

Non-Perturbative Quantum Gravity in Five Dimensions

Supriya Kar*

Department of Physics & Astrophysics, University of Delhi, New Delhi 110 007, India

Abstract

We briefly discuss a non-perturbative theory of quantum gravity in five dimensions, beneath a geometric torsion construction, on a vacuum created pair of Dirichlet 3-brane/anti-brane by a two form. Interestingly, the four dimensional gravity effects may see to re-appear with a Big Bang from a gauge theoretic vacuum on a D4-brane in a ten dimensional type IIA superstring theory. Generically it is argued that a non-perturbative correction, sourced by a non-linear electric/magnetic charge, couples to a flat metric and incorporates quantum effects into various Einstein vacua. A geometric torsion may provoke thought to glimpse into the conjectured M-theory in eleven dimensions.

Keywords: Superstring theories; D-branes; M-theory; Non-commutative gauge theory

Introduction

Einstein gravity, or the General Theory of Relativity (GTR), is established as a successful classical theory of gravitation. It incorporates a space-time curvature into a vacuum via a dynamical metric tensor on a manifold. GTR is primarily based on an intrinsic notion of geometry unlike to that in a Quantum Field Theory (QFT). Most importantly, the metric field equations of motion in GTR confirm a need for an appropriate non-linear matter field presumably sourced by a dark energy underlying a super symmetry. The search, for an appropriate non-linear QFT or a gauge theory to source the space-time deformation geometries, has been in the folklore of theoretical physics during last four decades [1-3].

In the context a perturbative correction in GTR, motivated by the QFT technique, is known to be non-renormalizable due to the running of its coupling G_N (Newton's constant). Nevertheless a ten dimensional superstring theory potentially resolves the dimensional renormalization issue in a perturbative GTR by replacing G_N by a perturbation string parameter α' in an effective description. It may be recalled that a fundamental string is a one dimensional extended dynamical object defined with a finite string tension $T=(2\pi\alpha')^{-1}$. Generically, the propagation of a string may alternately be viewed by a number of mass less scalar(s) QFT on a two dimensional world-sheet in presence of nontrivial mass less background fields [4]. The non-linear interactions are switched into the simplest scalar QFT via the background fields (a metric tensor, a two form, a scalar dilation) in a closed string bulk as well as with a gauge field at the open string boundaries. Most strikingly, a perturbative closed superstring theory is known to describe various p-dimensional non-perturbative Dirichlet (D_p)-branes [5] sourced by an appropriate form field in the stringy background. In fact, a D_p -brane is described by a non-linear U(1) gauge theory [6] and may see to possess a potential tool to explore a non-perturbative domain of quantum gravity in superstring theory. Interestingly a non-perturbative M-theory in eleven dimensions has been conjectured [7], which attempts to unify five distinct superstring theories in ten dimensions. M-theory, in its low energy limit, is known to describe a super gravity theory. However, it requires a complete non-perturbative theoretical description leading to quantum gravity.

In the article we primarily revisit an inspiring thought, with a renewed interest, leading to a plausible field theoretic construction for the M-theory. The formal idea is based on our recent analysis and results [8-16] leading to non-perturbative quantum space-time

curvature K in five dimensions. In particular, the irreducible curvature scalar K may seen to be sourced by a two form which is Poincare dual to a one form on a D_4 -brane underlying a non-linear U(1) gauge theory. One of the striking features in the formalism is bestowed with a fact that the Einstein gravity or a Riemann curvature tensor emerges for a non-propagating torsion in a lower dimension than the original space-time dimension of a non-perturbative torsion dynamics. As a result, various quantum vacua may formally be re-expressed by a non-perturbative correction to the respective Einstein vacua in presence of an extra dimension. The hidden fifth dimension may seem to be intrinsic to a four dimensional non-perturbative quantum correction which couples to a flat metric. The extra dimension initiates a quantum tunneling among the de Sitter vacua sourced by the potentials or fluxes [10]. In addition, an extra dimension has been shown to incorporate a large degeneracy in the quantum Kerr (Newman) vacua due to the charge independence of its event horizon [14]. A lower energy truncates the degeneracy at the expense of a significant electric non-linear charge to describe a quantum Reissner-Nordstrom black hole in four dimensions for an S_2 restoring geometry. A magnetic non-linear charge can be absorbed by a renormalized mass to describe a quantum Schwarzschild black hole. In a lowest energy limit, a non-linear electric charge reduces to its linear counter-part and gains significance to describe a Kerr-Newman and a Reissner-Nordstrom black hole in Einstein gravity. The quantum Schwarzschild in the limit reduces to a Schwarzschild black hole.

