

New Adomian's Polynomials Formulas for the Non-linear and Non-autonomous Ordinary Differential Equations

Zaouagui IN^{1*} and Badredine T²

¹Department of Physics, University El-Hadj Lakhdar, Batna 1, Batna 05000, Algeria

²Department of Mathematics, University Ferhat Abbas Setif 1, Setif 19000, Algeria

Abstract

In this paper, Adomian decomposition method has been adopted to resolve the non-linear and non-autonomous ordinary differential equations. It has been proved that this technique permits to give new expressions for the Adomian's polynomials (??) and (??).

Keywords: Decomposition method; Adomian's polynomials; Ordinary differential equations; Cauchy's problems

Introduction

The principle of the Adomian decomposition method consists in decomposing the nonlinear operator into a series of functions (where each term is called Adomian's Polynomial) and then to calculate the solution as a series of function where each term can be easily determined with Adomian's algorithm [1-15].

The previous works concerning this method deal only with the case of non-linear autonomous differential equation see [16-27]. In this paper, this technique is applied to non-linear and non-autonomous differential equations. By using a classical theorem for the n^{th} derivate of the compositions of functions, new expressions of Adomian's polynomials are determined [28-37].

Numerical examples treated in this work in order to compare the Adomian decomposition method with other classical methods such as Euler, Heun and the Runge Kutta method of the 4th order.

Application of the Decomposition Method to Non-autonomous and Non-linear Differential Equations

Let us consider the following non-linear and non-autonomous differential equation:

$$\frac{du}{dt} = f(t, u) \quad (1)$$

$$u(t_0) = c \quad (2)$$

where f is the non-linear term.

By integrating both sides the equation (??) we obtain:

$$u = \int_{t_0}^t f(s, u) ds + c \quad (3)$$

The canonical form [11,14] of Cauchy's problem of the eqn. (3) is:

$$u = \lambda \int_{t_0}^t f(s, u) ds + c \quad (4)$$

The Adomian's method consists in setting the solution in a series form:

$$u = \sum_{n \geq 0} \lambda^n u_n \quad (5)$$

and the non-linear term $f(t, u)$ is written by the form:

$$f(t, u) = \sum_{n \geq 0} \lambda^n A_n \quad (6)$$

where the A_n 's are terms dependent on t, u_0, u_1, \dots, u_n and are called Adomian's polynomials

Inserting eqns. (5) and (6) into eqn. (4) leads to:

$$\sum_{n \geq 0} \lambda^n u_n = c + \sum_{n \geq 0} \int_{t_0}^t \lambda^{n+1} A_n ds \quad (7)$$

which implies:

$$u_0 = c \quad \text{and} \quad u_n = \int_{t_0}^t A_{n-1} ds \quad \text{for} \quad n \geq 1 \quad (8)$$

Remark

In practice it's difficult to calculate all the terms of the series (5) that's why an approximation of the solution is used by truncating the series: $\phi_n = \sum_{i=0}^{n-1} \lambda^i u_i$.

Notations

Let $n \in \mathbb{N}, k = (k_1, k_2, \dots, k_n) \in \mathbb{N}^n, t \in \mathbb{R}$ and f is a multivariate function, we note:

$$\begin{cases} |k| = k_1 + k_2 + \dots + k_n \\ nk_n = k_1 + 2k_2 + \dots + nk_n \\ \partial_t^n f = \frac{\partial^n f}{\partial t^n} \end{cases} \quad (9)$$

Adomian's Polynomials

Theorem

The terms A_n are given by the formulas:

$$\begin{cases} A_0(t, u_0) = f(t_0, u_0) \\ A_n(t, u_0, \dots, u_n) = \frac{1}{n!} \frac{d^n}{d\lambda^n} f \left(t_0 + \lambda(t - t_0), \sum_{m \geq 0} \lambda^m u_m \right) \Big|_{\lambda=0} \end{cases} \quad \text{for} \quad n \geq 1 \quad (10)$$

Proof

For convenience we define two operators F and L as follows:

