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More-For-Less Paradox in a Transportation Problem under Fuzzy Environments

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Abstract

In this paper, we discuss more-for-less paradox in a transportation problem under fuzzy environment with linear constraints. In some cases of the transportation problem, an increase in the supplies and demands or in other words, increase in the flow results a decrease in the optimum transportation cost. This type of behavior which means paradoxical, is called transportation paradox. Thereby, we establish a sufficient condition for the existence of paradox in a transportation problem under fuzzy environment. Also we illustrate a numerical example in support of the theory.

Key words: Fuzzy number; Fuzzy transportation problem; Paradox; Paradoxical range of flow

Introduction

The basic transportation problem is one of the special class of linear programming problem, which was first formulated by Hitchcook [1] Charnes et al. [2] Appa [3] Klingman and Russel [4] developed further the basic transportation problem. Basically, the papers of Charnes and Klingman [5] and Szwarc [6] are treated as the sources of transportation paradox for the researchers. In the paper of Charnes and Klingman, they name it "morefor-less" paradox and wrote "The paradox was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some (e.g. A.Charnes and W.W.Cooper), but unknown to the great majority of workers in the field of linear programming". Subsequently, in the paper of Appa, he mentioned that this paradox is known as "Doig Paradox" at the London School of Economics, named after Alison Doig. Gupta et al. [7] established a sufficient condition for a paradox in a linear fractional transportation problem with mixed constraints. Adlakha and Kowalski [8] derived a sufficient condition to identify the cases where the paradoxical situation exists.

Ryan [9] developed a goal programming approach to the representation and resolution of the more for less and more for nothing paradoxes in the distribution problem. Deineko et al. [10] developed a necessary and sufficient condition for a cost matrix which is immuned against the transportation paradox. Dahiya and Verma [11] considered paradox in a nonlinear capacitated transportation problem. Adlakha et al. [12] developed a simple heuristic algorithm to identify the demand destinations and the supply points to ship more for less in fixed-charge transportation problems. Storoy [13] considered the classical transportation problem and studied the occurrence of the so-called transportation paradox (also called the more-for-less paradox). Joshi and Gupta [14] studied an efficient heuristic algorithm for solving more-for-less paradox and algorithm for finding the initial basic feasible solution for linear plus linear fractional transportation problem. Schrenk et al. [15] analyzed degeneracy characterizations for two classical problems (1) the transportation paradox in linear transportation problems and (2) the pure constant fixed charge transportation problem. asu et al. [16] considered the algorithm of finding all paradoxical pairs in a linear transportation problem.

Fuzzy sets and fuzzy logic were introduced by Lotfi A. Zadeh in 1965. Zadeh [17] was almost single-handedly responsible for the early development in this field. A fuzzy transportation problem is an extension of linear transportation problem, where at least one of the transportation costs, supply and demand quantities are fuzzy quantities. The objective function of the fuzzy transportation problem is to determined the total fuzzy minimum transportation cost by shipping the fuzzy supply and fuzzy demand. Bellman and Zadeh [18], Liu [19] developed further. Dinagar and Palanivel [20] investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers. Dutta and Murthy [21] investigated the transportation problem with additional impurity restrictions where costs are not deterministic numbers but imprecise ones, also the elements of the cost matrix are subnormal fuzzy intervals with strictly increasing linear membership functions. Ojha et al. [22] considered capacitated-multi-objective, solid transportation problem which formulated in fuzzy environment with non-linear varying transportation charge and an extra cost for transporting the amount to an interior place through small vehicles. In this paper, we present more-foe-less paradox in a transportation problem under fuzzy environment with linear constraints. To solve such type of problem we consider the transporting cost per unit product, supply and demand quantities are described in trapezoidal fuzzy. Thereby, we state the sufficient condition of existence of paradox. We also justify the theory by illustrating a numerical example.