On the other hand, a generic torsion curvature has been argued to begin with a vacuum created pair of $(D\bar{D})$ -instant on at the Big Bang by a two form [10]. A pair of lower dimensional bran arguably leads to one higher dimensional pair and the vacuum creation for a brane and anti-bran goes on until $p=8$. A pair of $(D\bar{D})_8$ -brane with space-time curvature underlie a gauge theory on a D_9 -brane and fills the superstring space-time. In principle a pair of $(D\bar{D})_9$ -brane may define an M_{10} brane in an eleven dimensional M-theory. Analysis suggests that a space-filling

*Corresponding author: Supriya Kar, Department of Physics & Astrophysics, University of Delhi, New Delhi 110 007, India, Tel: +91 9911918174; E-mail: skkar@physics.du.ac.in

Received December 03, 2013; Accepted December 03, 2013; Published December 04, 2013

Citation: Kar S (2013) Non-Perturbative Quantum Gravity in Five Dimensions. J Astrophys Aerospace Technol 2:e106. doi:10.4172/2329-6542.1000e106

Copyright: © 2013 Kar S. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

brane in M-theory is presumably described by a generalized irreducible curvature scalar $K^{(11)}$ underlying a geometric torsion. However on S1 the local degrees in torsion may freeze, at the expense of the local degrees in a non-perturbative correction, to describe a Riemannian curvature scalar in a ten dimensional type IIA superstring effective theory. Presumably, all the local degrees in an eleven dimensional torsion manifest themselves into a non-perturbative correction to a perturbative superstring vacuum.

Non-Perturbative Space-Time Curvature

It is inspiring to explore the quanta of a dynamical two form in a non-linear U (1) gauge theory on a D_4 -brane. Nevertheless a two form is Poincare dual to a one form in a non-linear gauge theory. A two form source to a string world-sheet is known to yield an effective theory of quantum gravity perturbatively [4]. It provokes thought to believe that a two form (or a one form) non-linear quanta at a gauge theoretic vacuum on a D_4 -brane creates a pair of $(D\bar{D})_3$ -brane presumably with a Big Bang [10]. The vacuum created brane and anti-brane, respectively, carry an appropriate new Ramond-Ramond charge and an anti-charge and hence signify a non-perturbative brane-Universe. They began to move in opposite directions to each other immediately after their creations. The quantum phenomenon may be viewed analogous to a pair of particle/anti-particle creation at a black hole vacuum [1] or otherwise in QFT. In other words, a nontrivial space-time curvature in a four dimensions began from a curvature singularity at a Big Bang. An extra dimension between a pair of $(D\bar{D})_3$ -brane becomes significant in a low energy limit where Einstein vacuum emerges.

A priori the required modification, to incorporate a geometric notion on a D_4 -brane, may be viewed through a modified covariant derivative defined with a completely anti-symmetric gauge connection: $H_{\mu\nu}^\lambda$. The appropriate covariant derivative may be given by

$$D_\lambda B_{\mu\nu} = \nabla_\lambda B_{\mu\nu} + \frac{1}{2} H_{\lambda\mu}{}^p B_{p\nu} - \frac{1}{2} H_{\lambda\nu}{}^p B_{p\mu} \quad (1)$$

