lambda_m^2 + d_m) & 0 & (-2\dfrac{c_m}{\beta^2}) & 0 \end{bmatrix} \tag{7}$$

The general solution of the ordinary differential equation is:

$$\{V_m\}_{K_j} = [Y_K]_i \{V_m\}_{K_{j-1}} \tag{8}$$

Where δ_i is the width of strip κ_j bounded by two nodal lines K_j, K_{j-1} inside region R_i.

The regional Transition Matrix $[Y_K]_i$ is calculated according to the Runge Kutta method such as:

$$[Y_K]_i = I + \sum_{r=1}^{4} \frac{[A_m]_i^r (\delta_i)^r}{r!} \tag{9}$$

Where I is the unit matrix?

Continuity conditions

The Magnitudes of displacement W slope $\frac{\partial W}{\partial v}$ Moment M_V and shear Q_v must satisfy the continuity conditions at the sudden step S_i so that:

$$\begin{Bmatrix} V_m \\ V'_m \\ V''_m \\ V'''_m \end{Bmatrix}_{\Re i} = \begin{Bmatrix} V_m \\ V'_m \\ \nu_x c_m(\gamma_i - 1)\beta^{-2} V_m + \gamma_i V'' \\ c_m(2-\nu_x)(\gamma_i - 1)\beta^{-2} V'_m + \gamma_i V''' \end{Bmatrix}_{Li} \tag{10}$$

The symbols $\Re_i, L_i$ denote positions immediately after and before the sudden step S_i respectively.

Where $\gamma_i = (\frac{h_i}{h_{i+1}})^3$ is step thickness ratio at step $S_i ; i = 1,2$ and ν_x is Poisson's ratio.

A right step vector $\{V_m\}_{\Re_i}$ of the sudden step S_i is achieved by updating the step left vector $\{V_m\}_{L_i}$ due to Eq. (10).

Boundary and initial conditions

Applying the proposed boundary conditions at $\eta = 0$, one can reduce the four initial unknowns $V_m(0), V'_m(0), V''_m(0) and\ V'''_m(0)$ to only two unknowns. Consequently, by satisfying the boundary conditions of plate at $\eta = 1$, two characteristic equations for the plate vibration are established. The natural frequency parameters of plate are the Eigen values of the characteristic matrix of these equations. The corresponding Eigen vectors create the mode shapes. Boundary conditions at the edges $\eta = 0,1$ are considered for various types of edges [22], such as simply supported S, clamped C, and free F, elastically restrained against rotation E_R. The restrained coefficients of rotation $\Phi\ \eta = 0,1$ are usually applied to vary from 0 to ∞.

The initial conditions are expressed as initial values $V_m(0), V'_m(0), V''_m(0) and\ V'''_m(0)$ which are the components of initial vector $\{V_m\}_o$ of the first region R_1. This initial vector $\{V_m\}_o$ can be derived according to the known boundary conditions at the first nodal edge $\eta = 0$ [23].

Results and Discussions

Different cases of plates composed of different panels with unequal thicknesses and panel widths are investigated by the present technique. This study takes in account the variation of the aspect ratios β and various magnitudes of panel thickness ratio $\alpha_i;\ i = 1,2,3$ and the coefficients of partially restrained boundary conditions. The natural frequency parameter λ_{mn} is calculated for every case where Poisson's ratio is taken as $\nu = 0.3$.

The natural frequencey coefficients $\sigma_{mn} = \frac{\lambda_{mn}}{\pi^2}$ based on the present technique are compared as shown in Table 1 with those obtained by Xiang et al. [5] using an exact solution and using closed form solution [4].

The comparisons are available only for the case of double panel square plate with panel width ratio $\ell_1 : (\ell_1 + \ell_2) = 1:2$ as shown in Table 1. Calculations are carried out in two cases of boundary conditions of square plates SSSS and CSCS where S, C mention to the simply supported and clamped edges respectively. The general method of restrained boundary conditions is applied when $\Phi_1 = \Phi_2 = \infty$, $\varphi_o = \varphi_F = \infty$ to posses the case of full simply supported plate SSSS. Another case of plate CSCS is obtained where $\Phi_1 = \Phi_2 = \infty$ and $\varphi_o = \varphi_F = 0$. The third case is full clamped plate CCCC where $\Phi_1 = \Phi_2 = 0$ and $\varphi_o = \varphi_F = 0$. The results are obtained for panel thickness ratios equal to 0.5 and 2/3. The comparisons show excellent agreement.

