

Non-Perturbative Quantum Gravity in Five Dimensions

Supriya Kar*

Department of Physics & Astrophysics, University of Delhi, New Delhi 110 007, India

Abstract

We briefly discuss a non-perturbative theory of quantum gravity in five dimensions, beneath a geometric torsion construction, on a vacuum created pair of Dirichlet 3-brane/anti-brane by a two form. Interestingly, the four dimensional gravity effects may see to re-appear with a Big Bang from a gauge theoretic vacuum on a D4-brane in a ten dimensional type IIA superstring theory. Generically it is argued that a non-perturbative correction, sourced by a non-linear electric/magnetic charge, couples to a flat metric and incorporates quantum effects into various Einstein vacua. A geometric torsion may provoke thought to glimpse into the conjectured M-theory in eleven dimensions.

Keywords: Superstring theories; D-branes; M-theory; Non-commutative gauge theory

Introduction

Einstein gravity, or the General Theory of Relativity (GTR), is established as a successful classical theory of gravitation. It incorporates a space-time curvature into a vacuum via a dynamical metric tensor on a manifold. GTR is primarily based on an intrinsic notion of geometry unlike to that in a Quantum Field Theory (QFT). Most importantly, the metric field equations of motion in GTR confirm a need for an appropriate non-linear matter field presumably sourced by a dark energy underlying a super symmetry. The search, for an appropriate non-linear QFT or a gauge theory to source the space-time deformation geometries, has been in the folklore of theoretical physics during last four decades [1-3].

In the context a perturbative correction in GTR, motivated by the QFT technique, is known to be non-renormalizable due to the running of its coupling G_N (Newton's constant). Nevertheless a ten dimensional superstring theory potentially resolves the dimensional renormalization issue in a perturbative GTR by replacing G_N by a perturbation string parameter α' in an effective description. It may be recalled that a fundamental string is a one dimensional extended dynamical object defined with a finite string tension $T=(2\pi\alpha')^{-1}$. Generically, the propagation of a string may alternately be viewed by a number of mass less scalar(s) QFT on a two dimensional world-sheet in presence of nontrivial mass less background fields [4]. The non-linear interactions are switched into the simplest scalar QFT via the background fields (a metric tensor, a two form, a scalar dilation) in a closed string bulk as well as with a gauge field at the open string boundaries. Most strikingly, a perturbative closed superstring theory is known to describe various p-dimensional non-perturbative Dirichlet (D_p)-branes [5] sourced by an appropriate form field in the stringy background. In fact, a D_p -brane is described by a non-linear U (1) gauge theory [6] and may see to possess a potential tool to explore a non-perturbative domain of quantum gravity in superstring theory. Interestingly a non-perturbative M-theory in eleven dimensions has been conjectured [7], which attempts to unify five distinct superstring theories in ten dimensions. M-theory, in its low energy limit, is known to describe a super gravity theory. However, it requires a complete non-perturbative theoretical description leading to quantum gravity.

In the article we primarily revisit an inspiring thought, with a renewed interest, leading to a plausible field theoretic construction for the M-theory. The formal idea is based on our recent analysis and results [8-16] leading to non-perturbative quantum space-time

curvature K in five dimensions. In particular, the irreducible curvature scalar K may seen to be sourced by a two form which is Poincare dual to a one form on a D_4 -brane underlying a non-linear U (1) gauge theory. One of the striking features in the formalism is bestowed with a fact that the Einstein gravity or a Riemann curvature tensor emerges for a non-propagating torsion in a lower dimension than the original space-time dimension of a non-perturbative torsion dynamics. As a result, various quantum vacua may formally be re-expressed by a non-perturbative correction to the respective Einstein vacua in presence of an extra dimension. The hidden fifth dimension may seem to be intrinsic to a four dimensional non-perturbative quantum correction which couples to a flat metric. The extra dimension initiates a quantum tunneling among the de Sitter vacua sourced by the potentials or fluxes [10]. In addition, an extra dimension has been shown to incorporate a large degeneracy in the quantum Kerr (Newman) vacua due to the charge independence of its event horizon [14]. A lower energy truncates the degeneracy at the expense of a significant electric non-linear charge to describe a quantum Reissner-Nordstrom black hole in four dimensions for an S_2 restoring geometry. A magnetic non-linear charge can be absorbed by a renormalized mass to describe a quantum Schwarzschild black hole. In a lowest energy limit, a non-linear electric charge reduces to its linear counter-part and gains significance to describe a Kerr-Newman and a Reissner-Nordstrom black hole in Einstein gravity. The quantum Schwarzschild in the limit reduces to a Schwarzschild black hole.

