

Nonlinear Robust Power Control of Uncertain DC-DC Converter with Bilinear Dynamics

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Abstract

The DC-DC converter with chopper method is the briefest DC-DC converters. Chopper means cutting, and it comes from that cutting electric current by switching with the chopper and convert the voltage. A coil works importantly by the chopper method. The electric current flowing through the circuit at every ON/OFF of the switching element suddenly changes, but the coil generates electromotive force to disturb a current change and produces an induced current (Lenz's law). From this property that this coil behavior as if breath comes to be jam-packed in the electric current like resistance for the alternating current repeating a current change, the coil is called a choke coil. DC-DC converter with the chopper method is the simple circuit which contains a switching element a choke coil, a capacitor, and a diode together, and step down or boosts DC voltage. The purpose of this study is robust control in consideration of the nonlinearity of the DC-DC converter. Switching-mode DC-DC converter controls electricity by opening and shutting of the semiconductor switch. This type of DC-DC converter is small size, light weight and high efficiency. To do output voltage control in consideration of the nonlinearity of the DC-DC converter, the model formula is derived to use state space averaging method. The control system is designed in consideration of the model-type nonlinearity using the operator theory that was one of the nonlinear control theories and confirmed the effectiveness of the control system by MATLAB and experiments with a breadboard.

Keywords: DC-DC converter; Buck boost converter; Power control; Right co-prime factorization; Nonlinear control

Introduction

Various electric apparatuses which we use every day require various voltages. For example, a DC motor works at DC 12 volt, a microcomputer works at DC 5 volt, and a CPU fan works at DC 24 V. In voltage conversion, a DC-DC converter is used to convert direct current power.

DC-DC converters are the most popular devices to adapt voltage and current levels between DC sources and DC loads, since the switching-type DC-DC converter has few losses and high efficiency in the conversion process and can downsize a part by increasing switching frequency [1-3].

When the switching element becomes switch on, electric current flows into the choke coil and the coil saves electric energy. When the switching element become switch off, the coil release saved energy and drain an induced current into the direction disturbing an electric current change. The base of the transistor is connected to a control circuit, and a sent square pulse from control circuit carries out switching. The output voltage rises so that time of on is long, and fall down so that time of off is long. Therefore, the desired output voltage is provided by controlling at time of on/off (duty cycle). It is a capacitor (electrolysis capacitor) and a choke coil to occupy the big space on a board. DC-DC converter with the chopper method includes the type called back boost converter which can step down and boost together too. This is what reversed the direction of the diode of the back converter, and the polar of the output voltage turns over. This is also called the polar inversion type. This research treats back boost converter.

Because of parametric uncertainty and current or voltage disturbances, DC-DC converters require a control system to maintain the desired levels of current or voltage. Such a control system needs to assure the stability of the converter and tracking a target value and to resist the disturbance.

Pulse Width Modulation (PWM) is the most extended modulation

method in DC-DC converters. The power stage of the converter with PWM modulation is usually expressed in state space averaging method. They can take into account the inherent nonlinearity of the converter, as bilinear terms and saturations of the control input and the states [3,4].

In this paper, nonlinear control system for a DC-DC converter using robust right co-prime factorization is designed. The detailed explanation is as follows. The nonlinear control system for the model formula of DC-DC converter considering uncertainties and the nonlinearity is designed by using operator-based robust right co-prime factorization. Then, the simulation and experimental results are given to show the effectiveness of the proposed scheme.

In this study, the type of DC-DC converter to treat is buck boost converter. A schema of buck boost converter is shown in Figure 1.

Buck boost converter is the DC-DC converter circuit which can drop and boost input voltage. In addition, polarity of the output voltage is opposite to the input voltage. Output voltage of the buck boost converter in the steady state V_o is expressed in (1).

$$V_o = -V_i \frac{D}{1-D} \quad (1)$$

V_i is input voltage, D is the duty ratio of the switching element. Since duty ratio D changes in the range of $0 < D < 1$, output voltage V_o changes in the range of $-\infty < V_o < 0$.