The natural frequencey parameters λ_{mn} in the first six modes are obtained in Table 2 for the cases of 3-panels square full clamped plates CCCC with panel width ratio $\ell_1 : \ell_2 : \ell_3 = 0.25 : 0.50 : 0.25$. The results are calculated under the variation of α_2 from 1.0 to 0.1 where $\alpha_1 = \alpha_3 = 1$. The results show that the natural frequency parametrer inreases by decreasing α_2.

Table 3 shows the natural frequencey parameters λ_{mn} for the cases of 3- panels square full plates CE_RCE_R with two opposite edges clamped and other edges ellastically restrained against rotation with restrained coefficient varying from 0 to ∞. The results are obtained for cases of panel width ratio $\ell_1 : \ell_2 : \ell_3 = 0.25 : 0.50 : 0.25$ and α_2 varying from 1.0 to 0.25, where $\alpha_1 = \alpha_3 = 1$. The natural frequencey parameter is obtained for the first three modes. The results show that the natural frequency parametrer decreases by increasing the coefficient of restrained φ.

This case is carried out for 3-panel square plate, SE_RSE_R, simply supported at two opposite edges and partially restrained against rotation at the other edges (Table 4). The results are obtained for panel width ratios $\ell_1 : \ell_2 : \ell_3 = 0.25 : 0.50 : 0.25$ and panel thickness ratio $1 : \alpha_2 : 1$ where α_2 varies from 1.0 to 0.25. Table 4 shows that the natural frequency parameter increases by decreasing the restrained coefficient φ. For the double panel isotropic, square, clamped supported plate at all edges CCCC the relationships of the variation of natural frequency parameters due to the change of thickness ratio α_2 are plotted in figures 2-a,b,c. For the panel width ratio $\ell_1 : (\ell_1 + \ell_2) = 1:3$, the natural frequency parameters λ_{mn} are obtained for the first three modes where $m = 1, 2$ or 3 and $n = 1, 2\ or\ 3$.

Similarly, the case of square isotropic stepped plate CSCS with two opposite edges clamped and other edges simply supported is carried out as shown in figures 3-a,b,c. The frequency parameters λ_{mn} are recorded for the case of panel width ratio $\ell_1 : (\ell_1 + \ell_2) = 1:2$ when the thickness ratio α_2 changes from 0.1 to 1.0. The results show that the natural frequency parameters increase by decreasing the thickness ratio α_2 for all modes.

The case of full clamped stepped rectangular plate CCCC with thickness ratio $\alpha_2 = .5$, panel width ratio $\ell_1 : (\ell_1 + \ell_2) = 1:2$ and aspect ratio β of the intire plate varying from 1 to 2 is investigated as shown in figures 4-a,b,c. The resuls show that the parameter λ_{mn} increases by increasing the aspect ratio β for all recorded modes.

Conclusion

The finite strip transition matrix method FSTM described here involvoes a numerical solution of stepped paneled plate with classical and restrained boundary conditions. This method is a combination between the strip and transition matrix method to solve the vibration problem of stepped plates as an initial value proplem. Transition matrix method is a semi analytical method relying on estmiating the numerical solution of the intial value problem by means of Range Kutta method. The plate domain is divided into paneled regoins consisting of strips bounded by nodal lines. Each strip is governed by the transition matrix formula which transite from one strip to another via nodal vectors until the final edge is reached. Several cases of double and triple panel plates are investigated for the variation of thickness ratio, aspect ratio, panel width ratio and boundary conditions. To show the accuracy of the present method, the results have been summerized and compared with those obtained by other methods. A good exteremely agreement of results is found for all compared cases.

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