On the other hand, a generic torsion curvature has been argued to begin with a vacuum created pair of $(D\bar{D})$ -instant on at the Big Bang by a two form [10]. A pair of lower dimensional bran arguably leads to one higher dimensional pair and the vacuum creation for a brane and anti-bran goes on until $p=8$. A pair of $(D\bar{D})_8$ -brane with space-time curvature underlie a gauge theory on a D_9 -brane and fills the superstring space-time. In principle a pair of $(D\bar{D})_9$ -brane may define an M_{10} brane in an eleven dimensional M-theory. Analysis suggests that a space-filling

*Corresponding author: Supriya Kar, Department of Physics & Astrophysics, University of Delhi, New Delhi 110 007, India, Tel: +91 9911918174; E-mail: skkar@physics.du.ac.in

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brane in M-theory is presumably described by a generalized irreducible curvature scalar $K^{(11)}$ underlying a geometric torsion. However on S1 the local degrees in torsion may freeze, at the expense of the local degrees in a non-perturbative correction, to describe a Riemannian curvature scalar in a ten dimensional type IIA superstring effective theory. Presumably, all the local degrees in an eleven dimensional torsion manifest themselves into a non-perturbative correction to a perturbative superstring vacuum.

Non-Perturbative Space-Time Curvature

It is inspiring to explore the quanta of a dynamical two form in a non-linear U (1) gauge theory on a D_4 -brane. Nevertheless a two form is Poincare dual to a one form in a non-linear gauge theory. A two form source to a string world-sheet is known to yield an effective theory of quantum gravity perturbatively [4]. It provokes thought to believe that a two form (or a one form) non-linear quanta at a gauge theoretic vacuum on a D_4 -brane creates a pair of $(D\bar{D})_3$ -brane presumably with a Big Bang [10]. The vacuum created brane and anti-brane, respectively, carry an appropriate new Ramond-Ramond charge and an anti-charge and hence signify a non-perturbative brane-Universe. They began to move in opposite directions to each other immediately after their creations. The quantum phenomenon may be viewed analogous to a pair of particle/anti-particle creation at a black hole vacuum [1] or otherwise in QFT. In other words, a nontrivial space-time curvature in a four dimensions began from a curvature singularity at a Big Bang. An extra dimension between a pair of $(D\bar{D})_3$ -brane becomes significant in a low energy limit where Einstein vacuum emerges.

A priori the required modification, to incorporate a geometric notion on a D_4 -brane, may be viewed through a modified covariant derivative defined with a completely anti-symmetric gauge connection: $H_{\mu\nu}^\lambda$. The appropriate covariant derivative may be given by

$$D_\lambda B_{\mu\nu} = \nabla_\lambda B_{\mu\nu} + \frac{1}{2} H_{\lambda\mu}{}^p B_{p\nu} - \frac{1}{2} H_{\lambda\nu}{}^p B_{p\mu} \quad (1)$$

Under an iteration $H_3 \rightarrow H_3$, the geometric torsion in a second order formalism may be defined with all order corrections in B_2 in a gauge theory. Formally, a geometric torsion may be expressed in terms of gauge theoretic torsion and its coupling to two forms. It is given by

$$H_{\mu\nu\lambda} = 3D_{[\mu} B_{\nu\lambda]} = 3\nabla_{[\mu} B_{\nu\lambda]} + 3H_{[\mu\nu}{}^\alpha B_{\lambda]\alpha} g_{\alpha\beta} \quad (2)$$

An exact covariant derivative in a perturbative gauge theory may see to define a non-perturbative covariant derivative in a second order formalism. Thus, a geometric torsion, constructed through a non-perturbative covariant derivative (1), may equivalently be described by an appropriate curvature tensor $K_{\mu\nu\lambda}{}^p$ as worked out in [10]. Explicitly,

$$K_{\mu\nu\lambda}{}^p - \frac{1}{2} \partial_\mu H_{\nu\lambda}{}^p - \frac{1}{2} \partial_\nu H_{\mu\lambda}{}^p + \frac{1}{4} H_{\mu\lambda}{}^\sigma H_{\mu\sigma}{}^p - \frac{1}{4} H_{\nu\lambda}{}^\sigma H_{\mu\sigma}{}^p \quad (3)$$