***Corresponding author:** Zaouagui IN, Department of Physics, University El-Hadj Lakhdar Batna 1, Batna 05000, Algeria, Tel: 21333319134; E-mail: idriss.zaouagui@univ-batna.dz

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$$\begin{cases} F(U) = (1, f(t, u)) \\ L(U) = \frac{d}{dt}U = (1, u') \end{cases} \text{ where } U = (t, u)$$

From equation (??) we have: $L(U)=F(U)$, so $L^{-1}(L(U))= L^{-1}(F(U))$ and it follows that:

$$\left(\int_{t_0}^t ds, \int_{t_0}^t u'(s) ds \right) = L^{-1}(F(U))$$

We obtain an equation of the form:

$$U=N(U)+C \text{ where } N(U)=L^{-1}F(U) \text{ and } C=(t_0, c)$$

According to Abbaoui et al. and Himoun et al. [11,14], the Adomian's polynomials A_n associated to $N(U)$ are given by:

$$\begin{cases} A_0(U_0) = N(U_0) \\ A_n(U_0, \dots, U_n) = \frac{1}{n!} \frac{d^n}{d\lambda^n} N \left(\sum_{m=0}^n U_m \lambda^m \right) \Big|_{\lambda=0} \text{ for } n \geq 1 \end{cases}$$

where again

$$\begin{cases} U_0 = C \\ U_1 = A_0(U_0) = N(U_0) \\ U_n = A_{n-1}(U_0, \dots, U_{n-1}) = \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} N \left(\sum_{m=0}^n U_m \lambda^m \right) \Big|_{\lambda=0} \text{ for } n \geq 2 \end{cases}$$

This translates into:

$$\begin{cases} U_0 = (t_0, c) \\ U_1 = L^{-1}(F(U_0)) = L^{-1}(1, f(t_0, c)) \\ U_n = \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} L^{-1} \left(F \left(\sum_{m=0}^n U_m \lambda^m \right) \right) \Big|_{\lambda=0} \\ = \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} L^{-1} \left(1, f \left(\sum_{m=0}^n U_m \lambda^m \right) \right) \Big|_{\lambda=0} \text{ for } n \geq 2 \end{cases}$$

As a result of this:

$$\begin{cases} U_0 = (t_0, c) \\ U_1 = \left(t-t_0, \int_{t_0}^t f(t_0, c) ds \right) \\ U_n = \frac{1}{(n-1)!} \left(\frac{d^{n-1}}{d\lambda^{n-1}} (t-t_0) \Big|_{\lambda=0}, \int_{t_0}^t \frac{d^{n-1}}{d\lambda^{n-1}} f \left(\sum_{m=0}^n U_m \lambda^m \right) \Big|_{\lambda=0} ds \right) \\ = \left(0, \int_{t_0}^t \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} f \left(U_0 + U_1 \lambda + \sum_{m=2}^n U_m \lambda^m \right) \Big|_{\lambda=0} ds \right) \text{ for } n \geq 2 \end{cases}$$

Considering the fact that the sequence $(U_n)_{n \in \mathbb{N}}$ is defined by: $U_0=(t_0, u_0), U_1=(t-t_0, u_1)$ and $U_n=(0, u_n)$ for $n \geq 2$, we obtain:

$$\begin{cases} u_0 = c \\ u_1 = \int_{t_0}^t f(t_0, c) ds \\ u_n = \int_{t_0}^t \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} f \left((t_0, u_0) + (t-t_0, u_1) \lambda + \sum_{m=2}^n (0, u_m) \lambda^m \right) \Big|_{\lambda=0} ds \\ = \int_{t_0}^t \frac{1}{(n-1)!} \frac{d^{n-1}}{d\lambda^{n-1}} f \left(t_0 + (t-t_0) \lambda, \sum_{m=0}^n u_m \lambda^m \right) \Big|_{\lambda=0} ds \text{ for } n \geq 2 \end{cases}$$

which ends the proof by a simple identification with eqn. (8).

