

Multiple Regression on the Example of the Presidential Elections in the United States during the Period from 1916 to 2000

Taenvat MM*

Vladivostok Postgraduate of the Department of State and Municipal Management Institute of Law and Governance, Vladivostok State University Economics and Service, Russia

Abstract

Systematization of knowledge about multiple regression: the withdrawal of the regression equation 1) Through a system of normal equations and 2) Matrix method; solution 1) Using packet analysis EXCEL, 2) Matrix calculator and "matrix arithmetic"; 3) Solution in MATLAB as the normal equations, and directly – matrix method; controversy about the statistical reliability - University of Vladivostok against the University of Indiana; graphical and tabular analysis of residues; concluded by differentiating the normal equations; Study regression with z-statistics and graphs standard normal probability.

Keywords: Multiple regression-systematization; Controversial statistical reliability; Regression equation; MATLAB; EXCEL; Matrix calculator; Analysis of residues; Concluded by differentiating the normal equations; z-Statistics

Introduction

Multiple regressions are studied in Russia and abroad. So Chetyrkin [1] describe a method of constructing it manually. Winston [2,3] described the quickest way to build it in the program Microsoft Office Excel, and taken them an example of submission of multiple regression, as well as details of its interpretation of some of the best works on the statistics. Sachs [4] not a lot of concerns of multiple regression, but look at how he described the manual methods of investigation regression really understand what the German precision [5].

But outside attention the authors were:

1. Systematization of knowledge about the multiple regressions;
2. Construction of the system of normal equations for mathematical models with more than two explanatory variables; conclusion of these equations is considered redundant because, firstly, the need to build an auxiliary table, and secondly, the solution of these equations requires a lot of work if it is to perform manually. But if you examine the sum of squared errors, standard errors of prediction and covariance, and you'll need. A solution of large systems is now easier MATLAB. Plus the consideration of such a system shows the interaction of variables;
3. Conclusion of normal equations of the functional equation by differentiation;
4. The decision referred to in MATLAB as the normal equations, and direct-matrix method;
5. The decision of using a calculator and matrix "matrix arithmetic";
6. Graphical analysis of residues;
7. Investigation of multiple regressions with z - statistics and graphs standard normal probability. The regression statistics - the most difficult section of statistics. In turn, the multiple regression - one of the most difficult in the regression statistics. The formula is as follows:

$$y_i = a_0 + a_1 \times x_1 + a_2 \times x_2 + \dots + a_m \times x_m + \varepsilon_i \quad (1)$$

Chetyrkin [1] wrote: "If selected as the independent variable is the dominant factor, respectively, the corresponding pair regression

adequately describes the mechanism of causation. The most common change of y due to the influence of several factors (sometimes acting in opposite directions). In these cases, naturally the desire to enter some explanatory variables. This is called multiple regressions. Multiple regression equation to better to explain the behavior of the dependent variable than steam regression, in addition, it makes it possible to compare the effectiveness of various factors".

Multiple Regressions

Wayne Winston – Indiana University professor who advises the company Ford Motor corporation General Motors, Intel, Microsoft, Proctor and Gamble, the US Army, US Department of Defense and other organizations, a graduate of Yale University with a Ph.D. and the Faculty of Mathematics at MIT, who among universities in the world by Times Higher Education World Reputation Rankings 2015 took 4th place, refers to a book economist Roy Fair [6] that the economy has a major impact on the results of the presidential elections. How can I predict U.S. presidential elections?

Presidential advisor James Carville said "It's the economy" when asked about which factors drive presidential elections. Yale economist Roy Fair showed that Carville was right in thinking that the state of the economy has a large influence on the results of presidential elections. Fair's dependent variable for each election (1916–2000) was the % of the two party vote (ignoring votes received by third party candidates) that went to the incumbent party. He tried to predict the incumbent party's % of the two party votes by using the following independent variables:

- 1) Party in power. In our data, we use a 1 to denote that the

***Corresponding author:** Taenvat MM, Vladivostok Postgraduate of the Department of State and Municipal Management Institute of Law and Governance, Vladivostok State University Economics and Service, Russia, Tel: +79502845660; E-mail: hh.dd.00@mail.ru

Received November 23, 2015; **Accepted** December 09, 2015; **Published** December 15, 2015

Citation: Taenvat MM (2015) Multiple Regression on the Example of the Presidential Elections in the United States during the Period from 1916 to 2000. J Appl Computat Math 4: 274. doi:10.4172/2168-9679.1000274

Copyright: © 2015 Taenvat MM. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Republican Party is in power and a 0 to denote that the Democratic Party is in power.

2) % growth in GNP during the first nine months of the election year.

3) Absolute value of the inflation rate during the first nine months of the election year. We use the absolute value because either a positive or a negative inflation rate is bad.

4) Number of quarters during the last four years in which economic growth has been strong. Strong economic growth is defined as growth at an annual level of 3.2% or more.

5) Time incumbent party has been in office. Fair used 0 to denote one term in office, 1 for two terms, 1.25 for three terms, 1.5 for four terms, and 1.75 for at least five terms. This definition implies that each term after the first term in office has less influence on the election results than the first term in office.

Is the election during wartime? The elections in 1920 (World War I), 1944 (World War II), and 1948 (World War II was still underway in 1945) were defined as wartime elections. Elections held during the Vietnam War were not considered wartime elections. During wartime years, the variables related to quarters of good growth and inflation was deemed irrelevant and was set to 0.

6) Is the current president running for re-election? If so, this variable is set to 1; otherwise, this variable is set to 0. In 1976, Gerald Ford was not considered a president running for re-election because he was not elected as either president or vice-president.

I've attempted to use the data from the elections in 1916 through

1996 to develop a multiple regression equation that can be used to forecast future presidential elections. I saved the infamous 2000 election as a «validation point».

In Table 1, you can see that the p-value for each independent variable is much less than.15, which indicates that each of our independent variables is helpful in predicting presidential elections. We can predict elections using an equation...:

$$\text{Projected percentage of votes in the presidential election} = 45,53 + 0,70_{\text{growth GNP}} - 0,71_{\text{abs.inf.}} + 0,90_{\text{quarter growth GNP}} - 3,33_{\text{terms of power}} + 5,66_{\text{Republicans}} + 4,71_{\text{war}} + 3,99_{\text{President of a new term}}$$

The coefficients of the independent variables can be interpreted as follows

(After adjusting for all other independent variables used in equation 2):

1. A 1% increase in the annual GNP growth rate during an election year is worth. 7% to the incumbent party.
2. A 1% deviation from the ideal (0% inflation) costs the incumbent party. 71% of the vote.
3. Every good quarter of growth during an incumbent's term increases his (maybe her someday soon!) vote by .90%.
4. Relative to having one term in office, the second term in office decreases the incumbent's vote by 3.33%, and each later term decreases the incumbent's vote by .25*(3.33%) = .83%.
5. A Republican has a 5.66% edge over a Democrat.

Applicants	Year	The share of government	Ruling party	The growth rate of GNP in the election year	The absolute value of the rate of inflation in an election year	The number of quarters with GNP growth of >3.2%	The number of time spent by the ruling party in power	The ruling party 1 = republics Kanz	War	Pull out whether the President for the next term?	Forecast
				1	2	3	4	5	6	7	
Cheats											
Uilkinson - Hyyuz	1916	51,7	D	2,2	4,3	3	0	0	0	1	50,71565364
Harding - Cox	1920	36,1	D	-11,5	0	0	1	0	1	0	38,86253503
Coolidge - Davis	1924	58,2	R	-3,9	5,2	10	0	1	0	1	57,79816261
Hoover -Smith	1928	58,8	R	4,6	0,2	7	1	1	0	0	57,28141426
Guver - FDR	1932	40,8	R	-14,9	7,1	4	1,25	1	0	1	39,14128587
FDR - Lendon	1936	62,5	D	11,9	2,4	9	0	0	0	1	64,29354145
FDR - Uilki	1940	55	D	3,7	0	8	1	0	0	1	56,019
FDR - Dewey	1944	53,8	D	4,1	0	0	1,25	0	1	1	52,94119656
Truman - Dyuy	1948	52,4	D	1,8	0	0	1,5	0	1	1	50,49626842
Ike - Stevenson	1952	44,6	D	0,6	2,3	6	1,75	0	0	0	43,91467348
Ike - Stevenson	1956	57,8	R	-1,5	1,9	5	0	1	0	1	57,30571419
Kennedy - Nixon	1960	49,9	R	0,1	1,9	5	1	1	0	0	51,10831446
Johnson - Goldwater	1964	61,3	D	5,1	1,2	10	0	0	0	1	61,2893759
Nixon - Humphrey	1968	49,6	D	4,8	3,2	7	1	0	0	0	49,62176954
Nixon - MakGovern	1972	61,8	R	6,3	4,8	4	0	1	0	1	59,7995525
Ford - Carter	1976	48,9	R	3,7	7,7	4	1	1	0	0	48,59420446
Carter - Reagan	1980	44,7	D	-3,8	8,1	5	0	0	0	1	45,61537819
Reagan - Mondale	1984	59,2	R	5,4	5,4	7	0	1	0	1	61,45627149
Bush - Dukakis	1988	53,9	R	2,1	3,3	6	1	1	0	0	52,41708877
Bush - Clinton	1992	46,5	R	2,3	3,7	1	1,25	1	0	1	50,89799139
Clinton - Dole	1996	54,7	D	2,9	2,3	3	0	0	0	1	52,63060779
Gore - Bush	2000	50,3	D	2,2	1,7	7	0	0	0	0	52,20336851

Table 1: Data on presidential elections.

6. A wartime incumbent president has a 4.71% edge over his opponent.

7. A sitting president running for re-election has a 3.99% edge over his opponent.

We find that 94% of the variation in the % received by an incumbent in a presidential election is explained by our independent variables. We have made no mention whether the candidates are “good or bad” candidates (Table 1).

The author of the article draws attention to the fact that the fall in the growth rate of GDP in the election year, almost always leads to a change of the ruling party (Table 1).

First, we decide through the normal equations.