Definition. Fuzzy Set: Let A be a classical set and $\mu A(x)$ be a function defined over $A \to [0, 1]$. A fuzzy set $A*$ with membership function $\mu A(x)$ is defined by $A* = \{(x, \mu A(x)) : x \in A \text{ and } \mu A(x) \in [0, 1]\}$ Definition 1.2. Fuzzy Number: A real fuzzy number $\tilde{a} \approx (a1, a2, a3, a4)$, where $a1, a2, a3, a4 \in R$ and two functions $f(x)$ and $g(x) : R \to [0, 1]$, where $f(x)$ is non-decreasing and $g(x)$ is non-increasing, such that

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we can define membership function $\mu \tilde{a}$ (x) satisfying the following conditions

$$\mu_{\tilde{a}}(x) = \begin{cases} f(x) \text{ if } a_1 \le x \le a_2 \\ 1 \text{ if } a_2 \le x \le a_3 \\ g(x) \text{ if } a_3 \le x \le a_4 \\ 0 \text{ otherwise} \end{cases}$$

Trapezoidal Membership Function: The trapezoidal membership function of trapezoidal fuzzy number $\tilde{a} \approx$ (a1, a2, a3, a4) is defined by

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} \text{ for } a_1 \le x \le a_2 \\ 1 \text{ for } a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} \text{ for } a_3 \le x \le a_4 \\ 0 \text{ otherwise} \end{cases}$$

Arithmetic operations: Let $\tilde{a} \approx (a_1, a_2, a_3, a_4)$ and $\tilde{b} \approx (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers, where a1, a_2, a_3, a_4, b_1, b_2, b_3, $b_4 \in R$ then the arithmetic operation on $\tilde{a}$ and $\tilde{b}$ are:

Addition: The addition of two fuzzy numbers and $\tilde{b}$ is $\tilde{a} \oplus \tilde{b} \approx (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$.

Subtraction: The negative fuzzy number of $\tilde{b}$ is $\ominus \tilde{b} \approx (-b_4,-b_3,-b_2,-b_1)$, then the

subtraction of two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ is $\tilde{a} \ominus \tilde{b} \approx (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$.

Multiplication:

(i) The multiplication of an arbitrary number _ and a fuzzy number $\tilde{a}$ is

$$\rho \otimes \tilde{a} \begin{cases} (\rho a_1, \rho a_2, \rho a_3, \rho a_4) \text{ for } \rho \ge 0 \\ (\rho a_4, \rho a_3, \rho a_2, \rho a_1) \text{ for } \rho \le 0 \end{cases}$$

(ii) The multiplication of two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ is $\tilde{a} \otimes \tilde{b} \approx$ (t1, t2, t3, t4),

where t1 = min{a1b1, a1b4, a4b1, a4b4}, t2 = min{a_2b_2, a_2b_3, a_3b_2, a_3b_3},

t3 = max{a_2b_2, a_2b_3, a_3b_2, a_3b_3}, t_4 =max{a_1b_1, a_1b_4, a_4b_1, a_4b_4}.

Definition: The magnitude of the trapezoidal fuzzy number $\tilde{a} \approx (a_1, a_2, a_3, a_4)$ is defined by $\text{Mag}(\tilde{a}) = \dfrac{a_1+2a_2+2a_3+a_4}{6}$

Problem Formulation

Let $\tilde{x}$ ij$\approx(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is the uncertain number of units transported from the ith origin to the jth destination, $\tilde{c}$ ij $\approx (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$ is the uncertain cost involved in transporting per unit product from the ith origin to the jth destination, $\tilde{a}$ i $\approx (a_i^1, a_i^2, a_i^3, a_i^4)$ is the uncertain number of units available at the ith origin, $\tilde{b}_j \approx |(b_j^1, b_j^2, b_j^3, b_j^4)$ is the uncertain number of units required at the jth destination. Then the cost minimizing fuzzy transportation problem be

$$P: Min\ \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

subject to the constraints,

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i;\ \forall\ i \in I = (1,2,....m)$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{b}_i;\ \forall\ j \in J = (1,2,....n)$$

