

Modern Differential Equations: Numerical, Theoretical, Applied

Daniel Cho*

Department of Computational Engineering, Seoul Western University, Seoul, South Korea

Introduction

Ahmad, Ali, and Khan introduced a novel numerical technique. This approach utilizes cubic B-spline functions to solve fractional-order partial differential equations. The method effectively discretizes and approximates solutions, demonstrating high accuracy and efficiency compared to existing techniques, particularly for complex fractional models in various scientific and engineering fields [1].

Alamro, Algehyne, and Abualnaja employed differential equations to model the spread of COVID-19. Their work carefully analyzed the impact of non-pharmaceutical interventions such as social distancing, alongside pharmaceutical interventions like vaccination. The resulting model offers valuable insights into disease dynamics, helping predict the effectiveness of public health strategies to control pandemics [2].

Li, Wu, and Fan developed a robust numerical scheme aimed at solving fractional stochastic differential equations with delay. This method addresses the complexities arising from both fractional derivatives and stochastic terms. It provides accurate and stable approximations, which are crucial for modeling systems with memory and inherent randomness observed in fields like finance and physics [3].

Chen, Li, and Wang investigated optimal control problems. These problems involved nonlinear fractional differential equations. Their work established the existence and uniqueness of solutions for such control systems. They also derived necessary conditions for optimality, thereby expanding the theoretical framework for effectively controlling systems exhibiting fractional dynamics and memory effects [4].

Jin, Cai, and Li explored the application of Physics-Informed Neural Networks (PINNs). These networks were used to solve both forward and inverse problems for partial differential equations. The research highlights how PINNs integrate domain knowledge into deep learning architectures. This offers a powerful approach for high-dimensional and complex Partial Differential Equation (PDE) problems, significantly reducing the need for extensive labeled data [5].

Abdeljawad, Hussain, and Baleanu established the existence and uniqueness of solutions for a specific class of fractional differential equations. This was done within the Caputo-Fabrizio framework. Their theoretical analysis, which employed fixed-point theorems, contributes to the foundational understanding and solvability conditions for these non-local differential operators, clarifying their mathematical behavior [6].

Wang, Zhou, and Wu conducted a detailed stability analysis. Their focus was on reaction-diffusion systems that incorporate time delays and nonlinear source

terms. Using Lyapunov functional methods, they derived conditions for asymptotic stability. This offers critical insights into the long-term behavior of complex spatially extended systems, which are prevalent in fields like physics and biology [7].

Atanacković, Stanković, and Pilipović reviewed and applied fractional differential equations to model complex phenomena. Specifically, they addressed viscoelasticity and wave propagation. Their work highlights how these equations effectively capture material memory and non-local effects. This provides more accurate descriptions than classical integer-order models for various engineering and physics problems [8].

Mohseni, Aminzadeh, and Ghasemi introduced a numerical method based on Bernoulli wavelets. This method is designed for solving nonlinear fractional delay differential equations. The approach efficiently converts the problem into a system of algebraic equations. It demonstrates high accuracy and computational efficiency for complex systems that involve both fractional and delayed dynamics [9].

Liu, Wang, and Wang investigated the existence of solutions for impulsive fractional differential equations. These equations were subject to nonlocal conditions. Utilizing fixed-point theory, the authors established sufficient conditions for solvability. This provides crucial theoretical groundwork for modeling systems that exhibit abrupt changes and memory effects across various scientific disciplines [10].

Description

The realm of differential equations, particularly those of fractional order and those incorporating stochastic elements or delays, poses significant challenges for analytical solutions. Various studies address these complexities through advanced numerical techniques. Ahmad, Ali, and Khan, for example, developed a novel numerical approach leveraging cubic B-spline functions to effectively solve fractional-order partial differential equations. Their method excels in discretizing and approximating solutions, achieving high accuracy and efficiency, especially for complex fractional models in diverse scientific and engineering domains [1]. Expanding on numerical solutions, Li, Wu, and Fan introduced a robust scheme for fractional stochastic differential equations with delay. This method meticulously tackles the intricacies of both fractional derivatives and stochastic terms, yielding accurate and stable approximations vital for systems characterized by memory and inherent randomness, seen in fields like finance and physics [3]. Further advancements in numerical methodologies are observed in the work by Mohseni, Aminzadeh, and

Ghasemi, who presented a Bernoulli wavelets-based method for nonlinear fractional delay differential equations. This efficient approach transforms the problem into a system of algebraic equations, showcasing significant accuracy and computational efficiency for complex systems with both fractional and delayed dynamics [9].