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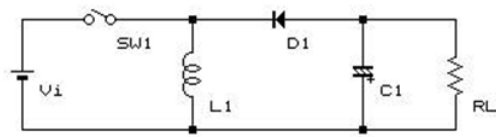


Figure 1: Schema of Buck Boost Converter.

$D[-]$:	Duty ratio
$L[H]$:	Inductance
$R[\Omega]$:	Load Resistance
$C[F]$:	Capacitance of a Capacitor
$v_g[V]$:	Input Voltage
$v_d[V]$:	Diode Voltage
$i_L[A]$:	Inductor Current
$v_c[V]$:	Capacitor Voltage
$v_o[V]$:	Output Voltage
$i_o[A]$:	Output Current

Table 1: Parameter Definition.

Modelling

In this section, the modelling of the DC-DC converter for a control system design is shown. Using state space averaging method, the output voltage is expressed in a time domain.

State space averaging method

This method is the way which weight charge account by the duty ratio in two circuit equations in switch on and off and this is a method to derive a model type [4]. Parameters to use for modeling are shown in Table 1.

At first, i_L and v_c are chosen as a state variable and circuit equations of the each state at the switch-on time and switch-off time are derived.

$$\text{switch-on time: } v_g = L \frac{di_L(t)}{dt}, -RC \frac{dv_c(t)}{dt} = v_c \quad (2)$$

$$\text{switch-on time: } v_c(t) = v_d + L \frac{di_L(t)}{dt}, i_L(t) = -C \frac{dv_c(t)}{dt} - \frac{v_c(t)}{R} \quad (3)$$

Successively, weighting by the duty ratio is done.

$$\text{switch-off time: } D \frac{di_L(t)}{dt} = \frac{D}{L} v_g, D \frac{dv_c(t)}{dt} = -\frac{D}{RC} v_c(t) \quad (4)$$

$$\text{switch-off time: } (1-D) \frac{di_L(t)}{dt} = \frac{1-D}{L} v_c(t) - \frac{1-D}{L} v_d \quad (5)$$

$$\text{switch-off time: } (1-D) \frac{dv_c(t)}{dt} = \frac{1-D}{C} i_L(t) - \frac{1-D}{RC} v_c(t) \quad (6)$$

Finally, (5) and (6) are added and the differential equation of the model formula of DC-DC converter is derived.

$$\frac{di_L(t)}{dt} = \frac{1-D}{L} v_c(t) + \frac{D}{L} v_g - \frac{1-D}{L} v_d \quad (7)$$

$$\frac{dv_c(t)}{dt} = \frac{1-D}{C} i_L(t) - \frac{1}{RC} v_c(t) \quad (8)$$

Simultaneous differential equations (7) and (8) are solved for v_c

$$v_c = Y \left(\frac{1}{Z} + 1 \right) e^{\frac{-1+Z}{X} t} + Y \left(-\frac{1}{Z} + 1 \right) e^{\frac{-1-Z}{X} t} - 2Y \quad (9)$$

Where, $X = 2RC$

Control System

Using the operator theory that is one of the nonlinear control theories, control system is designed. The conceptual diagram of control system is shown as follows (Figure 2).

Robust right co-prime factorization

Some definitions about robust right co-prime factorization are introduced. Consider nonlinear plant $P: U \rightarrow Y$ where U is input space and Y is output space. Right factorization is defined as $P \rightarrow ND^{-1}$ where stable operator $N: W \rightarrow Y$ and $D: W \rightarrow U$ such that D is invertible from U to W . The space W is called a quasi-state space of P . If P has a right co prime factorization, the Bezout identity $AN + BD = M$ is satisfied, where stable operator $A: Y \rightarrow U$, $B: U \rightarrow U$ such that B is invertible and M is unimodular operator. The actual plant has the uncertainties which contain the modeling error and the disturbance. The uncertainties are given as, ΔP where ΔP is unknown but upper and lower bounds are known. The robust right co-prime factorization of the actual plant $P + \Delta P$ which contains the uncertainties is shown as follows.