Under an iteration $H_3 \rightarrow H_3$, the geometric torsion in a second order formalism may be defined with all order corrections in B_2 in a gauge theory. Formally, a geometric torsion may be expressed in terms of gauge theoretic torsion and its coupling to two forms. It is given by

$$H_{\mu\nu\lambda} = 3D_{[\mu} B_{\nu\lambda]} = 3\nabla_{[\mu} B_{\nu\lambda]} + 3H_{[\mu\nu}{}^\alpha B_{\lambda]\alpha} g_{\alpha\beta} \quad (2)$$

An exact covariant derivative in a perturbative gauge theory may see to define a non-perturbative covariant derivative in a second order formalism. Thus, a geometric torsion, constructed through a non-perturbative covariant derivative (1), may equivalently be described by an appropriate curvature tensor $K_{\mu\nu\lambda}{}^p$ as worked out in [10]. Explicitly,

$$K_{\mu\nu\lambda}{}^p = \frac{1}{2} \partial_\mu H_{\nu\lambda}{}^p - \frac{1}{2} \partial_\nu H_{\mu\lambda}{}^p + \frac{1}{4} H_{\mu\lambda}{}^\sigma H_{\mu\sigma}{}^p - \frac{1}{4} H_{\nu\lambda}{}^\sigma H_{\mu\sigma}{}^p \quad (3)$$

The fourth order tensor is anti-symmetric within a pair of indices, i.e. $\mu \leftrightarrow \nu$ and $\lambda \leftrightarrow \rho$, which retains a property of Riemann tensor $R_{\mu\nu\lambda\rho}$. However the effective curvatures $K_{\mu\nu\lambda}{}^p$ do not satisfy the symmetric property, under an interchange of a pair of indices, as in Riemann tensor. Never the-less, for a constant torsion the generic tensor: $K_{\mu\nu\lambda}{}^p \rightarrow R_{\mu\nu\lambda}{}^p$. As a result, the effective curvature constructed in a non-perturbative formalism may be viewed as a generalized curvature tensor. It describes the propagation of a geometric torsion in a second order formalism. The gauge dynamics on a D_4 -brane, in presence of gauge connections, may be approximated by an irreducible gener-alized curvature scalar. Generically, a geometric action leading to non-perturbative quantum gravity may be given by

$$S_{QG} = \frac{1}{3C_4^2} \int d^5x \sqrt{-gK} \quad (4)$$

Where $C_4^2 = (8\pi^3 g_s) \alpha'^{3/2}$ is constant. The flat metric $g_{\mu\nu}$ on a D_4 -brane defines $g = \det g_{\mu\nu}$. A vacuum energy density $\Lambda \neq 0$ in the geometric action is sourced by a global two form in presence of its local modes.

Emergent Metric

Most importantly the U (1) gauge invariance of a constructed geometric torsion H_3 , in an appropriate combination, incorporates a notion of space-time and hence validates a second order formalism. It may imply that a space-time curvature on a brane-Universe began with the propagation of a geometric torsion. It may worth mentioning that a non-perturbative metric fluctuation, leading to a quantum gravity description, is significantly large and is very much unlike to a perturbative quantum fluctuation. A non-perturbative space-time quantum, sourced primarily by a two form flux, underlying a U (1) gauge invariance may be given by

$$f_{\mu\nu}^{NP} = C H_{\mu\alpha\beta} \bar{H}^{\alpha\beta}{}_\nu \quad (5)$$

Where C is an arbitrary constant and $\bar{H}_{\mu\nu\lambda} = (2\pi\alpha') H_{\mu\nu\lambda}$. Thus, the local degrees in a two form manifest themselves to incorporate a dynamical geometric torsion in a theory of non-perturbative quantum gravity. In other words, a non-perturbative correction significantly modifies a constant metric on a D_4 -brane. Nevertheless, the significance of a zero mode in two forms in the emergent geometry may not be ignored. They signify a non-zero vacuum energy density in the brane-Universe.

$$G_{\mu\nu} = g_{\mu\nu} + C_0 B_{\mu\lambda} B^\lambda{}_\nu + C \bar{H}_{\mu\lambda\rho} H^{\lambda\rho}{}_\nu \quad (6)$$

Various appropriate B_2 background fluctuations in the formalism presumably lead to a large number of quantum tunneling vacua and may incorporate a landscape scenario conjectured in a superstring theory. The background fluctuations in two forms may have their origin in a higher dimensional H_3 and hence their presence is vital to the non-perturbative quantum gravity in the formalism.