Beyond numerical computation, a fundamental aspect of differential equations involves establishing their theoretical underpinnings, such as the existence and uniqueness of solutions. Chen, Li, and Wang delved into optimal control problems involving nonlinear fractional differential equations. They successfully established the existence and uniqueness of solutions for the control system and derived necessary conditions for optimality, thereby broadening the theoretical framework for controlling systems exhibiting fractional dynamics and memory effects [4]. In a similar vein, Abdeljawad, Hussain, and Baleanu focused on the existence and uniqueness of solutions for a specific class of Caputo-Fabrizio fractional differential equations. Their rigorous theoretical analysis, utilizing fixed-point theorems, contributes significantly to the foundational understanding and solvability conditions pertinent to these non-local differential operators [6]. Further adding to the theoretical groundwork, Liu, Wang, and Wang investigated the existence of solutions for impulsive fractional differential equations under nonlocal conditions. By applying fixed-point theory, they established sufficient conditions for solvability, providing crucial theoretical insights for modeling systems that experience abrupt changes and memory effects across various scientific disciplines [10].

Differential equations are indispensable tools for modeling a wide array of complex phenomena across science and engineering. For example, Alamro, Algehyne, and Abualnaja employed differential equations to model the transmission dynamics of COVID-19. Their analysis focused on assessing the impact of both non-pharmaceutical interventions like social distancing and pharmaceutical interventions such as vaccination. The model yields valuable insights into disease dynamics and aids in predicting the effectiveness of public health strategies [2]. In another practical application, Atanacković, Stanković, and Pilipović presented a comprehensive review and application of fractional differential equations. Their work focused on modeling complex phenomena in viscoelasticity and wave propagation, emphasizing how these equations effectively capture material memory and non-local effects, offering more accurate descriptions than classical integer-order models for various engineering and physics challenges [8].

The pursuit of advanced methodologies for solving and analyzing differential equations is ongoing. Jin, Cai, and Li explored the innovative application of Physics-Informed Neural Networks (PINNs) to solve both forward and inverse problems for partial differential equations. This research underscores how PINNs integrate domain knowledge into deep learning architectures, presenting a powerful approach for high-dimensional and complex Partial Differential Equation (PDE) problems without the need for extensive labeled data [5]. Meanwhile, understanding the long-term behavior of complex systems is crucial. Wang, Zhou, and Wu conducted a detailed stability analysis for reaction-diffusion systems. These systems incorporated time delays and nonlinear source terms. Through the application of Lyapunov functional methods, they successfully derived conditions for asymptotic stability, providing vital insights into the long-term behavior of complex spatially extended systems commonly found in physics and biology [7].

Conclusion

This collection of research highlights significant advancements in the field of differential equations, with a strong emphasis on fractional-order and delayed systems. There's a clear focus on developing and applying sophisticated numerical techniques to tackle challenging problems. Methods such as cubic B-spline functions [1], robust schemes for fractional stochastic differential equations [3], and Bernoulli

wavelets [9] demonstrate efforts to achieve higher accuracy and efficiency in approximating solutions. These numerical advancements are crucial for modeling complex systems that exhibit memory and randomness across various scientific and engineering domains, including finance and physics.

Beyond computation, theoretical investigations into differential equations remain vital. Studies establish the existence and uniqueness of solutions for nonlinear fractional differential equations [4] and those within the Caputo-Fabrizio framework [6]. Additionally, the solvability conditions for impulsive fractional differential equations with nonlocal conditions are explored [10], laying essential groundwork for understanding systems with abrupt changes.

The practical applications of differential equations are also extensively covered. Models are developed to understand real-world phenomena, such as the spread of COVID-19 and the impact of public health interventions [2]. Fractional differential equations are also applied to describe viscoelasticity and wave propagation, capturing material memory more accurately than traditional models [8]. Innovative approaches like Physics-Informed Neural Networks (PINNs) are presented for solving Partial Differential Equation (PDE) problems [5], and detailed stability analyses are conducted for reaction-diffusion systems with delays [7]. Collectively, this research underscores the diverse theoretical and applied landscape of differential equations, pushing the boundaries of mathematical modeling and problem-solving.

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Conflict of Interest

None.

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***Address for Correspondence:** Daniel, Cho, Department of Computational Engineering, Seoul Western University, Seoul, South Korea, E-mail: d.cho@sce.swu.kr

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