Lamma

Let f be a function of two variables u and v . The Adomian's polynomials associated with f are given by the relationship [3,4]:

$$\begin{cases} A_0(u_0, v_0) = f(u_0, v_0) \\ A_n(u_0, \dots, u_n; v_0, \dots, v_n) = \sum_{|m_k|+|n_k|=n} \partial_u^{(k)} \partial_v^{(l)} f(u_0, v_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{v_1^{l_1}}{l_1!} \dots \frac{v_n^{l_n}}{l_n!} \text{ for } n \geq 1 \end{cases} \quad (11)$$

Theorem

The A_n 's are given by:

$$\begin{cases} A_0(t; u_0) = f(t_0, u_0) \\ A_n(t; u_0, \dots, u_n) = \sum_{m_1+|n_k|=n} \partial_t^{m_1} \partial_u^{(k)} f(t_0, u_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{(t-t_0)^{m_1}}{m_1!} \text{ for } n \geq 1 \end{cases} \quad (12)$$

Proof

It's sufficient to apply the eqn (11) to $f(t, u)$ by using the following decompositions: $t = \sum_{n \geq 0} t_n$ and $u = \sum_{n \geq 0} u_n$ where $t_1=t-t_0$ and $t_n=0$ for $n \geq 2$, so we have:

$$\begin{aligned} A_n(t; u_0, \dots, u_n) &= \sum_{|m_n|+|n_k|=n} \partial_t^{m_1} \partial_u^{(k)} f(t_0, u_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{t_1^{m_1}}{m_1!} \\ &= \sum_{\substack{|m_n|+|n_k|=n \\ m_1+n_k \neq n}} \partial_t^{m_1} \partial_u^{(k)} f(t_0, u_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{(t-t_0)^{m_1}}{m_1!} \frac{0^{m_2}}{m_2!} \dots \frac{0^{m_n}}{m_n!} \\ &\quad + \sum_{m_1+|n_k|=n} \partial_t^{m_1} \partial_u^{(k)} f(t_0, u_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{(t-t_0)^{m_1}}{m_1!} \\ &= \sum_{m_1+|n_k|=n} \partial_t^{m_1} \partial_u^{(k)} f(t_0, u_0) \frac{u_1^{k_1}}{k_1!} \dots \frac{u_n^{k_n}}{k_n!} \frac{(t-t_0)^{m_1}}{m_1!} \end{aligned}$$

Corollary

If $\alpha \in \mathbb{N}$ and $(\beta_i)_{i=1, \dots, n} \subset \mathbb{N}$ is a decreasing sequence of non-negative integers then:

$$\begin{cases} A_0(t, u_0) = f(t_0, u_0) \\ A_n(t; u_0, \dots, u_n) = \sum_{\alpha+|\beta_i|=n} \partial_t^\alpha \partial_u^{\beta_i} \frac{u_1^{\beta_1-\beta_2}}{(\beta_1-\beta_2)!} \dots \frac{u_{n-1}^{\beta_{n-1}-\beta_n}}{(\beta_{n-1}-\beta_n)!} \frac{u_n^{\beta_n}}{\beta_n!} \frac{(t-t_0)^\alpha}{\alpha!} \text{ for } n \geq 1 \end{cases}$$

Proof

It's sufficient to take: $m=\alpha, k_i=\beta_i-\beta_{i+1}$ for $i=1, \dots, n-1$ and $k_n=\beta_n$.

A_n 's Expressions for $n=5$

The following table shows the solution of the system:

$$\begin{cases} \alpha + \beta_1 + \dots + \beta_n = n \\ \beta_1 \geq \dots \geq \beta_n \end{cases}$$