$$\begin{aligned}
 a_0 \sum_{k=1}^n n + a_1 \sum_{k=1}^n x_1 + a_2 \sum_{k=1}^n x_2 + a_3 \sum_{k=1}^n x_3 + a_4 \sum_{k=1}^n x_4 + a_5 \sum_{k=1}^n x_5 + a_6 \sum_{k=1}^n x_6 + a_7 \sum_{k=1}^n x_7 &= \sum_{k=1}^n y; \\
 a_0 \sum_{k=1}^n x_1 + a_1 \sum_{k=1}^n x_1^2 + a_2 \sum_{k=1}^n x_2 x_1 + a_3 \sum_{k=1}^n x_3 x_1 + a_4 \sum_{k=1}^n x_4 x_1 + a_5 \sum_{k=1}^n x_5 x_1 + a_6 \sum_{k=1}^n x_6 x_1 + a_7 \sum_{k=1}^n x_7 x_1 &= \sum_{k=1}^n y x_1; \\
 a_0 \sum_{k=1}^n n + a_1 \sum_{k=1}^n x_1 + a_2 \sum_{k=1}^n x_2 + a_3 \sum_{k=1}^n x_3 + a_4 \sum_{k=1}^n x_4 + a_5 \sum_{k=1}^n x_5 + a_6 \sum_{k=1}^n x_6 + a_7 \sum_{k=1}^n x_7 &= \sum_{k=1}^n y; \\
 a_0 \sum_{k=1}^n x_1 + a_1 \sum_{k=1}^n x_1^2 + a_2 \sum_{k=1}^n x_2 x_1 + a_3 \sum_{k=1}^n x_3 x_1 + a_4 \sum_{k=1}^n x_4 x_1 + a_5 \sum_{k=1}^n x_5 x_1 + a_6 \sum_{k=1}^n x_6 x_1 + a_7 \sum_{k=1}^n x_7 x_1 &= \sum_{k=1}^n y x_1; \\
 a_0 \sum_{k=1}^n x_2 + a_1 \sum_{k=1}^n x_1 x_2 + a_2 \sum_{k=1}^n x_2^2 + a_3 \sum_{k=1}^n x_3 x_2 + a_4 \sum_{k=1}^n x_4 x_2 + a_5 \sum_{k=1}^n x_5 x_2 + a_6 \sum_{k=1}^n x_6 x_2 + a_7 \sum_{k=1}^n x_7 x_2 &= \sum_{k=1}^n y x_2; \\
 a_0 \sum_{k=1}^n x_3 + a_1 \sum_{k=1}^n x_1 x_3 + a_2 \sum_{k=1}^n x_2 x_3 + a_3 \sum_{k=1}^n x_3^2 + a_4 \sum_{k=1}^n x_4 x_3 + a_5 \sum_{k=1}^n x_5 x_3 + a_6 \sum_{k=1}^n x_6 x_3 + a_7 \sum_{k=1}^n x_7 x_3 &= \sum_{k=1}^n y x_3; \\
 a_0 \sum_{k=1}^n x_4 + a_1 \sum_{k=1}^n x_1 x_4 + a_2 \sum_{k=1}^n x_2 x_4 + a_3 \sum_{k=1}^n x_3 x_4 + a_4 \sum_{k=1}^n x_4^2 + a_5 \sum_{k=1}^n x_5 x_4 + a_6 \sum_{k=1}^n x_6 x_4 + a_7 \sum_{k=1}^n x_7 x_4 &= \sum_{k=1}^n y x_4; \\
 a_0 \sum_{k=1}^n x_5 + a_1 \sum_{k=1}^n x_1 x_5 + a_2 \sum_{k=1}^n x_2 x_5 + a_3 \sum_{k=1}^n x_3 x_5 + a_4 \sum_{k=1}^n x_4 x_5 + a_5 \sum_{k=1}^n x_5^2 + a_6 \sum_{k=1}^n x_6 x_5 + a_7 \sum_{k=1}^n x_7 x_5 &= \sum_{k=1}^n y x_5; \\
 a_0 \sum_{k=1}^n x_6 + a_1 \sum_{k=1}^n x_1 x_6 + a_2 \sum_{k=1}^n x_2 x_6 + a_3 \sum_{k=1}^n x_3 x_6 + a_4 \sum_{k=1}^n x_4 x_6 + a_5 \sum_{k=1}^n x_5 x_6 + a_6 \sum_{k=1}^n x_6^2 + a_7 \sum_{k=1}^n x_7 x_6 &= \sum_{k=1}^n y x_6; \\
 a_0 \sum_{k=1}^n x_7 + a_1 \sum_{k=1}^n x_1 x_7 + a_2 \sum_{k=1}^n x_2 x_7 + a_3 \sum_{k=1}^n x_3 x_7 + a_4 \sum_{k=1}^n x_4 x_7 + a_5 \sum_{k=1}^n x_5 x_7 + a_6 \sum_{k=1}^n x_6 x_7 + a_7 \sum_{k=1}^n x_7^2 &= \sum_{k=1}^n y x_7.
 \end{aligned}$$

The output of this system by differentiation:

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n (a + b \times x_1 + c \times x_2 + d \times x_3 + e \times x_4 + h \times x_5 + p \times x_6 + q \times x_7 - y)^2.$$

Since we have eight variables a, b, c, d, e, h, p, and q – we take the partial derivatives. Fix b, c, d, e, h, p and q and differentiate E (a, b, c, d, e, h, p, q) on a. Obtain

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial a} = \sum_{k=1}^n 1 \times a^{1-1} + 0 + 0 + 0 + 0 + 0 + 0 - 0 = \sum_{k=1}^n 1;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{2-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial a} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial a} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n 1 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (1) =$$

$$2 \sum_{k=1}^n (a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y); E'(a) = 0;$$

$$0 = \sum_{k=1}^n (a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) =$$

$$= a \sum_{k=1}^n 1 + b \sum_{k=1}^n x_1 + c \sum_{k=1}^n x_2 + d \sum_{k=1}^n x_3 + e \sum_{k=1}^n x_4 + h \sum_{k=1}^n x_5 + p \sum_{k=1}^n x_6 + q \sum_{k=1}^n x_7 - \sum_{k=1}^n y;$$

$$an + b \sum_{k=1}^n x_1 + c \sum_{k=1}^n x_2 + d \sum_{k=1}^n x_3 + e \sum_{k=1}^n x_4 + h \sum_{k=1}^n x_5 + p \sum_{k=1}^n x_6 + q \sum_{k=1}^n x_7 = \sum_{k=1}^n y.$$

We fix a, c, d, e, h, p and q and differentiate E (a, b, c, d, e, h, p, q) by b. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial a} = \sum_{k=1}^n 0 + 1 \times b^{1-1} \times x_1 + 0 + 0 + 0 + 0 + 0 - 0 = \sum_{k=1}^n x_1;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{2-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial b} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial b} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_1 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_1) =$$

$$2 \sum_{k=1}^n (ax_1 + bx_1^2 + cx_2 x_1 + dx_3 x_1 + ex_4 x_1 + hx_5 x_1 + px_6 x_1 + qx_7 x_1 - yx_1); E'(b) = 0;$$

$$0 = \sum_{k=1}^n (ax_1 + bx_1^2 + cx_2 x_1 + dx_3 x_1 + ex_4 x_1 + hx_5 x_1 + px_6 x_1 + qx_7 x_1 - yx_1) =$$

$$= a \sum_{k=1}^n x_1 + b \sum_{k=1}^n x_1^2 + c \sum_{k=1}^n x_2 x_1 + d \sum_{k=1}^n x_3 x_1 + e \sum_{k=1}^n x_4 x_1 + h \sum_{k=1}^n x_5 x_1 + p \sum_{k=1}^n x_6 x_1 + q \sum_{k=1}^n x_7 x_1 - \sum_{k=1}^n yx_1;$$

$$a \sum_{k=1}^n x_1 + b \sum_{k=1}^n x_1^2 + c \sum_{k=1}^n x_2 x_1 + d \sum_{k=1}^n x_3 x_1 + e \sum_{k=1}^n x_4 x_1 + h \sum_{k=1}^n x_5 x_1 + p \sum_{k=1}^n x_6 x_1 + q \sum_{k=1}^n x_7 x_1 = \sum_{k=1}^n yx_1.$$

We fix a, b, d, e, h, p and q and differentiate E (a, b, c, d, e, h, p, q) by c. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial c} = \sum_{k=1}^n 0 + 0 + 1 \times c^{1-1} \times x_2 + 0 + 0 + 0 + 0 - 0 = \sum_{k=1}^n x_2;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{2-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial c} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial c} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_2 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_2) =$$

$$2 \sum_{k=1}^n (ax_2 + bx_1 x_2 + cx_2^2 + dx_3 x_2 + ex_4 x_2 + hx_5 x_2 + px_6 x_2 + qx_7 x_2 - yx_2); E'(c) = 0;$$

$$0 = \sum_{k=1}^n (ax_2 + bx_1 x_2 + cx_2^2 + dx_3 x_2 + ex_4 x_2 + hx_5 x_2 + px_6 x_2 + qx_7 x_2 - yx_2) =$$

$$= a \sum_{k=1}^n x_2 + b \sum_{k=1}^n x_1 x_2 + c \sum_{k=1}^n x_2^2 + d \sum_{k=1}^n x_3 x_2 + e \sum_{k=1}^n x_4 x_2 + h \sum_{k=1}^n x_5 x_2 + p \sum_{k=1}^n x_6 x_2 + q \sum_{k=1}^n x_7 x_2 - \sum_{k=1}^n yx_2;$$

$$a \sum_{k=1}^n x_2 + b \sum_{k=1}^n x_1 x_2 + c \sum_{k=1}^n x_2^2 + d \sum_{k=1}^n x_3 x_2 + e \sum_{k=1}^n x_4 x_2 + h \sum_{k=1}^n x_5 x_2 + p \sum_{k=1}^n x_6 x_2 + q \sum_{k=1}^n x_7 x_2 = \sum_{k=1}^n yx_2.$$

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial d} = \sum_{k=1}^n 0 + 0 + 0 + 1 \times d^{1-1} \times x_3 + 0 + 0 + 0 - 0 = \sum_{k=1}^n x_3;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{2-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial d} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial d} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_3 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_3) =$$

$$2 \sum_{k=1}^n (ax_3 + bx_1 x_3 + cx_2 x_3 + dx_3^2 + ex_4 x_3 + hx_5 x_3 + px_6 x_3 + qx_7 x_3 - yx_3); E'(d) = 0;$$

We fix a, b, c, e, h, p and q and differentiate E (a, b, c, d, e, h, p, q) by d. Receive

$$\begin{aligned}
 &0 = \sum_{k=1}^n (ax_3 + bx_1x_3 + cx_2x_3 + dx_3^2 + ex_4x_3 + hx_5x_3 + px_6x_3 + qx_7x_3 - yx_3) = \\
 &= a \sum_{k=1}^n x_3 + b \sum_{k=1}^n x_1x_3 + c \sum_{k=1}^n x_2x_3 + d \sum_{k=1}^n x_3^2 + e \sum_{k=1}^n x_4x_3 + h \sum_{k=1}^n x_5x_3 + p \sum_{k=1}^n x_6x_3 + q \sum_{k=1}^n x_7x_3 - \sum_{k=1}^n yx_3; \\
 &a \sum_{k=1}^n x_3 + b \sum_{k=1}^n x_1x_3 + c \sum_{k=1}^n x_2x_3 + d \sum_{k=1}^n x_3^2 + e \sum_{k=1}^n x_4x_3 + h \sum_{k=1}^n x_5x_3 + p \sum_{k=1}^n x_6x_3 + q \sum_{k=1}^n x_7x_3 = \sum_{k=1}^n yx_3.
 \end{aligned}$$

We fix a, b, c, d, h, p and q and differentiate E (a, b, c, d, e, h, p, q) by e. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial e} = \sum_{k=1}^n 0 + 0 + 0 + 0 + 1 \times e^{-1} \times x_4 + 0 + 0 + 0 - 0 = \sum_{k=1}^n x_4;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial e} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial e} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_4 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_4) =$$

$$2 \sum_{k=1}^n (ax_4 + bx_1x_4 + cx_2x_4 + dx_3x_4 + ex_4^2 + hx_5x_4 + px_6x_4 + qx_7x_4 - yx_4); E'(e) = 0;$$

$$0 = \sum_{k=1}^n (ax_4 + bx_1x_4 + cx_2x_4 + dx_3x_4 + ex_4^2 + hx_5x_4 + px_6x_4 + qx_7x_4 - yx_4) =$$

$$= a \sum_{k=1}^n x_4 + b \sum_{k=1}^n x_1x_4 + c \sum_{k=1}^n x_2x_4 + d \sum_{k=1}^n x_3x_4 + e \sum_{k=1}^n x_4^2 + h \sum_{k=1}^n x_5x_4 + p \sum_{k=1}^n x_6x_4 + q \sum_{k=1}^n x_7x_4 - \sum_{k=1}^n yx_4;$$

$$a \sum_{k=1}^n x_4 + b \sum_{k=1}^n x_1x_4 + c \sum_{k=1}^n x_2x_4 + d \sum_{k=1}^n x_3x_4 + e \sum_{k=1}^n x_4^2 + h \sum_{k=1}^n x_5x_4 + p \sum_{k=1}^n x_6x_4 + q \sum_{k=1}^n x_7x_4 = \sum_{k=1}^n yx_4.$$