And

$$\tilde{x}_{ij} \succeq 0\ \ \forall\ \ (i,j) \in IxJ$$

Let B be the basis of the problem P and $X^0 \approx \{\tilde{x}_{ij}^0 \mid (i,j) \in IxJ\}$ be its basic feasible solution. The value of the objective function is $\tilde{Z}$ 0 and the flow $\tilde{F}$ 0 corresponding to the basic feasible solution $\tilde{x}$ 0 are $\tilde{Z}$ 0 $\approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}^0$ and $\tilde{F} \approx \sum_{i \in I} \tilde{a}_i \approx \sum_{i \in I} \tilde{b}_j$ We consider the dual variables $\tilde{u}_i$ for $i \in I$ and $\tilde{V}_j$ for $j \in J$ such that $\tilde{u}_i \oplus \tilde{v}_j \approx \tilde{c}_{ij}$ corresponding to the basis B. Also $\forall$ (i, j)$\notin$ **B** let $\tilde{c}_{ij} \approx (\tilde{u}_i \oplus \tilde{v}_j) \odot \tilde{c}_{ij}$ and if $\tilde{c}_{ij} \preceq 0 \forall (i,j) \notin B$ then solution of the fuzzy transportation problem is optimum.

Definitions

Paradox in a fuzzy transportation problem

In a fuzzy transportation problem if we can obtain more flow ($\tilde{F}$ 1) with lesser transportation cost ($\tilde{Z}$ 1) than the optimum flow ($\tilde{F}$ 0) corresponding to the optimum transportation cost ($\tilde{Z}$ 0) i.e. $\tilde{F}$ 1 $\succ$ $\tilde{F}$ 0 and $\tilde{Z}$ 1 $\prec$ $\tilde{Z}$ 0, then we say that a paradox occurs in a fuzzy transportation problem.

Fuzzy cost-flow pair

If the value of the objective function is $\tilde{Z}$ i and the flow is $\tilde{F}$ i corresponding to the feasible solution $\tilde{x}$ i of a fuzzy transportation problem, then the pair ($\tilde{Z}$ i, $\tilde{F}$ i) is called the fuzzy cost-flow pair corresponding to the feasible solution $\tilde{x}$ i.

Fuzzy paradoxical pair

A fuzzy cost-flow pair ($\tilde{Z}$, $\tilde{F}$) of an objective function is called fuzzy paradoxical pair if $\tilde{Z} \prec \tilde{Z}$ 0 and $\tilde{F} \succ \tilde{F}$ 0 where $\tilde{Z}$ 0 is the optimum transportation cost and $\tilde{F}$ 0 is the optimum flow of the fuzzy transportation problem.

Best fuzzy paradoxical pair

The fuzzy paradoxical pair ($\tilde{Z}$ ∗, $\tilde{F}$ ∗) is called the best fuzzy paradoxical pair of a fuzzy transportation problem if for all fuzzy paradoxical pair ($\tilde{Z}$, $\tilde{F}$), either $\tilde{Z}* \prec \tilde{Z}$ or $\tilde{Z}* \approx \tilde{Z}$ but $\tilde{F}* \succ \tilde{F}$

Fuzzy paradoxical range of flow

If $\tilde{F}$ 0 be the optimum flow and $\tilde{F}$ ∗ be the flow corresponding to the best fuzzy paradoxical pair of a fuzzy transportation problem then [$\tilde{F}$ 0, $\tilde{F}$ ∗] is called fuzzy paradoxical range of flow.

Theorem 2.1. The sufficient condition for the existence of paradoxical solution of (P) is that in the optimum table of (P), ∃ at least one cell (r, s)$\notin$ B where we have $(\tilde{u}_r \oplus \tilde{v}_s) \prec 0$ if $\tilde{a}$ r and $\tilde{b}$ s are replaced by $\tilde{a}$ r $\oplus \tilde{l}$ and $\tilde{b}$ s $\oplus \tilde{l}$ ($\tilde{l} \succ \tilde{0}$) respectively.