M-theory: An attempt

The idea of a geometric torsion leading to a non-perturbative quantum curvature theory in five dimensions may formally be uplifted on a M_{10} -brane to address the bosonic sector of a conjectured M-theory in eleven dimensions. A priori, non-perturbative quantum gravity may be formalized by a geometric torsion in the conjectured M-theory. In addition to appropriate topological couplings, the dynamical content in the bosonic sector may be obtained from a generalization of a two form dynamics on a D_4 -brane. It may be given by

$$S_M \rightarrow \frac{1}{k^2} \int d^{11}x \sqrt{-g} \left(K^{11} - \frac{1}{2.4!} F_4^2 \right) \quad (7)$$

In a low energy limit, the local degrees in a geometric torsion may seen to be insignificant. In the limit, a large r dominates and hence the non perturbative quantum correction reduces drastically and leads to a (semi) classical vacuum in Einstein gravity [14]. In a gauge choice, for a non-propagating torsion, the formalism may reduce to a vacuum in an eleven dimensional super gravity theory. However, it remains to check some of the detailed intricacies of quantum fields to define an appropriate M-theory action. The formalism may be helpful to an enhance understanding of dark energy in a brane-Universe.

Acknowledgments

The work of S.K. is partially supported by a research grant-in-aid under the Department of Science and Technology, Government of India.

References

1. Hawking SW (1975) Particle creation by black holes. *Comm Math Phys* 43: 199-220.
2. Gross DJ (1984) Is Quantum Gravity Unpredictable? *Nucl Phys B*236.
3. Hoofdt GT (1985) On the Quantum Structure Of A Black Hole. *Nucl Phys B*256: 727-745.
4. Green MB, Schwarz JH, Witten E (1987) *Superstring theory-I*. Cambridge Univ Press.
5. Polchinski J (1995) Dirichlet-Branes and Ramond-Ramond Charges. *Phys Rev Lett* 75: 4724-4727.
6. Seiberg N, Witten E (1999) String Theory and Non commutative Geometry. *JHEP* 9909: 032.
7. Witten E (1995) String Theory Dynamics in Various Dimensions. *Nucl Phys B*443: 85-126.
8. Kar S, Pandey KP, Singh S, Singh AK (2013) Quantum Kerr(Newman) degenerate stringy vacua in 4D on a non-BPS brane.
9. Kar S, Pandey KP, Singh S, Singh AK (2011) *World Sci Proceeding C*10-02-24 (2011) 567.
10. Kar S, Pandey KP, Singh S, Singh AK (2013) Discrete torsion, de Sitter tunneling vacua and AdS brane: U(1) gauge theory on D 4-brane and an effective curvature.
11. Kar S, Pandey KP, Singh S, Singh AK (2013) Emergent Schwarzschild and Reissner-Nordstrom black holes in 4D: An effective curvature sourced by a B2-field on a D4-brane.
12. Kar S, Pandey KP, Singh S, Singh AK *Nucl Phys B Proceeding*.
13. Kar S, Pandey KP, Singh S, Singh AK (2013) Quantum Kerr tunneling vacua on a ($D^- D$)4-brane: An emergent Kerr black hole in five dimensions.
14. Kar S, Pandey KP, Singh S, Singh AK (2013) Quantum Kerr(Newman) degenerate stringy vacua in 4D on a non-BPS brane.
15. Kar S, Pandey KP, Singh S, Singh AK (2010) Gravity dual D3-braneworld and Open/Closed string duality.
16. Kar S, Pandey KP, Singh S, Singh AK (2010) Emergent gravity/Non-linear U(1) gauge theory correspondence.