We fix a, b, c, d, e, p and q and differentiate E (a, b, c, d, e, h, p, q) by h. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial h} = \sum_{k=1}^n 0 + 0 + 0 + 0 + 0 + 1 \times h^{-1} \times x_5 + 0 + 0 - 0 = \sum_{k=1}^n x_5;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial h} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial h} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_5 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_5) =$$

$$2 \sum_{k=1}^n (ax_5 + bx_1x_5 + cx_2x_5 + dx_3x_5 + ex_4x_5 + hx_5^2 + px_6x_5 + qx_7x_5 - yx_5); E'(h) = 0;$$

$$0 = \sum_{k=1}^n (ax_5 + bx_1x_5 + cx_2x_5 + dx_3x_5 + ex_4x_5 + hx_5^2 + px_6x_5 + qx_7x_5 - yx_5) =$$

$$= a \sum_{k=1}^n x_5 + b \sum_{k=1}^n x_1x_5 + c \sum_{k=1}^n x_2x_5 + d \sum_{k=1}^n x_3x_5 + e \sum_{k=1}^n x_4x_5 + h \sum_{k=1}^n x_5^2 + p \sum_{k=1}^n x_6x_5 + q \sum_{k=1}^n x_7x_5 - \sum_{k=1}^n yx_5;$$

$$a \sum_{k=1}^n x_5 + b \sum_{k=1}^n x_1x_5 + c \sum_{k=1}^n x_2x_5 + d \sum_{k=1}^n x_3x_5 + e \sum_{k=1}^n x_4x_5 + h \sum_{k=1}^n x_5^2 + p \sum_{k=1}^n x_6x_5 + q \sum_{k=1}^n x_7x_5 = \sum_{k=1}^n yx_5.$$

We fix a, b, c, d, e, h and q and differentiate E (a, b, c, d, e, h, p, q) by p. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial p} = \sum_{k=1}^n 0 + 0 + 0 + 0 + 0 + 0 + 1 \times p^{-1} \times x_6 + 0 - 0 = \sum_{k=1}^n x_6;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial p} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial p} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_6 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_6) =$$

$$2 \sum_{k=1}^n (ax_6 + bx_1x_6 + cx_2x_6 + dx_3x_6 + ex_4x_6 + hx_5x_6 + px_6^2 + qx_7x_6 - yx_6); E'(p) = 0;$$

$$0 = \sum_{k=1}^n (ax_6 + bx_1x_6 + cx_2x_6 + dx_3x_6 + ex_4x_6 + hx_5x_6 + px_6^2 + qx_7x_6 - yx_6) =$$

$$= a \sum_{k=1}^n x_6 + b \sum_{k=1}^n x_1x_6 + c \sum_{k=1}^n x_2x_6 + d \sum_{k=1}^n x_3x_6 + e \sum_{k=1}^n x_4x_6 + h \sum_{k=1}^n x_5x_6 + p \sum_{k=1}^n x_6^2 + q \sum_{k=1}^n x_7x_6 - \sum_{k=1}^n yx_6;$$

$$a \sum_{k=1}^n x_6 + b \sum_{k=1}^n x_1x_6 + c \sum_{k=1}^n x_2x_6 + d \sum_{k=1}^n x_3x_6 + e \sum_{k=1}^n x_4x_6 + h \sum_{k=1}^n x_5x_6 + p \sum_{k=1}^n x_6^2 + q \sum_{k=1}^n x_7x_6 = \sum_{k=1}^n yx_6.$$

We fix a, b, c, d, e, h and p and differentiate E (a, b, c, d, e, h, p, q) by q. Receive

$$E(a, b, c, d, e, h, p, q) = \sum_{k=1}^n u^2, \text{ if } u = a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y.$$

$$\text{Here } \frac{\partial u}{\partial q} = \sum_{k=1}^n 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 \times q^{-1} \times x_7 - 0 = \sum_{k=1}^n x_7;$$

$$\frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} = \sum_{k=1}^n 2u^{-1} = \sum_{k=1}^n 2u;$$

$$\text{Respectively: } \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial q} = \frac{\partial E(a, b, c, d, e, h, p, q)}{\partial u} \times \frac{\partial u}{\partial q} = \sum_{k=1}^n 2u^1 \times \sum_{k=1}^n x_7 =$$

$$= \sum_{k=1}^n 2(a + bx_1 + cx_2 + dx_3 + ex_4 + hx_5 + px_6 + qx_7 - y) \times (x_7) =$$

$$2 \sum_{k=1}^n (ax_7 + bx_1x_7 + cx_2x_7 + dx_3x_7 + ex_4x_7 + hx_5x_7 + px_6x_7 + qx_7^2 - yx_7); E'(q) = 0;$$

$$0 = \sum_{k=1}^n (ax_7 + bx_1x_7 + cx_2x_7 + dx_3x_7 + ex_4x_7 + hx_5x_7 + px_6x_7 + qx_7^2 - yx_7) =$$

$$= a \sum_{k=1}^n x_7 + b \sum_{k=1}^n x_1x_7 + c \sum_{k=1}^n x_2x_7 + d \sum_{k=1}^n x_3x_7 + e \sum_{k=1}^n x_4x_7 + h \sum_{k=1}^n x_5x_7 + p \sum_{k=1}^n x_6x_7 + q \sum_{k=1}^n x_7^2 - \sum_{k=1}^n yx_7;$$

$$a \sum_{k=1}^n x_7 + b \sum_{k=1}^n x_1x_7 + c \sum_{k=1}^n x_2x_7 + d \sum_{k=1}^n x_3x_7 + e \sum_{k=1}^n x_4x_7 + h \sum_{k=1}^n x_5x_7 + p \sum_{k=1}^n x_6x_7 + q \sum_{k=1}^n x_7^2 = \sum_{k=1}^n yx_7.$$

To find the coefficients of the system of equations we construct auxiliary (Tables 2a and 2b).

Finally got a model that reflects the policy of the United States for nearly a century:

$$\begin{aligned}
 &21a_0 + 26a_1 + 65a_2 + 104a_3 + 14a_4 + 10a_5 + 3a_6 + 14a_7 = 1102,2 \\
 &26a_0 + 737,58a_1 + 12,29a_2 + 240,5a_3 + 0,625a_4 + 4,2a_5 - 5,6a_6 + 21,6a_7 = 1990,94 \\
 &65a_0 + 12,29a_1 + 332,9a_2 + 342,2a_3 + 33,825a_4 + 41,2a_5 + 46,4a_7 = 3346,53 \\
 &104a_0 + 240,5a_1 + 342,2a_2 + 706a_3 + 53,75a_4 + 53a_5 + 69a_7 = 5695,4
 \end{aligned}$$

$$14a_0 + 0,625a_1 + 33,825a_2 + 53,75a_3 + 17a_4 + 6,5a_5 + 3,75a_6 + 6,25a_7 = 685,225$$

$$10a_0 + 4,2a_1 + 41,2a_2 + 53a_3 + 6,5a_4 + 10a_5 + 6a_7 = 535,81$$

$$3a_0 - 5,6a_1 + 3,75a_4 + 3a_6 + 2a_7 = 142,3$$

$$14a_0 + 21,6a_1 + 46,4a_2 + 69a_3 + 6,25a_4 + 6a_5 + 2a_6 + 14a_7 = 760,4$$

To solve the program will use MATLAB. First make sure that the matrix A - non-singular, showing that det (A) ≠ 0.

$$\begin{aligned}
 &A = [21.000 \ 26.000 \ 65.000 \ 104.000 \ 14.000 \ 10.000 \ 3.000 \ 14.000; \\
 &26.000 \ 737.580 \ 12.290 \ 240.500 \ 0.625 \ 4.200 \ -5.600 \ 21.600; \ 65.000 \ 12.290 \\
 &332.900 \ 342.200 \ 33.825 \ 41.200 \ 0.000 \ 46.400; \ 104.000 \ 240.500 \ 342.200 \\
 &706.000 \ 53.750 \ 53.000 \ 0.000 \ 69.000; \ 14.000 \ 0.625 \ 33.825 \ 53.750 \ 17.000 \\
 &6.500 \ 3.750 \ 6.250; \ 10.000 \ 4.200 \ 41.200 \ 53.000 \ 6.500 \ 10.000 \ 0.000 \ 6.000; \\
 &3.000 \ -5.600 \ 0.000 \ 0.000 \ 3.750 \ 0.000 \ 3.000 \ 2.000; \ 14.000 \ 21.600 \ 46.400 \\
 &69.000 \ 6.250 \ 6.000 \ 2.000 \ 14.000];
 \end{aligned}$$

$$\det(A) \text{ ans} = 1.8924e + 10$$

Decision systems AX = B represented in the form X = A⁻¹B.

x1*x2	x2*x3	x3*x4	x4*x5	x5*x6	x6*x7	x1^2	x2^2	x3^2	x4^2	x5^2	x6^2	x7^2	x2*x4	x1*x4	x3*x4	x4*x6	x4*x7
9,46	12,9	0	0	0	0	4,84	18,49	9	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	132,25	0	0	1	0	1	0	0	-11,5	0	1	0
-20,28	52	0	0	0	0	15,21	27,04	100	0	1	0	1	0	0	0	0	0
0,92	1,4	7	1	0	0	21,16	0,04	49	1	1	0	0	0,2	4,6	7	0	0
-105,79	28,4	5	1,25	0	0	222,01	50,41	16	1,5625	1	0	1	8,875	-18,625	5	0	1,25
28,56	21,6	0	0	0	0	141,61	5,76	81	0	0	0	1	0	0	0	0	0
0	0	8	0	0	0	13,69	0	64	1	0	0	1	0	3,7	8	0	1
0	0	0	0	0	1	16,81	0	0	1,5625	0	1	1	0	5,125	0	1,25	1,25
0	0	0	0	0	1	3,24	0	0	2,25	0	1	1	0	2,7	0	1,5	1,5
1,38	13,8	10,5	0	0	0	0,36	5,29	36	3,0625	0	0	0	4,025	1,05	10,5	0	0
-2,85	9,5	0	0	0	0	2,25	3,61	25	0	1	0	1	0	0	0	0	0
0,19	9,5	5	1	0	0	0,01	3,61	25	1	1	0	0	1,9	0,1	5	0	0
6,12	12	0	0	0	0	26,01	1,44	100	0	0	0	1	0	0	0	0	0
15,36	22,4	7	0	0	0	23,04	10,24	49	1	0	0	0	3,2	4,8	7	0	0
30,24	19,2	0	0	0	0	39,69	23,04	16	0	1	0	1	0	0	0	0	0
28,49	30,8	4	1	0	0	13,69	59,29	16	1	1	0	0	7,7	3,7	4	0	0
-30,78	40,5	0	0	0	0	14,44	65,61	25	0	0	0	1	0	0	0	0	0
29,16	37,8	0	0	0	0	29,16	29,16	49	0	1	0	1	0	0	0	0	0
6,93	19,8	6	1	0	0	4,41	10,89	36	1	1	0	0	3,3	2,1	6	0	0
8,51	3,7	1,25	1,25	0	0	5,29	13,69	1	1,5625	1	0	1	4,625	2,875	1,25	0	1,25
6,67	6,9	0	0	0	0	8,41	5,29	9	0	0	0	1	0	0	0	0	0
12,29	342,2	53,75	6,5	0	2	737,58	332,9	706	17	10	3	14	33,825	0,625	53,75	3,75	6,25

Table 2a: Auxiliary table.