We can theirs list the Adomian's polynomials for: $n=0, \dots, 5$, (Table 1)

$$\begin{aligned} A_0(t, u_0) &= f(t_0, u_0) \\ A_1(t, u_0, u_1) &= \partial_t f(t_0, u_0)(t-t_0) + \partial_u f(t_0, u_0) u_1 \\ A_2(t, u_0, u_1, u_2) &= \frac{1}{2} \partial_t^2 f(t_0, u_0)(t-t_0)^2 + \partial_t \partial_u f(t_0, u_0) u_1(t-t_0) + \frac{1}{2} \partial_u^2 f(t_0, u_0) u_1^2 + \partial_u f(t_0, u_0) u_2 \\ A_3(t, u_0, u_1, u_2, u_3) &= \frac{1}{6} \partial_t^3 f(t_0, u_0)(t-t_0)^3 + \frac{1}{2} \partial_t^2 \partial_u f(t_0, u_0) u_1(t-t_0)^2 \\ &\quad + \left(\frac{1}{2} \partial_t \partial_u^2 f(t_0, u_0) u_1^2 + \partial_t \partial_u f(t_0, u_0) u_2 \right) (t-t_0) \\ &\quad + \frac{1}{6} \partial_u^3 f(t_0, u_0) u_1^3 + \partial_u^2 f(t_0, u_0) u_1 u_2 + \partial_u f(t_0, u_0) u_3 \\ A_4(t, u_0, u_1, u_2, u_3, u_4) &= \frac{1}{24} \partial_t^4 f(t_0, u_0)(t-t_0)^4 + \frac{1}{6} \partial_t^3 \partial_u f(t_0, u_0) u_1(t-t_0)^3 \\ &\quad + \left(\frac{1}{4} \partial_t^2 \partial_u^2 f(t_0, u_0) u_1^2 + \frac{1}{2} \partial_t^2 \partial_u f(t_0, u_0) u_2 \right) (t-t_0)^2 \\ &\quad + \left(\frac{1}{6} \partial_t \partial_u^3 f(t_0, u_0) u_1^3 + \partial_t \partial_u^2 f(t_0, u_0) u_1 u_2 + \partial_t \partial_u f(t_0, u_0) u_3 \right) (t-t_0) \\ &\quad + \frac{1}{24} \partial_u^4 f(t_0, u_0) u_1^4 + \frac{1}{2} \partial_u^3 f(t_0, u_0) u_1^2 u_2 + \partial_u^2 f(t_0, u_0) \left(\frac{1}{2} u_2^2 + u_1 u_3 \right) \\ &\quad + \partial_u f(t_0, u_0) u_4 \end{aligned}$$

α	β_1	β_2	β_3	β_4	β_5
	0	0	0	0	0
	1	0	0	0	0
	2	0	0	0	0
	1	1	0	0	0
	3	0	0	0	0
	2	1	0	0	0
	1	1	1	0	0
	4	0	0	0	0
	3	1	0	0	0
	2	2	0	0	0
	2	1	1	0	0
	1	1	1	1	0
	5	0	0	0	0
	4	1	0	0	0
	3	2	0	0	0
	3	1	1	0	0
	2	2	1	0	0
	2	1	1	1	0
	1	1	1	1	1

Table 1: A_n 's expressions for n=5.

i	t_i	u_i Euler	u_i Heun	u_i M. Euler	u_i R. K. 4	$\Phi_7(t_i)$	$u(t_i)$
	1.0	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000
	0.9	1.000000000	1.005525855	1.005263555	1.005360533	1.005360500	1.005360516
	0.8	1.011051709	1.023501284	1.022873305	1.023143611	1.023141334	1.023143551
	0.7	1.035751233	1.057260082	1.056129656	1.056675110	1.056632500	1.056674944
	0.6	1.077720962	1.111680314	1.109801825	1.110826058	1.110464000	1.110825624
	0.5	1.142216782	1.194310397	1.191233436	1.193148329	1.191145834	1.193147181
	0.4	1.237251263	1.317746592	1.312558209	1.316293980	1.307728000	1.316290732
	0.3	1.375851781	1.505396365	1.495957568	1.503983311	1.472580500	1.503972804
	0.2	1.581126058	1.808637928	1.788667163	1.809481031	1.702293334	1.809437912
	0.1	1.899476467	2.379025091	2.320967096	2.402810915	2.018696500	2.402585093

Table 2: Adomian decomposition approximation of the order of Taylor's expansion.