Proof: Let $\tilde{Z}$ 0 be the value of the objective function and $\tilde{F}$ 0 be the optimum flow corresponding to the optimum solution ($\tilde{x}$ 0) of the problem (P). The dual variables $\tilde{u}$ i and $\tilde{v}$ j are given by $\tilde{u}$ i $\oplus$ $\tilde{v}$ j $\approx$ $\tilde{c}$ ij ∀ (i, j) ∈ B Then the value of the objective function in terms of the dual variables is given by

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$$\tilde{Z}^0 \approx \sum_i \sum_j \tilde{c}_{ij}\tilde{x}_{ij}^{\ 0}$$
$$\approx \sum_i \sum_j (\tilde{u}_i \oplus \tilde{v}_j)\tilde{x}_{ij}^{\ 0}$$
$$\approx \sum_i (\sum_j \tilde{x}_{ij}^{\ 0})\tilde{u}_i \oplus \sum_j (\sum_i \tilde{x}_{ij}^{\ 0})\tilde{v}_j$$
$$\approx \sum_i \tilde{a}_i\tilde{u}_i \oplus \sum_j \tilde{b}_j\tilde{v}_j$$

And $\tilde{F}^0 \approx \sum_i \tilde{a}_i \approx \sum_j \tilde{b}_j$

Now, let ∃ at least one cell (r, s) ∉ B, where if we replace $\tilde{a}$ r and $\tilde{b}$ s by $\tilde{a}$ r $\oplus \tilde{l}$ and $\tilde{b}$ s $\oplus \tilde{l}$ respectively ($\tilde{l} \succ \tilde{0}$), in such a way that the optimum basis remains same, then the value of the objective function $\hat{\tilde{Z}}$ is given by

$$\hat{\tilde{Z}} \approx [\sum_{i,i\neq r} \tilde{a}_i\tilde{u}_i \oplus \sum_{j,j\neq s} \tilde{b}_j\tilde{v}_j \oplus \tilde{u}_r(\tilde{a}_r \oplus \tilde{l}) \oplus \tilde{v}_s(\tilde{b}_s \oplus \tilde{l})]$$
$$\approx [\tilde{Z}^0 \oplus \tilde{l}(\tilde{u}_r \oplus \tilde{v}_s)]$$

The new flow $\hat{\tilde{F}}$ is given by

$$\hat{\tilde{F}} \approx \sum_i \tilde{a}_i \oplus \tilde{l} \approx \sum_j \tilde{b}_j \oplus \tilde{l} \approx \tilde{F}^0 \oplus \tilde{l}$$
$$\hat{\tilde{F}} \odot \tilde{F}^0 \approx \tilde{l} \succ \tilde{0}$$

Therefore, for the existence of paradox we must have $\hat{\tilde{Z}} \ominus Z^0 \prec \tilde{0}$ since $\hat{\tilde{F}} \ominus \tilde{F}^0 \succ \tilde{0}$ i.e. $\tilde{u}_r \oplus \tilde{v}_s \prec \tilde{0}$ because $\tilde{l} \succ \tilde{0}$

Hence, the theorem.

Now we state the following algorithm to find all the paradoxical pairs of the problem

(P).

3 Algorithm : To obtain all the paradoxical pairs

Step 1: i = 0.

Step 2: Find the cost-flow pair ($\tilde{Z}$ 0, $\tilde{F}$ 0) for the optimum solution $\tilde{x}$ 0.

Step 3: Find all cells (r, s) ∉ B such that ($\tilde{u}$ r ⊕ $\tilde{v}$ s) ≺ $\tilde{0}$ if it exists, otherwise go to step 8.

Step 4: Find min flow for $\tilde{l}$ ≈ (1, 0, 0, 0), $\tilde{l}$ ≈ (0, 1, 0, 0), $\tilde{l}$ ≈ (0, 0, 1, 0), $\tilde{l}$ ≈ (0, 0, 0, 1) or $\tilde{l}$ ≈ (1, 1, 1, 1) which enters into the existing basis whose corresponding cost is minimum. Let ($\tilde{Z}$ i, $\tilde{F}$ i) be the new cost flow pair corresponding to the optimum solution Xi.

Step 5: i = i + 1.

Step 6: Write ($\tilde{Z}$ i, $\tilde{F}$ i).

Step 7: Find all cells (r, s) ∉ B such that ($\tilde{u}$ r ⊕ ⊕ s) ≺ $\tilde{0}$ if it exists go to step 4, otherwise go to step 9.

Step 8: Write paradox does not exist and go to step 10.