x1*x5	x1*x6	x1*x7	x2*x7	x3*x7	x5*x7	x2*x5	x5*x6	x1*x3	y	y*x1	y*x2	y*x3	y*x4	y*x5	y*x6	y*x7
0	0	2,2	4,3	3	0	0	0	6,6	51,7	113,74	222,31	155,1	0	0	0	51,7
0	-11,5	0	0	0	0	0	0	0	36,1	-415,15	0	0	36,1	0	36,1	0
-3,9	0	-3,9	5,2	10	1	5,2	0	-39	58,2	-226,98	302,64	582	0	58,2	0	58,2
4,6	0	0	0	0	0	0,2	0	32,2	58,8	270,48	11,76	411,6	58,8	58,8	0	0
-14,9	0	-14,9	7,1	4	1	7,1	0	-59,6	40,8	-607,92	289,68	163,2	51	40,8	0	40,8
0	0	11,9	2,4	9	0	0	0	107,1	62,5	743,75	150	562,5	0	0	0	62,5
0	0	3,7	0	8	0	0	0	29,6	55	203,5	0	440	55	0	0	55
0	4,1	4,1	0	0	0	0	0	0	53,8	220,58	0	0	67,25	0	53,8	53,8
0	1,8	1,8	0	0	0	0	0	0	52,4	94,32	0	0	78,6	0	52,4	52,4
0	0	0	0	0	0	0	0	3,6	44,6	26,76	102,58	267,6	78,05	0	0	0
-1,5	0	-1,5	1,9	5	1	1,9	0	-7,5	57,8	-86,7	109,82	289	0	57,8	0	57,8
0,1	0	0	0	0	0	1,9	0	0,5	49,9	4,99	94,81	249,5	49,9	49,9	0	0
0	0	5,1	1,2	10	0	0	0	51	61,3	312,63	73,56	613	0	0	0	61,3
0	0	0	0	0	0	0	0	33,6	49,6	238,08	158,72	347,2	49,6	0	0	0
6,3	0	6,3	4,8	4	1	4,8	0	25,2	61,8	389,34	296,64	247,2	0	61,8	0	61,8
3,7	0	0	0	0	0	7,7	0	14,8	48,9	180,93	376,53	195,6	48,9	48,9	0	0
0	0	-3,8	8,1	5	0	0	0	-19	44,7	-169,86	362,07	223,5	0	0	0	44,7
5,4	0	5,4	5,4	7	1	5,4	0	37,8	59,2	319,68	319,68	414,4	0	59,2	0	59,2
2,1	0	0	0	0	0	3,3	0	12,6	53,9	113,19	177,87	323,4	53,9	53,9	0	0
2,3	0	2,3	3,7	1	1	3,7	0	2,3	46,5	106,95	172,05	46,5	58,125	46,5	0	46,5
0	0	2,9	2,3	3	0	0	0	8,7	54,7	158,63	125,81	164,1	0	0	0	54,7
4,2	-5,6	21,6	46,4	69	6	41,2	0	240,5	1102,2	1990,94	3346,53	5695,4	685,225	535,8	142,3	760,4

Table 2b: Auxiliary table.

$X = \text{inv}(A) * [1102.200 \ 1990.94 \ 3346.53 \ 5695.400 \ 685.225 \ 535.810 \ 142.300 \ 760.400]'$

$X = 45.5373; \ 0.7006; \ -0.7124; \ 0.9049; \ -3.3347; \ 5.6655; \ 4.7162; \ 3.9840$

We test the solution, making sure that $AX = B$.

$B = A * X$

$B = 1.0e + 03 *; \ 1.1022; \ 1.9909; \ 3.3465; \ 5.6954; \ 0.6852; \ 0.5358; \ 0.1423; \ 0.7604$

With an increasing number of independent variables are more cumbersome recording [normal] equations and increases the complexity of the data. Since the observation of each variable provides not one but a number of its values, it is convenient to record every such number with a vector and the set of values of the independent variables - with the help of a matrix.

MNCs in matrix form. Introduce appropriate vector and matrix notation. Let

$\alpha = (\alpha_i), i = 0, 1, \dots, m$ - vector of unknown parameters;

$a = (a_i)$ - vector parameters estimates;

$y = (y_i), i = 1, \dots, n$ - vector of a dependent variable;

$X = (x_{ij})$ - matrix of values of the independent variables dimension $n \times m$;

$\varepsilon = (\varepsilon_i)$ - error vector;

$e = (e_i)$ - error vector.

$$y_i = \alpha_0 + \alpha_1 x_{i1} + \alpha_2 x_{i2} + \dots + \alpha_m x_{im} + \varepsilon_i \quad (3)$$

Rewrite the linear model (3), Using matrix notation adopted.

$$Y = X\alpha + \varepsilon.$$

Multiple regression equation with the estimated parameters

$$y_i = a_0 + a_1 x_{i1} + a_2 x_{i2} + \dots + a_m x_{im} + \varepsilon_i \quad (4)$$

Now we write a

$$Y = Xa + \varepsilon.$$

The sum of squared deviations obtained as follows:

$$Q = \sum e_i^2 = e^T e = (y - Xa)^T (y - Xa) = y^T y - a^T X^T y - y^T Xa + a^T X^T Xa.$$

Here and below T denotes the transpose of a vector or matrix. So how

$$a^T X^T y = y^T Xa,$$

then

$$Q = y^T y - 2a^T X^T y + a^T X^T Xa.$$

Differentiating Q for a and we get

$$a^T a = a^2;$$

$$\frac{dQ}{da} = 0 - 2 \times 1 \times (a^{1-1})^T X^T y + 2a X^T X = -2X^T y + 2aX^T X.$$

We equate the result to zero. After that, we can easily find the system of normal equations that in matrix form is written as

$$X^T y = X^T Xa \quad \text{omciyoda} \quad a = (X^T X)^{-1} \times X^T y.$$

Formula solutions matrix method takes the form:

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1m} \\ 1 & x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix}; \quad X^T X = \begin{bmatrix} n & \sum x_{i1} & \dots & \sum x_{im} \\ \sum x_{i1} & \sum x_{i1}^2 & \dots & \sum x_{i1} x_{im} \\ \vdots & \vdots & \dots & \vdots \\ \sum x_{im} & \sum x_{i1} x_{im} & \dots & \sum x_{im}^2 \end{bmatrix};$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}; \quad X^T y = \begin{bmatrix} \sum y_i \\ \sum y_i x_{i1} \\ \vdots \\ \sum y_i x_{im} \end{bmatrix};$$

$$a = (X^T X)^{-1} \times X^T y = (X^T X)^{-1} \times \begin{bmatrix} \sum y_i \\ \sum y_i x_{i1} \\ \vdots \\ \sum y_i x_{im} \end{bmatrix}.$$

Now to solve MATLAB.

```
X = [1 2.2 4.3 3 0 0 0 1;
1 -11.5 0 0 1 0 1 0;
1 -3.9 5.2 10 0 1 0 1;
1 4.6 0.2 7 1 1 0 0;
1 -14.9 7.1 4 1.25 1 0 1;
1 11.9 2.4 9 0 0 0 1;
1 3.7 0 8 1 0 0 1;
1 4.1 0 0 1.25 0 1 1;
1 1.8 0 0 1.5 0 1 1;
1 0.6 2.3 6 1.75 0 0 0;
1 -1.5 1.9 5 0 1 0 1;
1 0.1 1.9 5 1 1 0 0;
1 5.1 1.2 10 0 0 0 1;
1 4.8 3.2 7 1 0 0 0;
1 6.3 4.8 4 0 1 0 1;
1 3.7 7.7 4 1 1 0 0;
1 -3.8 8.1 5 0 0 0 1;
1 5.4 5.4 7 0 1 0 1;
1 2.1 3.3 6 1 1 0 0;
1 2.3 3.7 1 1.25 1 0 1;
1 2.9 2.3 3 0 0 0 1];
```

```
X' ans = 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000
2.2000 -11.5000 -3.9000 4.6000 -14.9000 11.9000 3.7000
4.1000 1.8000 0.6000 -1.5000 0.1000 5.1000 4.8000 6.3000
0.6000 -1.5000 0.1000 5.1000 0.6000 1.7500 0.0000 0.0000
-1.5000 0.1000 1.9000 0.1000 5.1000 3.7000 3.2000 6.3000
3.7000 5.4000 2.1000 2.3000 2.9000 2.3000 3.0000 2.3000
```

3.7000 -3.8000 5.4000 2.1000 2.3000 2.9000
 4.3000 0 5.2000 0.2000 7.1000 2.4000 0 0 0
 2.3000 1.9000 1.9000 1.2000 3.2000 4.8000 7.7000 8.1000
 5.4000 3.3000 3.7000 2.3000
 3.0000 0 10.0000 7.0000 4.0000 9.0000 8.0000 0 0
 6.0000 5.0000 5.0000 10.0000 7.0000 4.0000 4.0000 5.0000
 7.0000 6.0000 1.0000 3.0000
 0 1.0000 0 1.0000 1.2500 0 1.0000 1.2500 1.5000
 1.7500 0 1.0000 0 1.0000 0 1.0000 0 0 1.0000
 1.2500 0
 0 0 1.0000 1.0000 1.0000 0 0 0 0 0
 1.0000 1.0000 0 0 1.0000 1.0000 0 1.0000 1.0000
 1.0000 0
 0 1.0000 0 0 0 0 0 1.0000 1.0000 0
 0 0 0 0 0 0 0 0 0 0
 1.0000 0 1.0000 0 1.0000 1.0000 1.0000 1.0000
 1.0000 0 1.0000 0 1.0000 0 1.0000 0 1.0000
 1.0000 0 1.0000 1.0000
 X^*X ans = 21.0000 26.0000 65.0000 104.0000 14.0000
 10.0000 3.0000 14.0000
 26.0000 737.5800 12.2900 240.5000 0.6250 4.2000
 -5.6000 21.6000
 65.0000 12.2900 332.9000 342.2000 33.8250 41.2000 0
 46.4000
 104.0000 240.5000 342.2000 706.0000 53.7500 53.0000 0
 69.0000
 14.0000 0.6250 33.8250 53.7500 17.0000 6.5000 3.7500
 6.2500
 10.0000 4.2000 41.2000 53.0000 6.5000 10.0000 0
 6.0000
 3.0000 -5.6000 0 0 3.7500 0 3.0000 2.0000
 14.0000 21.6000 46.4000 69.0000 6.2500 6.0000 2.0000
 14.0000
 $(X^*X)^{-1}$ ans = 1.4952 -0.0124 -0.0763 -0.1012 -0.3846 -0.1691
 -0.7932 -0.3668
 -0.0124 0.0019 0.0019 -0.0001 0.0033 0.0027 0.0121
 -0.0004
 -0.0763 0.0019 0.0145 0.0037 0.0126 -0.0097 0.0668
 -0.0040
 -0.1012 -0.0001 0.0037 0.0119 0.0155 0.0067 0.0751 0.0097
 -0.3846 0.0033 0.0126 0.0155 0.2550 -0.0106 -0.0324
 0.1567
 -0.1691 0.0027 -0.0097 0.0067 -0.0106 0.2586 0.1644
 0.0343
 -0.7932 0.0121 0.0668 0.0751 -0.0324 0.1644 1.2213
 -0.0475
 -0.3668 -0.0004 -0.0040 0.0097 0.1567 0.0343 -0.0475
 0.3265
 $Y = [51.7000; 36.1000; 58.2000; 58.8000; 40.8000; 62.5000;$

55.0000; 53.8000; 52.4000; 44.6000; 57.8000;
 49.9000; 61.3000; 49.6000; 61.8000; 48.9000; 44.7000; 59.2000;
 53.9000; 46.5000; 54.7000];
 X^*y ans = 1.0e+03 *; 1.1022; 1.9909; 3.3465; 5.6954; 0.6852;
 0.5358; 0.1423; 0.7604
 $(X^*X)^{-1}(X^*y)$ ans = 45.5390; 0.7006; -0.7123; 0.9049;
 -3.3346; 5.6629; 4.7145; 3.9837

We can solve the problem and using the calculator matrices [7] and "matrix arithmetic". Enter the original data into the calculator (Figure 1). We find the transposed matrix (Figure 2). Go to the window "matrix arithmetic" (Figure 3). We get work and transpose of the original matrix (Figure 4). We find the inverse matrix (Figures 5 and 6). In the same window, find the product of the transposed matrix of the vector and the column y. And by multiplying the inverse matrix that happened, we obtain a matrix of unknown coefficients (Figures 7 and 8) (Tables 3-7).