The fourth order tensor is anti-symmetric within a pair of indices, i.e. $\mu \leftrightarrow \nu$ and $\lambda \leftrightarrow \rho$, which retains a property of Riemann tensor $R_{\mu\nu\lambda\rho}$. However the effective curvatures $K_{\mu\nu\lambda}{}^p$ do not satisfy the symmetric property, under an interchange of a pair of indices, as in Riemann tensor. Never the-less, for a constant torsion the generic tensor: $K_{\mu\nu\lambda}{}^p \rightarrow R_{\mu\nu\lambda}{}^p$. As a result, the effective curvature constructed in a non-perturbative formalism may be viewed as a generalized curvature tensor. It describes the propagation of a geometric torsion in a second order formalism. The gauge dynamics on a D_4 -brane, in presence of gauge connections, may be approximated by an irreducible gener-alized curvature scalar. Generically, a geometric action leading to non-perturbative quantum gravity may be given by

$$S_{QG} = \frac{1}{3C_4^2} \int d^5x \sqrt{-gK} \quad (4)$$

Where $C_4^2 = (8\pi^3 g_s) \alpha'^{3/2}$ is constant. The flat metric $g_{\mu\nu}$ on a D_4 -brane defines $g = \det g_{\mu\nu}$. A vacuum energy density $\Lambda \neq 0$ in the geometric action is sourced by a global two form in presence of its local modes.

Emergent Metric

Most importantly the U (1) gauge invariance of a constructed geometric torsion H_3 , in an appropriate combination, incorporates a notion of space-time and hence validates a second order formalism. It may imply that a space-time curvature on a brane-Universe began with the propagation of a geometric torsion. It may worth mentioning that a non-perturbative metric fluctuation, leading to a quantum gravity description, is significantly large and is very much unlike to a perturbative quantum fluctuation. A non-perturbative space-time quantum, sourced primarily by a two form flux, underlying a U (1) gauge invariance may be given by

$$f_{\mu\nu}^{NP} = C H_{\mu\alpha\beta} \bar{H}^{\alpha\beta}{}_\nu \quad (5)$$

Where C is an arbitrary constant and $\bar{H}_{\mu\nu\lambda} = (2\pi\alpha') H_{\mu\nu\lambda}$. Thus, the local degrees in a two form manifest themselves to incorporate a dynamical geometric torsion in a theory of non-perturbative quantum gravity. In other words, a non-perturbative correction significantly modifies a constant metric on a D_4 -brane. Nevertheless, the significance of a zero mode in two forms in the emergent geometry may not be ignored. They signify a non-zero vacuum energy density in the brane-Universe.

$$G_{\mu\nu} = g_{\mu\nu} + C_0 B_{\mu\lambda} B^\lambda{}_\nu + C \bar{H}_{\mu\lambda\rho} H^{\lambda\rho}{}_\nu \quad (6)$$

Various appropriate B_2 background fluctuations in the formalism presumably lead to a large number of quantum tunneling vacua and may incorporate a landscape scenario conjectured in a superstring theory. The background fluctuations in two forms may have their origin in a higher dimensional H_3 and hence their presence is vital to the non-perturbative quantum gravity in the formalism.

M-theory: An attempt

The idea of a geometric torsion leading to a non-perturbative quantum curvature theory in five dimensions may formally be uplifted on a M_{10} -brane to address the bosonic sector of a conjectured M-theory in eleven dimensions. A priori, non-perturbative quantum gravity may be formalized by a geometric torsion in the conjectured M-theory. In addition to appropriate topological couplings, the dynamical content in the bosonic sector may be obtained from a generalization of a two form dynamics on a D_4 -brane. It may be given by

$$S_M \rightarrow \frac{1}{k^2} \int d^{11}x \sqrt{-g} \left(K^{11} - \frac{1}{2.4!} F_4^2 \right) \quad (7)$$

In a low energy limit, the local degrees in a geometric torsion may seen to be insignificant. In the limit, a large r dominates and hence the non perturbative quantum correction reduces drastically and leads to a (semi) classical vacuum in Einstein gravity [14]. In a gauge choice, for a non-propagating torsion, the formalism may reduce to a vacuum in an eleven dimensional super gravity theory. However, it remains to check some of the detailed intricacies of quantum fields to define an appropriate M-theory action. The formalism may be helpful to an enhance understanding of dark energy in a brane-Universe.

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