$$P + \Delta P \rightarrow (N + \Delta N) D^{-1} \quad (10)$$

$$\text{The system is stable if (11) and } A(N + \Delta N) + BD = \hat{M} \quad (11)$$

$$\text{where } \hat{M} \text{ is unimodular, or } [A(N + \Delta N) - AN] M^{-1}_{Lip} < 1 \quad (12)$$

are satisfied. $\|A\|_{Lip}$ is called Lipschitz norm of A and defined as follows [5-8].

$$\|A\|_{Lip} = \sup_{T \in [0, \infty]} \sup_{\substack{x, \tilde{x} \in D \\ x_T \neq \tilde{x}_T}} \frac{[A(x)]_T - [A(\tilde{x})]_T}{x_T - \tilde{x}_T} \quad (13)$$

As input signal u is duty ratio D and output signal y is capacitor voltage v_c , control system design using the robust right co-prime factorization based on an operator theory is done. The block diagram of the nonlinear feedback control system is shown in Figure 3.

Compensation of stability

To begin with, a nominal model $y = P(u)$ which doesn't include uncertainties transform into N and D^{-1} . Furthermore, the nonlinear closed loop is made BIBO stable by designing each operator to satisfy Bezout equation $AN + BD = M$. N and D^{-1} become a right co-prime factorization. Where controller A is stable, B is stable and invertible controller, and M is unimodular operator.

Operator corresponding to the nominal model is found.

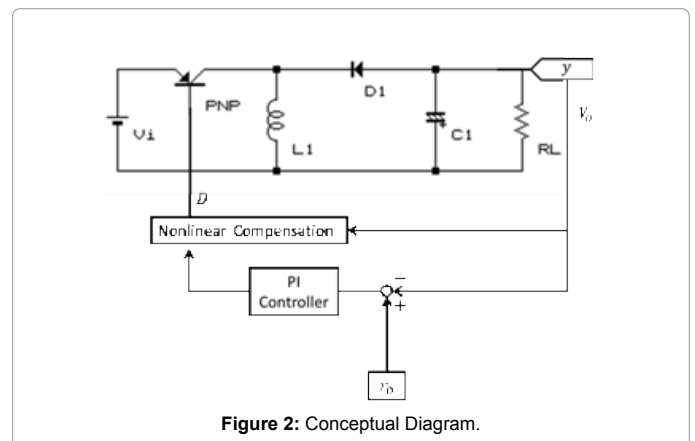


Figure 2: Conceptual Diagram.

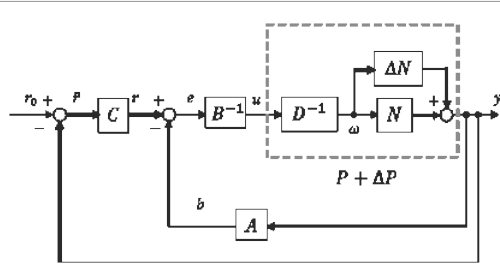


Figure 3: Nonlinear Feedback Control System.

load resistance	20 Ω
Inductance	1.0x10 ⁻² H
Capacitance of a Capacitor	3.3x10 ⁻⁶ F
Diode Voltage	0.67V
Input Voltage	3V
Target value	-3V
Sampling Time	0.05s
Design Parameter	K=10
	K _i =8
	K _p =0.1

Table 2: Simulation Condition.

$$B : e(t) = \log K + \log \left\{ \left(-\frac{4R^2C}{L} (1-u(t))^2 \right)^2 \right\} \quad (14)$$

$$A : b(t) = \log \left\{ \left(\frac{y}{y + 2Xy + X^2y} \right)^2 \right\} \quad (15)$$

$$D : u(t) = \frac{\omega(t)}{1 + \omega(t)} \quad (16)$$

$$N : y(t) = Y(\omega(t)) \left(1 + \frac{1}{Z(\omega(t))} \right) e^{\frac{-1+Z(\omega(t))}{X}t}$$

$$+ Y(\omega(t)) \left(1 - \frac{1}{Z(\omega(t))} \right) e^{\frac{-1-Z(\omega(t))}{X}t} - 2Y(\omega(t)) \quad (17)$$

$$Y(\omega(t)) = \frac{1}{2} (v_s \omega(t) - v_D) \quad (18)$$

$$Z(\omega(t)) = \sqrt{1 - \frac{4R^2C}{L} \left(\frac{1}{1 + \omega(t)} \right)^2} \quad (19)$$

Meanwhile, K is design parameter.