1. Since $29.4043 \geq 2.832097502$ and $5.48E-07 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;
2. Since $7.458777 \geq 2.160368652$, a $4.77E-06 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

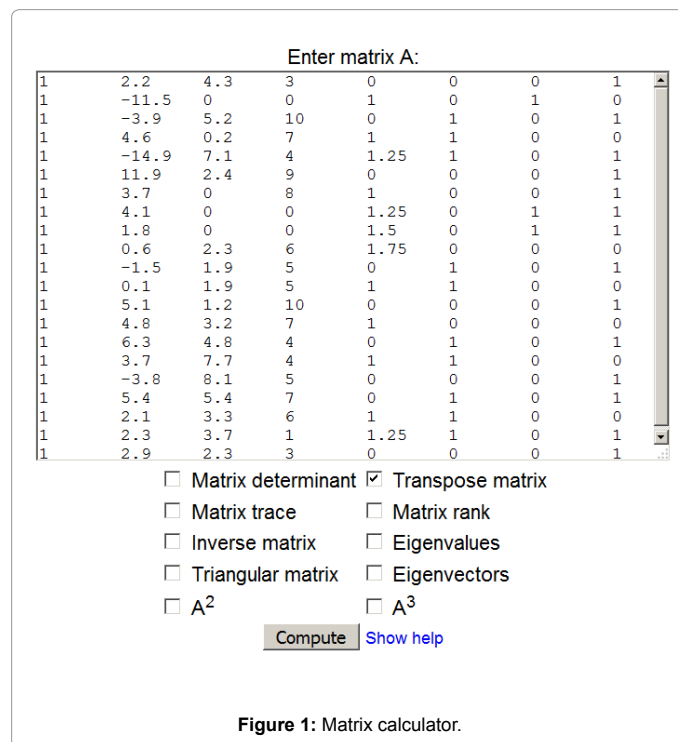


Figure 1: Matrix calculator.

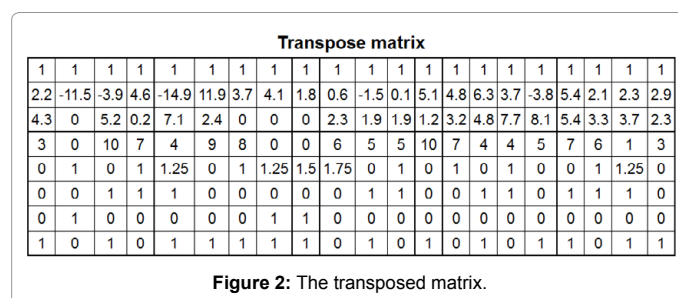


Figure 2: The transposed matrix.

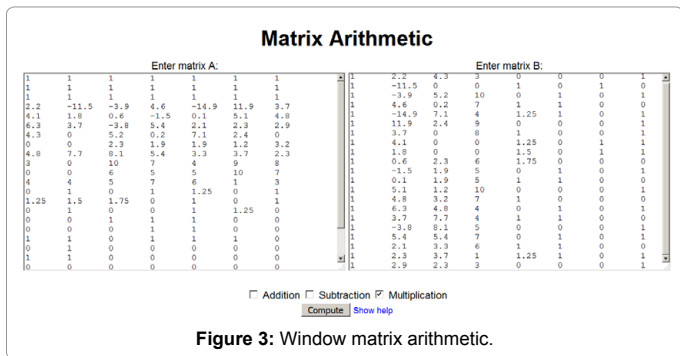


Figure 3: Window matrix arithmetic.

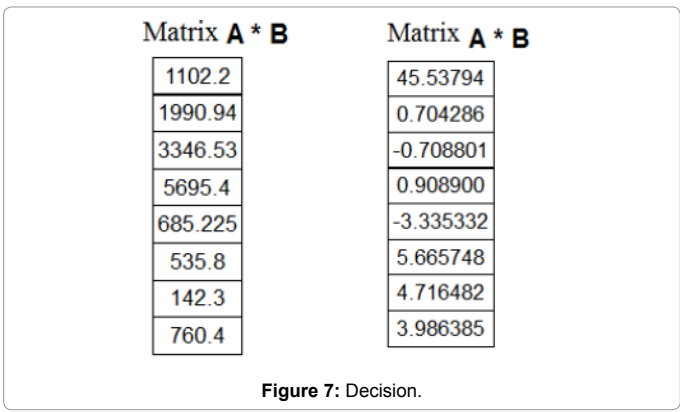


Figure 7: Decision.

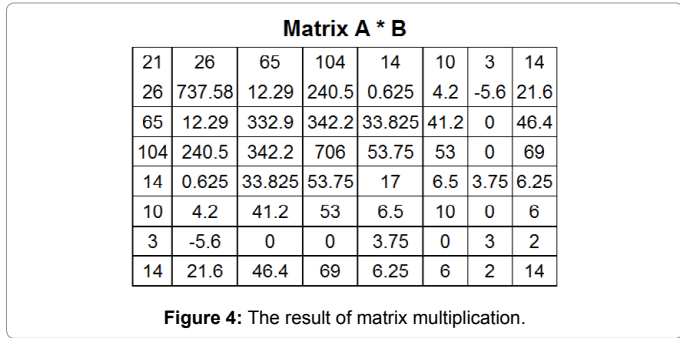


Figure 4: The result of matrix multiplication.

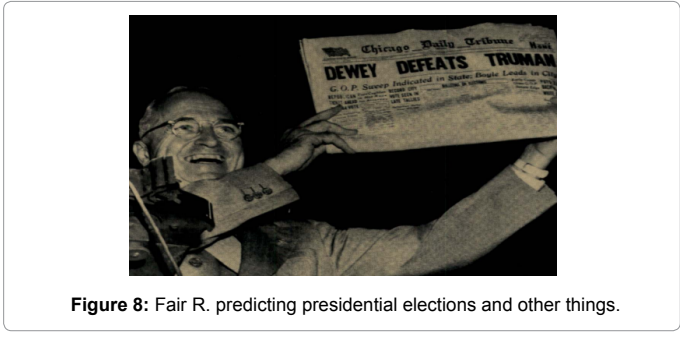


Figure 8: Fair R. predicting presidential elections and other things.

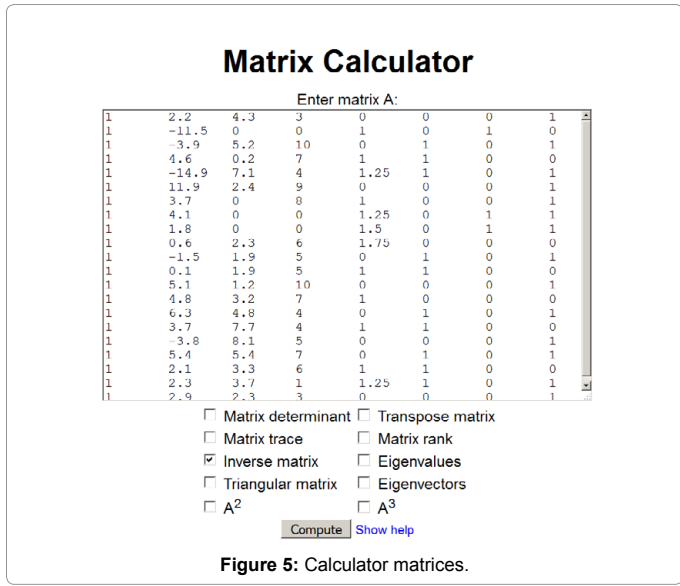


Figure 5: Calculator matrices.



Figure 9: US presidential elections.

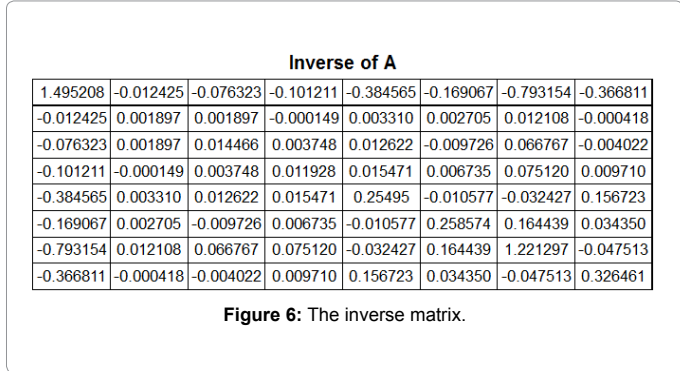


Figure 6: The inverse matrix.

Regression Statistics

multiple R	0,969842
R- square	0,940593
Normalized R-squared	0,908605
standard error	215,654
Observations	21

Table 3: Output multiple regression.

	df	SS	MS	F	Relevance F
Regression	7	9,572,471	1,367,496	294,043	5,48E-07
The residue	13	6,045,865	4,650,666		
Total	20	1,017,706			

Table 4: Output and outcome.

	Odds	Standard error	t- statistics	P- Value	Lower 95%	Top 9 5%	Lower 95,0%	Top 95,0%
Y- intersection	4,553,899	263,699	1,726,931	2,4E-10	3,984,212	5,123,586	3,984,212	5,123,586
The growth rate of GNP in the election year	0,700554	0,093923	7,458,777	4,77E-06	0,497645	0,903463	0,497645	0,903463
Abs. the value of the rate of inflation in an election year	-0,71228	0,259375	-274,615	0,01666	-127,263	-0,15194	-127,263	-0,15194
The number of quarters with GNP growth of > 3.2%	0,904863	0,235524	384,191	0,002039	0,396044	1,413,681	0,396044	1,413,681
The number of periods spent by the ruling party in power	-333,461	1,088,893	-306,239	0,009081	-568,703	-0,9822	-568,703	-0,9822
The ruling party 1 = Republicans	5,662,906	1,096,604	5,164,039	0,000182	3,293,837	8,031,975	3,293,837	8,031,975
War	4,714,529	2,383,243	1,978,199	0,069497	-0,43415	9,863,212	-0,43415	9,863,212
Pull out whether the president for the next term ?	3,983,671	1,232,177	3,233,035	0,006538	1,321,714	6,645,628	1,321,714	6,645,628

Table 5: Analysis of variance.

Surveillance	Predicted share in Government	Remains
1	5,071,565	0,984346
2	3,886,254	-276,254
3	5,779,816	0,401837
4	5,728,141	1,518,586
5	3,914,129	1,658,714
6	6,429,354	-179,354
7	56,019	-1,019
8	529,412	0,858803
9	5,049,627	1,903,732
10	4,391,467	0,685327
11	5,730,571	0,494286
12	5,110,831	-120,831
13	6,128,938	0,010624
14	4,962,177	-0,02177
15	5,979,955	2,000,448
16	485,942	0,305796
17	4,561,538	-0,91538
18	6,145,627	-225,627
19	5,241,709	1,482,911
20	5,089,799	-439,799
21	5,263,061	2,069,392

Table 6: Withdrawal of balance.

One-Tail F-Test	
Critical Value	2,832,097,502
Two-Tail Test	
Lower Critical Value	-2,160,368,652
Upper Critical Value	2,160,368,652

Table 7: Limit values of F and t-statistics.