$$\begin{aligned}
 A_5(t, u_0, u_1, u_2, u_3, u_4, u_5) = & \frac{1}{120} \partial_x^5 f(t_0, u_0)(t-t_0)^5 + \frac{1}{24} \partial_x^4 \partial_u f(t_0, u_0) u_1 (t-t_0)^4 \\
 & + \left(\frac{1}{12} \partial_x^3 \partial_u^2 f(t_0, u_0) u_1^2 + \frac{1}{6} \partial_x^3 \partial_u f(t_0, u_0) u_2 \right) (t-t_0)^3 \\
 & + \left(\frac{1}{12} \partial_x^2 \partial_u^3 f(t_0, u_0) u_1^3 + \frac{1}{2} \partial_x^2 \partial_u^2 f(t_0, u_0) u_1 u_2 + \frac{1}{2} \partial_x^2 \partial_u f(t_0, u_0) u_3 \right) (t-t_0)^2 \\
 & + \left(\frac{1}{24} \partial_x \partial_u^4 f(t_0, u_0) u_1^4 + \frac{1}{2} \partial_x \partial_u^3 f(t_0, u_0) u_1^2 u_2 + \partial_x \partial_u^2 f(t_0, u_0) \left(\frac{1}{2} u_2^2 + u_1 u_3 \right) \right. \\
 & \left. + \partial_x \partial_u f(t_0, u_0) u_4 \right) (t-t_0) + \frac{1}{120} \partial_x^5 f(t_0, u_0) u_1^5 + \frac{1}{6} \partial_x^4 f(t_0, u_0) u_1^4 u_2 \\
 & + \frac{1}{2} \partial_x^3 f(t_0, u_0) (u_1 u_2^2 + u_1^2 u_3) + \partial_x^2 f(t_0, u_0) (u_1 u_3 + u_1 u_4) + \partial_x f(t_0, u_0) u_5
 \end{aligned}$$

Numerical Example

Let us solve the following equation:

$$\begin{cases} u' = (t-1)\exp(u-t) \\ u(1) = 1 \end{cases}$$

The exact solution of this equation is the function: $u(t) = t - \ln t$. By the Adomian decomposition method, we obtain:

$$\begin{cases} A_0(t) = 0 \\ A_1(t) = t-1 \\ A_2(t) = -(t-1)^2 \\ A_3(t) = (t-1)^3 \\ A_4(t) = -(t-1)^4 \\ A_5(t) = (t-1)^5 \end{cases} \Rightarrow \begin{cases} u_1(t) = 0 \\ u_2(t) = \frac{1}{2}(t-1)^2 \\ u_3(t) = -\frac{1}{3}(t-1)^3 \\ u_4(t) = \frac{1}{4}(t-1)^4 \\ u_5(t) = -\frac{1}{5}(t-1)^5 \\ u_6(t) = \frac{1}{6}(t-1)^6 \end{cases}$$

from where, it follows that:

$$\Phi_7(t) = 1 + \frac{1}{2}(t-1)^2 - \frac{1}{3}(t-1)^3 + \frac{1}{4}(t-1)^4 - \frac{1}{5}(t-1)^5 + \frac{1}{6}(t-1)^6$$

It's easy to see that $\Phi_7(t)$ is an approximation of the 6th order of Taylor's expansion of in the neighborhood of 1.

With the Euler, Heun, Modified Euler and Runge Kutta methods within the interval [0,1], by Considering the subdivision: $t_{i+1} = t_i + 0.1, i=0, \dots, 8$ and $t_0=1$, we obtain by Maple the following results (Table 2).

Conclusion

In this paper we have generalized the application of the decomposition method to non-linear and non-autonomous differential equations. We have obtained new expressions for the Adomian polynomials (??) and (??). The case of autonomous non-linear differential equation [2] becomes, of course, a particular case of this work. Moreover, we note that the Adomian method is more general than some classical methods because the terms of truncated series can be easily deduced in a recursive way. However, the solution coincides quite often with that of another classical method (in the example studied, the solution coincides with the Taylor series).

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