Improvement of following

To let an output signal follows the targeted value, controller C is designed as follows.

$$C \left(\overset{i}{r} \right) (t) = K_i \int_0^t \overset{i}{r}(\tau) d\tau + K_p \overset{i}{r}(t) \quad (20)$$

Simulation

On the output voltage control of the DC-DC converter, numerical simulation in MATLAB is done. The conditions of the simulation are shown in Table 2.

Figure 4 is the output voltage of the DC-DC converter under nonlinearity compensation by the designed controller. Output voltage appears to follow -3 volts of target value without an overshoot. Figure 5 shows a control input (the duty ratio) under nonlinearity compensation by the designed controller. Control input falls in the correct range of the duty ratio $0 < D < 1$, the output voltage appears controlled definitely.

Figure 6 is the output voltage of the DC-DC converter without nonlinearity compensation by the designed controller, but with only PI control. Target value is -3\$ volts. However, output voltage in Figure 6 is varying widely. It appears difficult to control output voltage of DC-DC converter with only PI controller. Figure 7 shows a control input (the duty ratio) without nonlinearity compensation by the designed controller, but with only PI control. For safety, a range of duty ratio is restricted from 0.2 to 0.8. However, control input is varying widely within this limit. Therefore, the output voltage appears not to be controlled definitely.

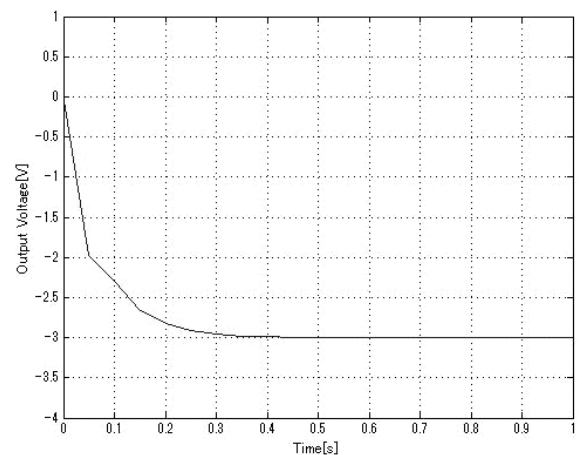


Figure 4: Output Voltage (with Compensation).

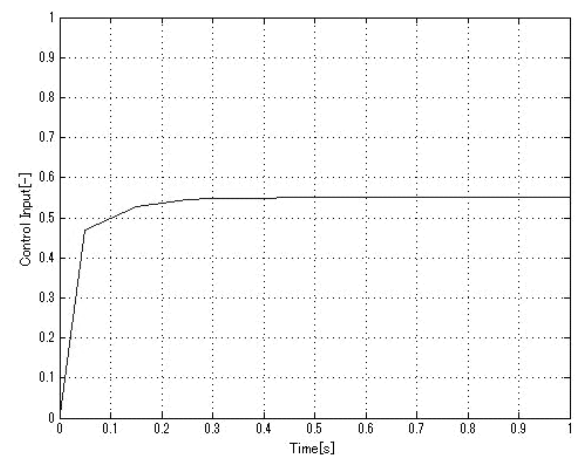


Figure 5: Control Input (with Compensation).