3. Since $-2.74615 \leq -2.160368652$, a $0.01666 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

4. Since $3.84191 \geq 2.160368652$, a $0.002039 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

5. Since $-3.06239 \leq -2.160368652$, a $0.009081 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

6. Since $5.164039 \geq 2.160368652$, a $0.000182 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

7. Since $1.978199 \leq 2.160368652$, a $0.069497 \geq 0.05$, zero gipotezas with 95% reliability is not rejected;

8. Since $3.233035 \geq 2.160368652$, a $0.006538 \leq 0.05$, with 95% the

Regression statistics	
multiple R	0,969842
R- square	0,940593
normalized	0,908605
R- square	
standard error	215,654
Observations	21

Table 8: Regression statistics.

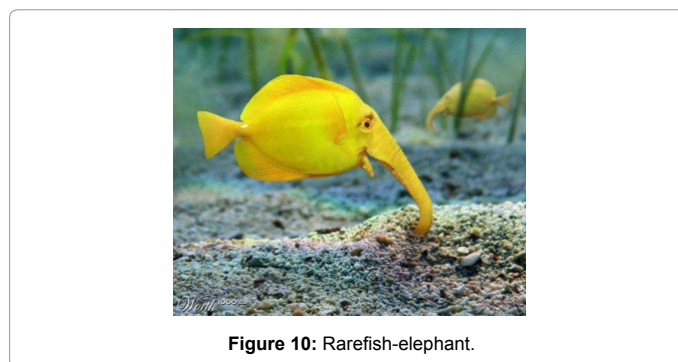


Figure 10: Rarefish-elephant.

reliability of the null hypothesis is rejected.

Limit values of F and t- statistics

1. Since $29.4043 \geq 2.542144362$ and $5.48E - 07 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

2. Since $7.458777 \geq 1.978175101$, a $4.77E - 06 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

3. Since $-2.74615 \leq -1.978175101$, a $0.01666 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

4. Since $3.84191 \geq 1.978175101$, a $0.002039 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

5. Since $-3.06239 \leq -1.978175101$, a $0.009081 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

6. Since $5.164039 \geq 1.978175101$, a $0.000182 \leq 0.069497$, with 93.0503% the reliability of the null hypothesis is rejected;

7. Since $1.978199 \geq 1.978175101$, a $0.069497 \leq 0.069497$, null hypothesis 93.0503% reliability NOW rejected;

8. As $3.233035 \geq 2.160368652$, a $0.006538 \leq 0.0695$, with 93.05% the reliability of the null hypothesis is rejected (Figures 9 and 10) (Tables 8-12).

	df	SS	MS	F	Relevance F
regression	7	9,572,471	1,367,496	294,043	5,48E-07
The residue	13	6,045,865	4,650,666		
Total	20	1,017,706			

Table 9: Analysis of variance.

	Odds	Standard errors	t- statistics	P- Value	Lower 95%	Top 95%	Lower 93,05%	Top 93,05%
Y- intersection	4,553,899	263,699	1,726,931	2,4E-10	3,984,212	5,123,586	4,032,257	5,075,542
The growth rate of GNP in the election year	0,700554	0,093923	7,458,777	4,77E-06	0,497645	0,903463	0,514757	0,886351
Abs. the value of the rate of inflation in an election year	-0,71228	0,259375	-274,615	0,01666	-127,263	-0,15194	-122,537	-0,19919
The number of quarters with GNP growth of > 3.2%	0,904863	0,235524	384,191	0,002039	0,396044	1,413,681	0,438955	137,077
The number of periods spent ruling the party in power	-333,461	1,088,893	-306,239	0,009081	-568,703	-0,9822	-548,864	-118,059
Ruling party1 = republics - end	5,662,906	1,096,604	5,164,039	0,000182	3,293,837	8,031,975	3,493,631	7,832,181
War	4,714,529	2,383,243	1,978,199	0,069497	-0,43415	9,863,212	5,75E-05	9,429
Pull out whether the president for the next term?	3,983,671	1,232,177	3,233,035	0,006538	1,321,714	6,645,628	1,546,209	6,421,134

Table 10: Analysis of variance.

Surveillance	Predicted Share in Government	Remains
1	5,071,565	0,984346
2	3,886,254	-276,254
3	5,779,816	0,401837
4	5,728,141	1,518,586
5	3,914,129	1,658,714
6	6,429,354	-179,354
7	56,019	-1,019
8	529,412	0,858803
9	5,049,627	1,903,732
10	4,391,467	0,685327
11	5,730,571	0,494286
12	5,110,831	-120,831
13	6,128,938	0,010624
14	4,962,177	-0,02177
15	5,979,955	2,000,448
16	485,942	0,305796
17	4,561,538	-0,91538
18	6,145,627	-225,627
19	5,241,709	1,482,911
20	5,089,799	-439,799
21	5,263,061	2,069,392

Table 11: Withdrawal of balance.

One-Tail F-Test	
Critical Value	2,542,144,362
Two-Tail Test	
Lower Critical Value	-1,978,175,101
Upper Critical Value	1,978,175,101

Table 12: Limit values of F and t- statistics.

Regression statistics	
Multiple R	0,960578
R- square	0,92271
normalized R- square	0,889586
standard error	2,370,322
Observations	21

Table 13: Output outcome.



Figure 11: Scene from the movie "Sholay" in 1975 India.

Again repeat $29.4043 \geq 2.542169216$ and $5.48E-07 \leq 0.069498$, with 93.0503% the reliability of the null hypothesis is rejected. And further, since $1.978 \geq 1.978$, a $0.069497 \leq 0.069497$, null hypothesis 93.0503% reliability definitively rejected. Why is this important? Eventually chocolate weighs 100 grams, 95 or about 93? Consider the game of Russian roulette with one bullet in the drum. Do not we would like to see the drum, for example, 1,000 or a million rounds of ammunition? Where is the limit in medicine temperature 37°C? Almost all authors agree on the fact that 95% - a statistical threshold beyond which - a disease. And, secondly, if the counting carried out with the reverse end, the error is increased from 5% to about 7% and 15% (three times). That is, at 95% reliability a failure on 20 attempts, and at 93.0503% reliability a failure already at 14.38911032. When reliability of 85%, which is offered by Professor Winston, single failure accounted for 6.67 attempts. In a game of Russian roulette we get 20 and 14 and the 6-7 Chargers revolvers charged 1 cartridge. Even in the first case, we cannot risk it. Sachs [4] points out: "In special cases, especially when the test poses a risk to human life, you should take less than $\alpha = 0,001$ probability of errors." In the second case, the risk has increased. In the third your chances are dangerous. The offender Gabbar Singh has played in Russian roulette, spinning drum six-shooter revolver. Thus, Mr. Gabbar would fulfill the conditions of a statistical reliability of 85%, if he had a 6.67 charging revolver with one cartridge (Figure 11).

We have no right to risk, said Holmes. Ten to one that they go down to the river, rather than up. And yet, we must take into account

this possibility. Lips hero Sir Arthur waives 90% reliability.

In Tables 3-7 we have made the correction, whose outcome in Table 12. The last two columns are not the same way. In this way Table 3 contains, first, a mathematical error instead of the 95% reliability should indicate the reliability of 93.0502%. Second – under certain conditions and statistical. What conditions? If you are willing to risk – the coefficient of reliability of 93.0502% and 85% is statistically true. But if the risk is not acceptable – the coefficient of 4.714529 must be zero. Thus Tables 3 and 6 after the both patches reconstructed in Table 8. In support of Sax in [4] causes test the significance of the regression coefficient: "Checks null hypothesis $H_0: \beta_{yx} = 0$, i.e checks whether significantly different estimates of regression coefficients from zero. The border is set on the basis of the significance of the t-distribution [8].

$$\hat{t} = \frac{|\hat{b}_{yx}|}{s_{b_{yx}}} \quad (5)$$

C (n - 2) degrees of freedom. If statistics are greater than or equal to the limit of the significance of it, to β_{yx} significantly different from zero".

Table 3 received:

$$\hat{a}_{War} = 4,714529; s_{a_{War}} = 2,383243; n = 21; S = 95\%, i.$$

$$\hat{t}_{19,0.05} = 2,160368652; \hat{t} = \frac{4,714529}{2,383243} = 1,978199034 \leq 2,160368652.$$

Coefficient a_{War} not significantly different from zero ($P \geq 0.05$).

$$y = 0,700554 \times x_1 - 0,71228 \times x_2 + 0,904863 \times x_3 - 3,33461 \times x_4 + 5,662906 \times x_5 + 4,714529 \times x_6 + 3,983671 \times x_7 + 45,53899 \quad (6)$$

In addition, Sachs [4] points out: Let's device consists of 300 complex elements. If these elements, for example, 284 completely smoothly, 12 have the reliability of 99%, and 4 – 98% reliability, the reliability of the device, provided independence is reliable elements.

$$1,00^{284} \times 0,99^{12} \times 0,98^4 \times 1 \times 0,8864 \times 0,9224 = 0,8176 \approx 82\%;$$

This instrument no one would buy it".

Instead of equation 2, we can adopt a new mathematical model of the 6:

$$\text{Projected\% of votes in the presidential election} = 48.60078 + 0.653815_{\text{growthGNP}} - 0.97002_{\text{abs.inf.}}$$

$$+ 0.614881_{\text{quarter growth GNP}} - 3.20944 + \text{terms of powers} + 5.028129_{\text{Republicans}} + 0_{\text{War}} + 4.167085_{\text{President for a new term}} \quad (7)$$

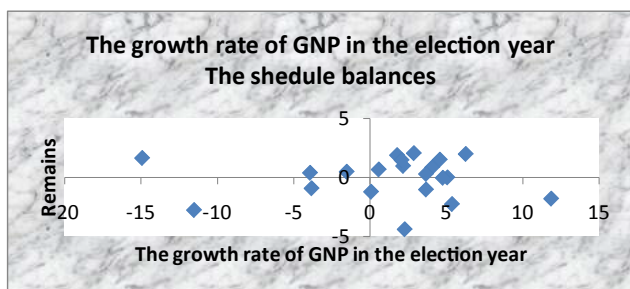


Figure 12: Schedule residues.

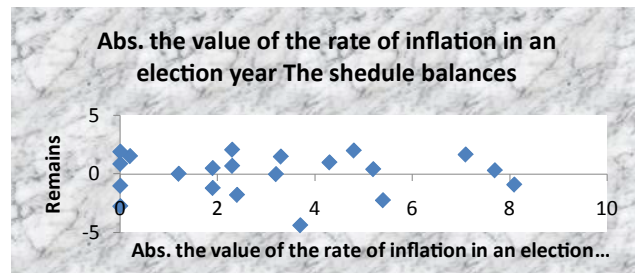


Figure 13: Residual plot.

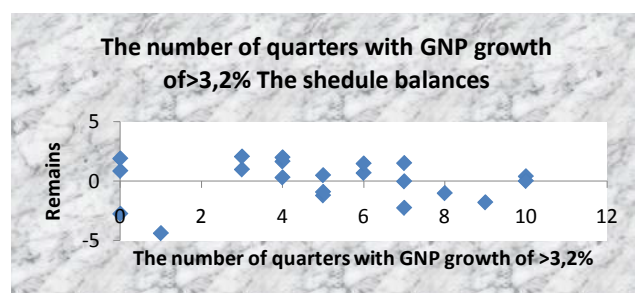


Figure 14: Residual Plot.

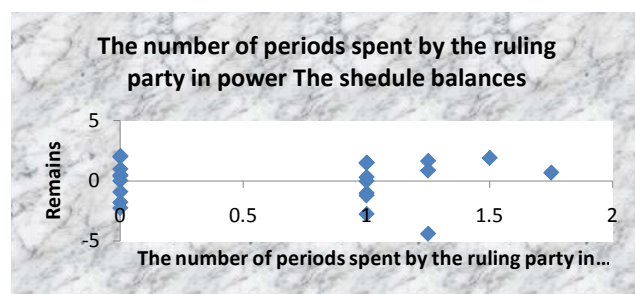


Figure 15: Residual plot.