Step 9: Write paradox exists and the best paradoxical pair ($\tilde{Z}$ ∗, $\tilde{F}$ ∗) ≈ ($\tilde{Z}$ i, $\tilde{F}$ i) for the optimum solution $\tilde{x}$ ∗ ≈ $\tilde{x}$ i.

Step 10: End.

Numerical Example

We consider a numerical example which consists of three origins and four destinations, the uncertain numbers of supply, demand and cost per unit are tabulated in Table 1.

Solving the fuzzy transportation problem given in Table 1, the optimum solution is given in Table 2.

Table 2 gives the optimum solution $\tilde{x}$ 0 = {$\tilde{x}$ 11 = (18, 19, 21, 22), $\tilde{x}$ 13 = (6, 8, 12, 14), $\tilde{x}$ 14 = (4, 7, 13, 16), $\tilde{x}$ 22 = (23, 24, 26, 27), $\tilde{x}$ 24 = (21, 23, 27, 29), $\tilde{x}$ 33 = (38, 39, 41, 42)} and the cost-flow pair is ($\tilde{Z}$ 0, $\tilde{F}$ 0) = ((72, 216, 512, 714), (124, 127, 133, 136)).

Now we have $\tilde{u}$ 2 + $\tilde{v}$ 1 < 0 and $\tilde{u}$ 2 + $\tilde{v}$ 3 < 0 where the cells (2, 1) and (2, 3) are not in the basis, so paradox exists. We take $\tilde{l}$ = (1, 1, 1, 1) and we have; for the cell (2, 1) the optimum cost-flow pair is ($\tilde{Z}$, $\tilde{F}$) = ((68, 213, 511, 714), (123, 125, 135, 139)) given in Table 3, for the cell (2, 3) the optimum cost-flow pair is ($\tilde{Z}$, $\tilde{F}$) = ((69, 214, 512, 715), (123, 125, 135, 139)) given in Table 4, The paradoxical cost-flow pair is ($\tilde{Z}$ 1, $\tilde{F}$ 1) = ((68, 213, 511, 714), (125, 128, 134, 137)) given in Table 4.

Some of the fuzzy paradoxical pairs and best fuzzy paradoxical pair obtained by algorithm in section 3, are given in Table 5.

Dest → ↓Orifagin	D_1	D_2	D_3	D_4	$\tilde{a}$ i
O_1	(0, 1, 3, 4)	(5, 6, 8, 9)	(1, 2, 4, 5)	(6, 7, 9, 10)	(38, 39, 41, 42)
O_2	(4, 5, 7, 8)	(0, 1, 1, 2)	(7, 8, 10, 11)	(2, 3, 5, 6)	(48, 49, 51, 52)
O_3	(1, 2, 4, 5)	(6, 7, 9, 10)	(0, 1, 3, 4)	(8, 9, 11, 12)	(38, 39, 41, 42)
	(18, 19, 21, 22)	(23, 24, 26, 27)	(48, 49, 51, 52)	(33, 34, 36, 37)	

Table 1: Demand and cost per unit.

Dest → ↓Origin	D_1	D_2	D_3	D_4	$\tilde{a}$ i $\tilde{u}$ i
O_1	(0, 1, 3, 4) [18,19,21,22]	(5, 6, 8, 9)	(1, 2, 4, 5) [6,8,12,14]	(6, 7, 9, 10) [4, 7, 13, 16]	(38, 39, 41, 42) (0, 0, 0, 0)
O_2	(4, 5, 7, 8)	(0, 1, 1, 2) [23,24,26,27]	(7, 8, 10, 11)	(2, 3, 5, 6) [21,23,27,29]	(48, 49, 51, 52) (−8,−6,−2, 0)
O_3	(1, 2, 4, 5)	(6, 7, 9, 10)	(0, 1, 3, 4) [38,39,41,42]	(8, 9, 11, 12)	(38, 39, 41, 42) (−5,−3, 1, 3)
	(18, 19, 21, 22) (0, 1, 3, 4)	(23, 24, 26, 27) (0, 3, 7, 10)	(48, 49, 51, 52) (1, 2, 4, 5)	(33, 34, 36, 37) (6, 7, 9, 10)	

Table 2: The optimum solution.