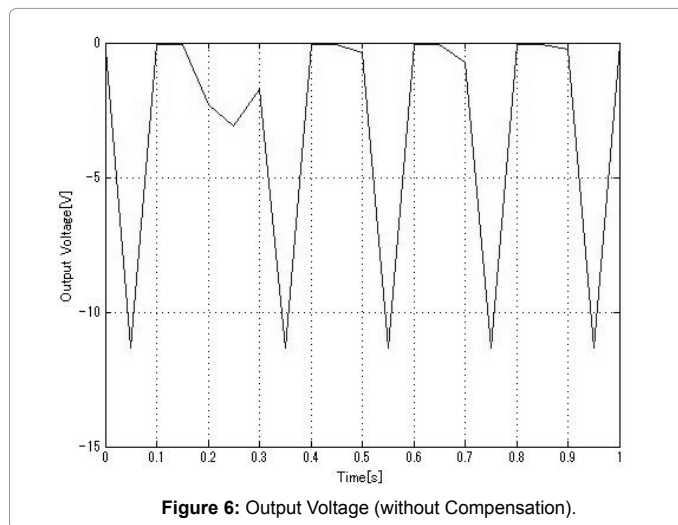


Figure 6: Output Voltage (without Compensation).

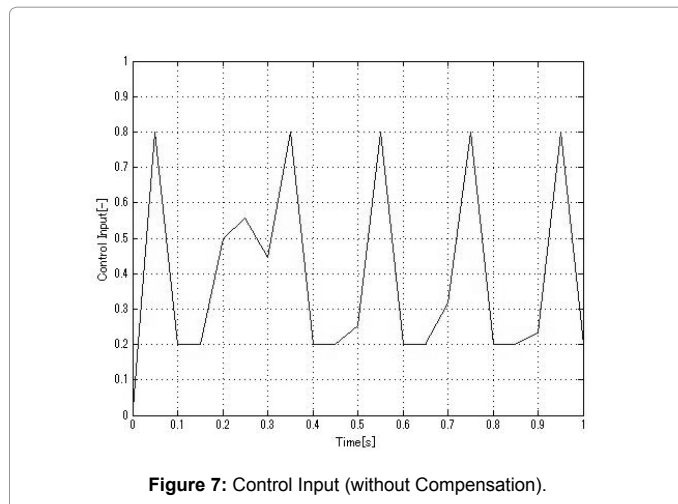


Figure 7: Control Input (without Compensation).

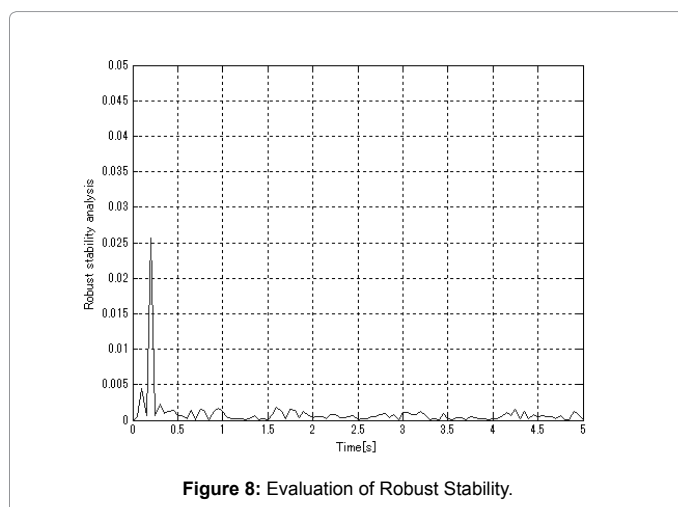


Figure 8: Evaluation of Robust Stability.

Then, by using (21), evaluation of robust stability is done. (21) is introduced again.

$$\| [A(N + \Delta N) - AN] M^{-1} \|_{Lip} < 1 \quad (21)$$

-5V	Target Voltage
40 mH	Inductance
100Ω	Load Resistance
1000μF:	Capacitance of a Capacitor
3V	Input Voltage
0.67V	Diode Voltage
0.2s	sampling period
100 kHz	switching frequency

Table 3: Parameter in Experiment.