After calculation not 21a and 22 members of the general population, you can get a little more accurate formula, the summary table to which you can build yourself.

$$\text{Projected\% of votes in the presidential election} = 47.71879 + 0.661574_{\text{growthGNP}} - 0.94685_{\text{abs.inf.}} + 0.621092_{\text{quarter growthGNP}} - 2.80492 + \text{terms of power} + 5.214158_{\text{Republicans}} + 0_{\text{war}} + 4.669377_{\text{President for a new term}} \quad (8)$$

Finally, Professor Winston in his work is not conducted analysis of residues (Figures 12-19) (Tables 12-17).

Limit values of F and t- statistics

1. Since $27.85619 \geq 22.847725996$ and $5.16E-07 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;
2. Since $6.543745 \geq 2.144786681$, a $1.3E-05 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;
3. Since $-3.93503 \leq -2.144786681$, a $0.001495 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;
4. Since $3.034645 \geq 2.144786681$, a $0.008917 \leq 0.05$, with 95% the

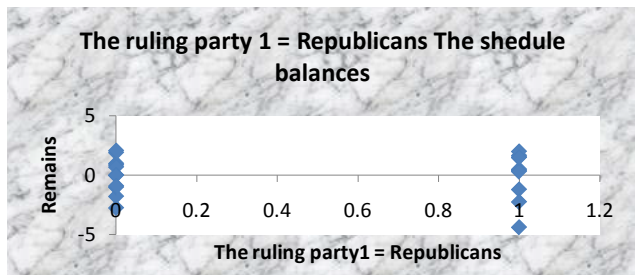


Figure 16: Residual plot.

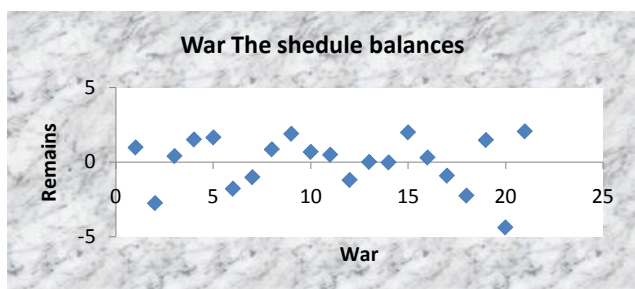


Figure 17: Residual plot.

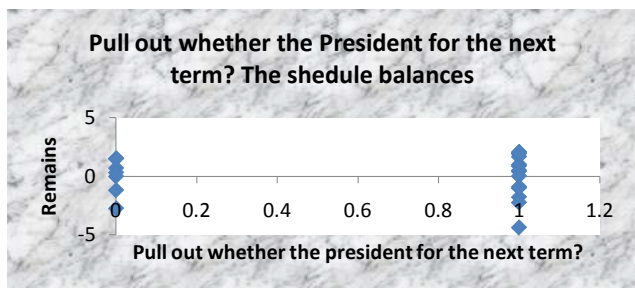


Figure 18: Residual plot.

	df	SS	MS	F	Significance F
Regression	6	9,390,477	156,508	2,785,619	5,16E-07
The residue	14	7,865,797	5,618,427		
Total	20	1,017,706			

Table 14: Analysis of variance.

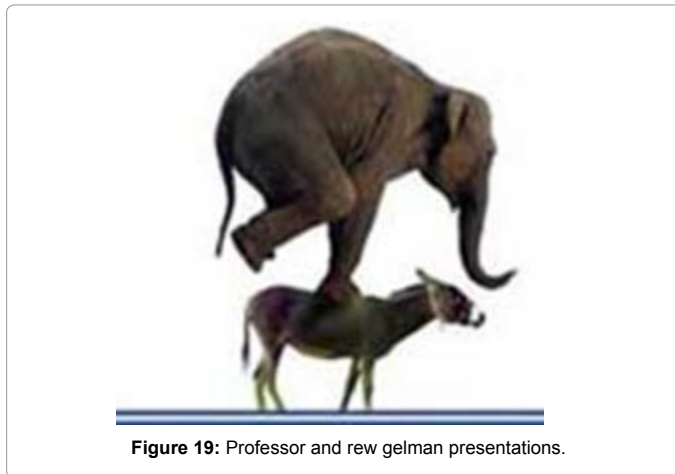


Figure 19: Professor and rew gelman presentations.

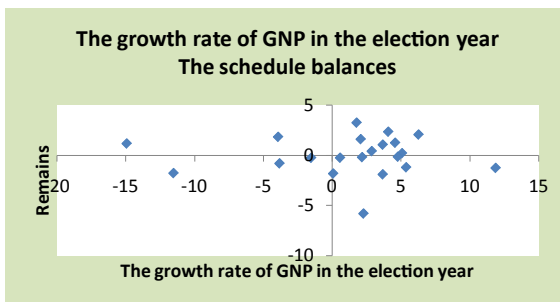


Figure 20: Residual plot.

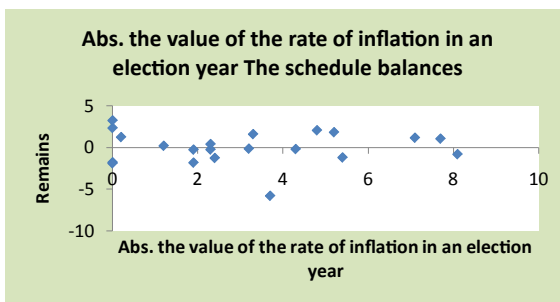


Figure 21: Residual plot.

	Coefficient β_1	standard error	t-Statistic a	P- Value	Lower 95%	Top 95%	Lower 95,0%	Top 95,0%
Y- intersection	4,860,078	2,346,626	2,071,092	6,69E-12	4,356,777	5,363,379	4,356,777	5,363,379
The growth rate of GNP in the election year	0,653815	0,099915	6,543,745	1,3E-05	0,43952	0,868111	0,43952	0,868111
Abs. the value of the rate of inflation in an election year	-0,97002	0,24651	-393,503	0,001495	-149,873	-0,44131	-149,873	-0,44131
The number of quarters with GNP growth of > 3.2%	0,614881	0,20262	3,034,645	0,008917	0,180303	1,049,459	0,180303	1,049,459
The number of time spent by the ruling party in power	-320,944	1,194,815	-268,614	0,017731	-577,206	-0,64682	-577,206	-0,64682
Ruling party 1 = Republicans	5,028,129	1,152,555	4,362,593	0,00065	2,556,144	7,500,115	2,556,144	7,500,115
Pull out whether the president for the next term?	4,167,085	1,350,486	308,562	0,008059	1,270,582	7,063,589	1,270,582	7,063,589

Table 15: Analysis of variance.

Surveillance	The predicted share in the governments e	Remains
1	518,798	-0,1798
2	3,787,246	-177,246
3	563,508	1,849,197
4	5,753,718	126,282
5	396,147	1,185,296
6	6,375,414	-125,414
7	5,689,659	-189,659
8	5,143,671	2,363,294
9	4,913,057	3,269,429
10	4,483,478	-0,23478
11	5,804,663	-0,24663
12	5,171,621	-181,621
13	610,871	0,212896
14	4,972,975	-0,12975
15	5,971,844	2,081,556
16	4,782,893	1,071,067
17	4,550,058	-0,80058
18	6,039,264	-119,264
19	5,228,069	161,931
20	5,231,376	-581,376
21	5,427,752	0,422482

Table 16: Predicted share in the governments.

One-Tail F-Test	
Critical Value	2,847,725,996
Two-Tail Test	
Lower Critical Value	-2,144,786,681
Upper Critical Value	2,144,786,681

Table 17: Limit values of F and t-statistics.

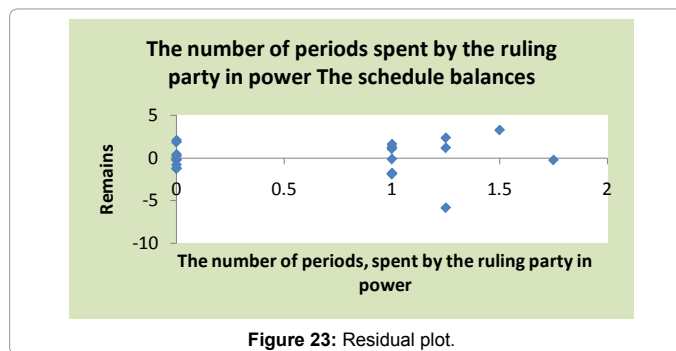


Figure 23: Residual plot.

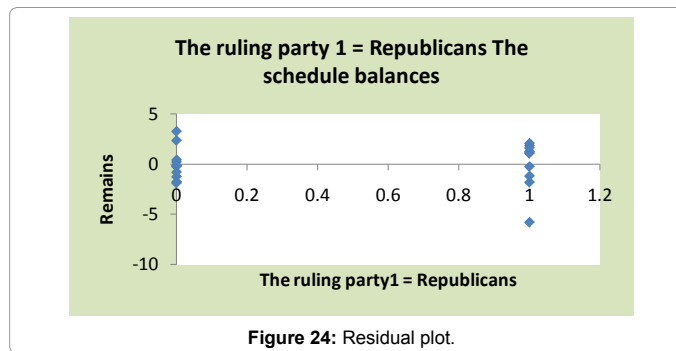


Figure 24: Residual plot.

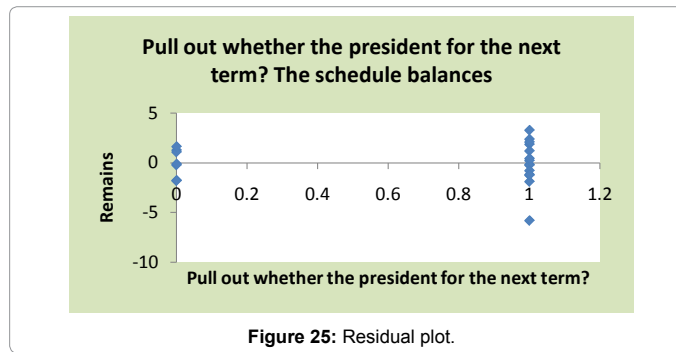


Figure 25: Residual plot.

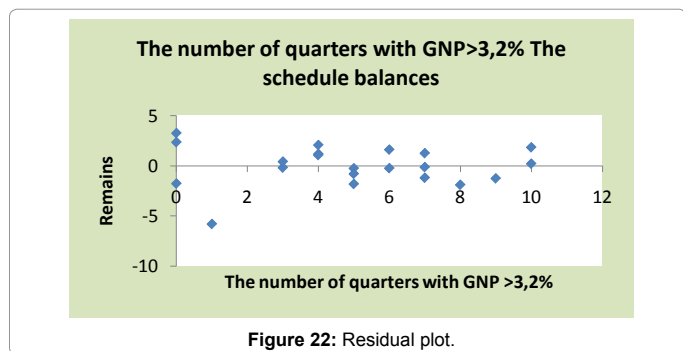


Figure 22: Residual plot.

reliability of the null hypothesis is rejected;

5. Since $-2.68614 \leq 2.144786681$, a $0.017731 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

6. Since $4.362593 \geq 2.144786681$, a $0.00065 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected;

7. Since $3.08562 \geq 2.144786681$, a $0.008059 \leq 0.05$, with 95% the reliability of the null hypothesis is rejected.

More precisely will – to 98.2269% reliability (Figures 20-31).