Dest → ↓Origin	D_1	D_2	D_3	D_4	$\tilde{a}$ i
O_1	(0, 1, 3, 4) (19, 20, 22, 23)	(5, 6, 8, 9)	(1, 2, 4, 5) (6, 8, 12, 14)	(6, 7, 9, 10) (3, 6, 12, 15)	(38, 39, 41, 42)
O_2	(4, 5, 7, 8)	(0, 1, 1, 2) (23, 24, 26, 27)	(7, 8, 10, 11)	(2, 3, 5, 6) (22, 24, 28, 30)	(48, 49, 51, 52)
O_3	(1, 2, 4, 5)	(6, 7, 9, 10)	(0, 1, 3, 4) (38, 39, 41, 42)	(8, 9, 11, 12)	(38, 39, 41, 42)
	(18, 19, 21, 22)	(23, 24, 26, 27)	(48, 49, 51, 52)	(33, 34, 36, 37)	

Table 3: For the cell (2, 3) the optimum cost-flow pair.

Dest → ↓Origin	D_1	D_2	D_3	D_4	$\tilde{a}$ i
O_1	(0, 1, 3, 4) (18, 19, 21, 22)	(5, 6, 8, 9)	(1, 2, 4, 5) (7, 9, 13, 15)	(6, 7, 9, 10) (3, 6, 12, 15)	(38, 39, 41, 42)
O_2	(4, 5, 7, 8)	(0, 1, 1, 2) (23, 24, 26, 27)	(7, 8, 10, 11)	(2, 3, 5, 6) (22, 24, 28, 30)	(48, 49, 51, 52)
O_3	(1, 2, 4, 5)	(6, 7, 9, 10)	(0, 1, 3, 4) (38, 39, 41, 42)	(8, 9, 11, 12)	(38, 39, 41, 42)
	(18, 19, 21, 22)	(23, 24, 26, 27)	(48, 49, 51, 52)	(33, 34, 36, 37)	

Table 4: The paradoxical cost-flow pair.

$\tilde{I}$	Cost $\tilde{Z}$ i	Mag ($\tilde{Z}$ i)	Flow $\tilde{F}$ i	Mag ($\tilde{Z}$ i)
(0, 0, 0, 0)	(72, 216, 512, 714)	373.67	(124, 127, 133, 136)	130
(1, 0, 0, 0)	(74, 216, 512, 704)	372.33	(125, 127, 133, 136)	130.17
(0, 1, 0, 0)	(72, 220, 503, 714)	372	(124, 128, 133, 136)	130.33
(1, 1, 1, 1)	(68, 213, 511, 714)	371.67	(125, 128, 134, 137)	131
(16, 0, , 0)	(64, 216, 512, 690)	368.3	(140, 127, 133, 136)	132.667
(0, 13, 0, 0)	(72, 181, 536, 714)	370	(124, 140, 133, 136)	134.33
(16, 0, 7, 0)	(64, 167, 568, 690)	370.667	(140, 127, 140, 136)	135
(0, 13, 7, 0)	(72, 209, 473, 714)	358.33	(124, 140, 140, 136)	136.667
(16, 0, 0, 4)	(72, 216, 512, 650)	363	(140, 127, 133, 140)	133.33
(0, 13, 0, 4)	(48, 181, 536, 754)	372.667	(124, 140, 133, 140)	135
……	…….	……..	……..	………….
(16, 13, 7, 4)	(72, 209, 473, 650)	347.67	(140, 140, 140, 140)	140

Table 5: Algorithm in section 3.

Conclusion

We have developed an efficient algorithm for finding paradoxical solution, if paradox exists, in a transportation problem under fuzzy environments. Adlakha and Kowalski demonstrated the practicality of identifying cases where the paradoxical situation exists in crisp environment. Klingman and Russel's approach, Adlakha and Kowalski absolute point procedure provide only best paradoxical pair whereas this method gives step by step development of this solution procedure for finding all paradoxical pairs. 6 Acknowledgements The authors would like to thank the referees for their useful comments which have improved the paper significantly.

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