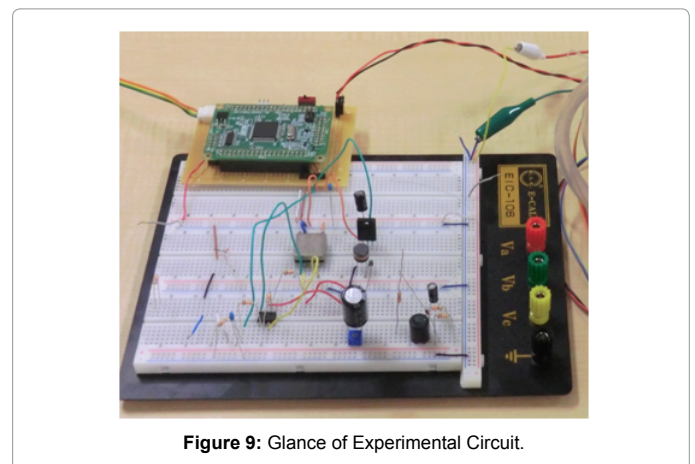


Figure 9: Glance of Experimental Circuit.

Each parameter of resistance, a condenser and the coil adds up to $\pm 10\%$ of errors in random as uncertainties. A result of evaluation of robust stability is shown as follow.

From the result of evaluation, the size of Lipschitz norm is much smaller than 1. In this verification, considered uncertainties change at random. Nevertheless, in any case, Lipschitz norm is much smaller than 1 (Figure 8).

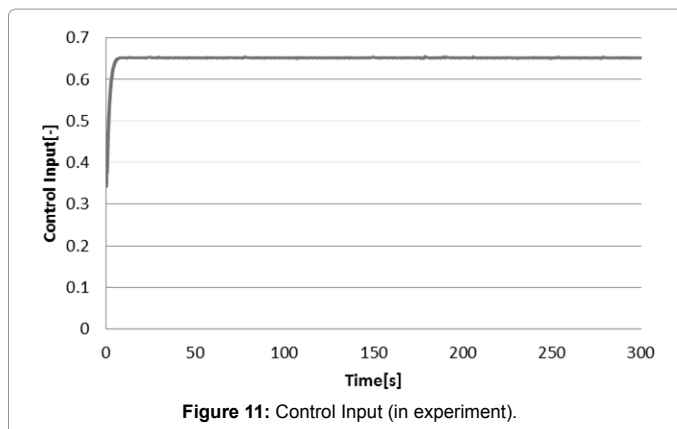
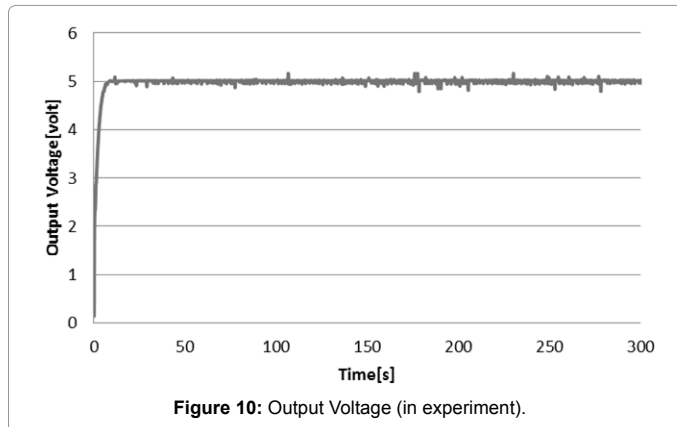
Experiment

An experimental circuit is mounted on a breadboard and voltage control experiment of back boost converter is done. Using microcomputer H8-3052. Parameters of circuit elements are shown as follows (Table 3).

Figure 9 shows an overview of mounted experimental circuit. With microcomputer H8-3052, measurement of voltage and calculation and generation of block pulse for control input are done. In addition, since a normal microcomputer cannot measure negative voltage, a polar character of output voltage inverts with an inverting amplifier whose gain is 1. Figure 10 shows output voltage of DC-DC converter with nonlinear compensation and Figure 11 shows control input. Output voltage is following to 5 V of target voltage. Moreover, control input varies within $0 < D < 1$ and becomes constant as output voltage follows.

Conclusion

In this paper, an operator-based nonlinear control of uncertain DC-DC converter with bilinear system by using robust right co-prime factorization is proposed. The nonlinear controllers based on robust right co-prime factorization for the nonlinear model with the uncertainties are designed. Finally, simulation and experimental results are given to show the effectiveness of the designed controller.



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