Thus, having considered the multivariate regression model of

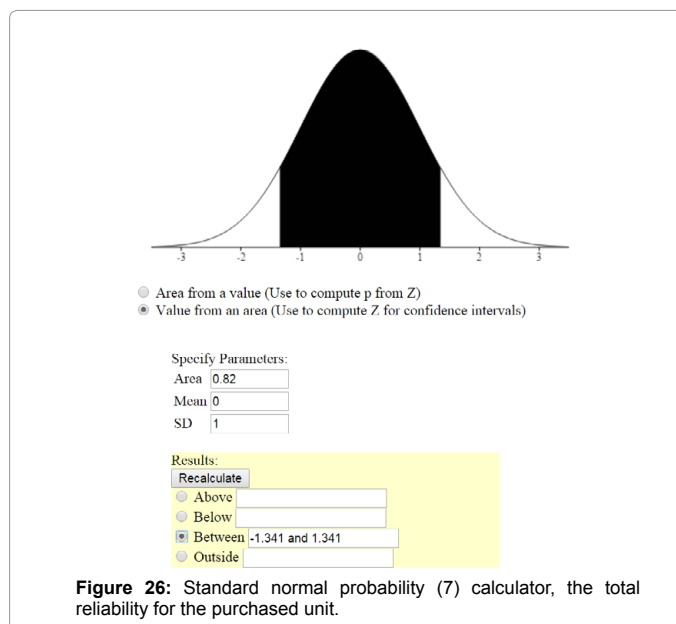
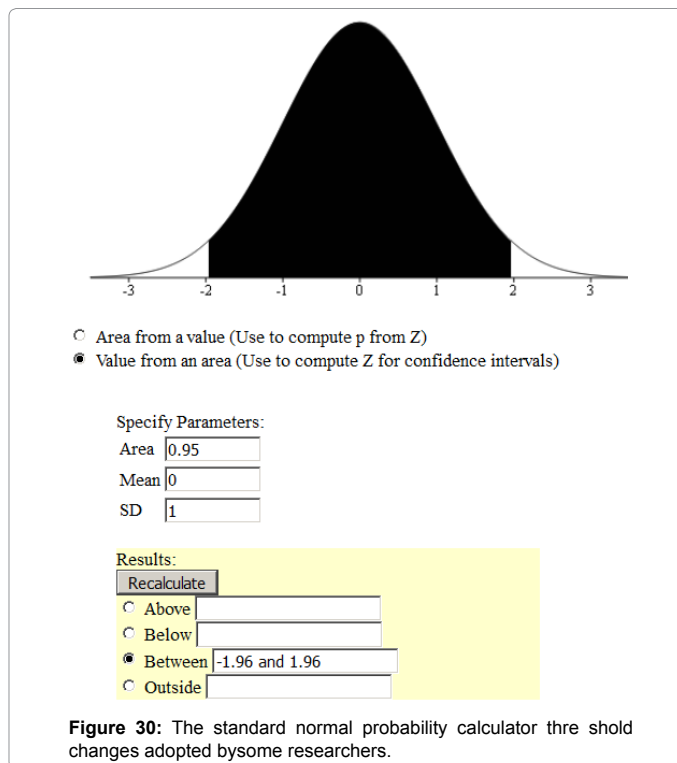
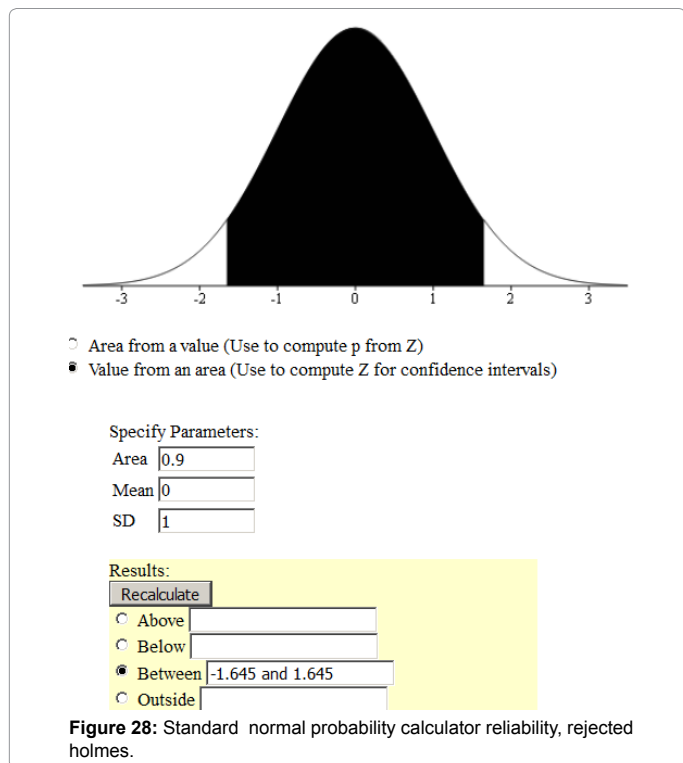
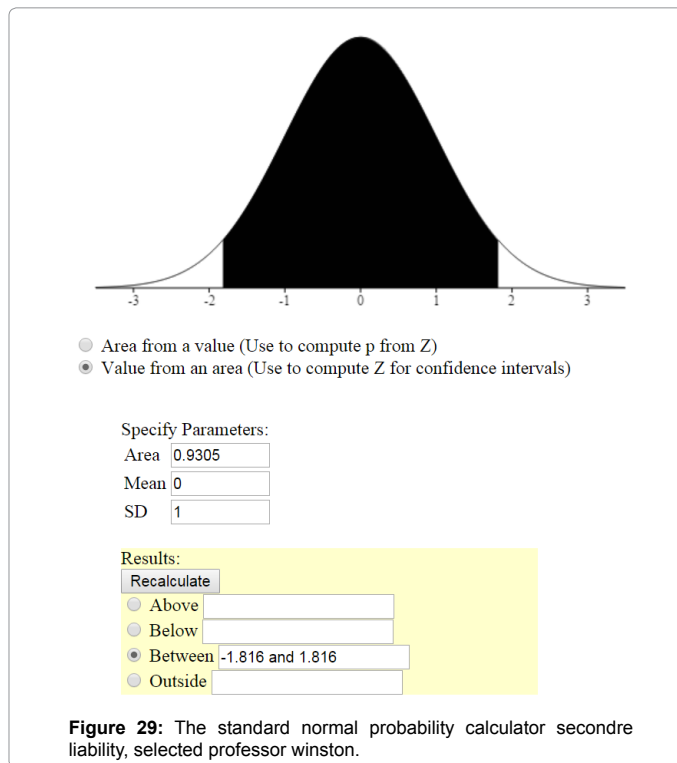
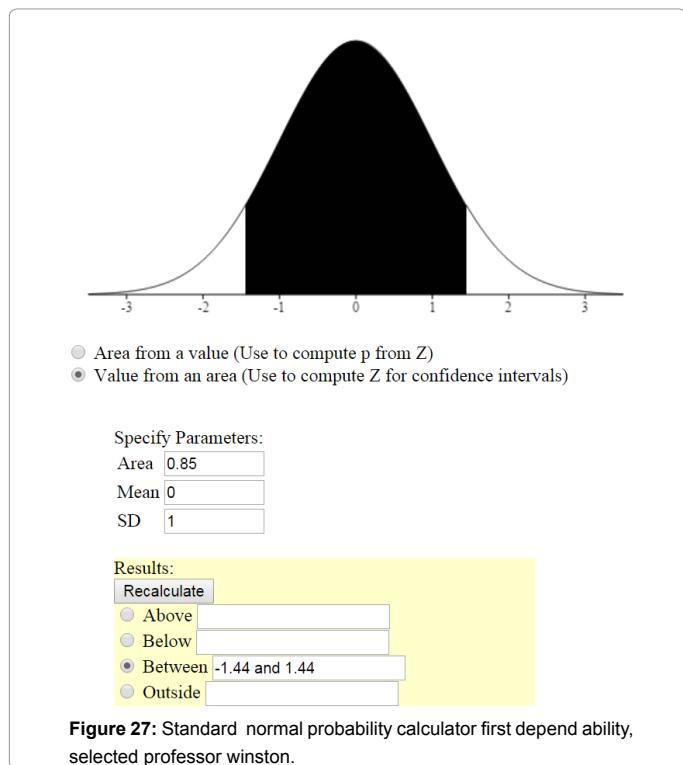


Figure 26: Standard normal probability (7) calculator, the total reliability for the purchased unit.



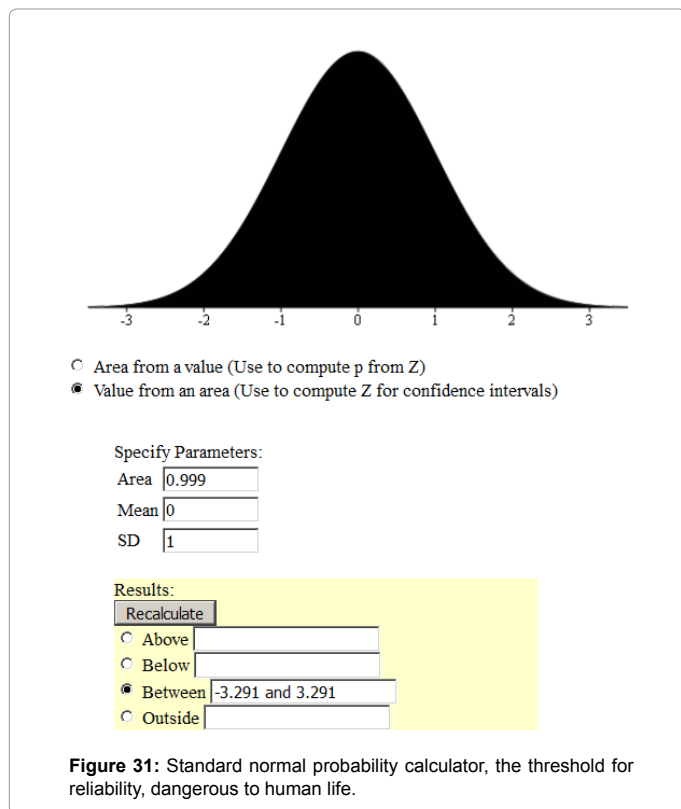


Figure 31: Standard normal probability calculator, the threshold for reliability, dangerous to human life.

Professor Winston, our output is divided into several mutually exclusive options:

1. 95% reliability in Table 3 – mechanical error interpreter file «president» in the CD-ROM, attached to the book [2]; 93.0503%

reliability must be in Table 3 instead of 95%;

2. 93.0503% reliability for this kind of data is sufficient and the model type 2 93.0503% reliability acceptable in life;

3. 93.0503% reliability, though not enough, but as there are managers as a risk-averse, and are not inclined, the first model to be adopted; Finally, we can recognize that the reliability of 93.0503% is inadequate and should be replaced by at least 95% reliability, and the model of the form (2) should lose factor a_6 and be re-constructed from the data in Table 7. The graphical analysis of residues also inclined to do so. The charts residues, relating to the Model (2) Professor Winston, the four of them has homoscedasticity. In the graphs relating to the model (6), we have constructed, homoscedasticity reduced to two, and the scatter of points is more favorable. Model (6) (as well as (7)) is 98.2269% reliability. Comparison of the standard normal probability plots gives the perception that a small reduction in reliability leads to a sharp change in the values of z-statistics and the number of standard deviations.

References

1. Chetyrkin E (1982) Probability and statistics. Finance and statistics.
2. Winston W (2005) Data analysis and business models, lane from English pp: 576.
3. Winston W (2004) Microsoft excel: Data analysis and business modeling. Microsoft Press pp: 593.
4. Sachs L (1976) Statistical estimators. Statistics, Moscow pp: 598.
5. Doyle AC (1994) Collected works: The 10th V.8 per from English. Hermes publishing house pp: 320.
6. Fair RC (2002) Predicting presidential elections and other things. Stanford University Press pp: 181.
7. The Number empire.
8. HyperStat online statistics textbook.