International Journal of Economics & Management Sciences

Rana, Int J Econ Manag Sci 2018, 7:2
DOI: 10.4172/2162-6359.1000515

Research Article Open Access

Modelling Value-at-Risk in Investment Banks: "Empirical Evidence of JP Morgan, Merrill Lynch and Bank of America"

Rajeev Rana*

APB Govt. PG College, Augustmuni, Rudparyag, Uttarakhand, India

Abstract

The objective of paper is to assess the efficiency of financial model to capture increasing volatilities across asset class markets of the three investment banks. For which data will be collect to forecast the credit risk, and to know how well our standard tools forecast volatility, particularly during the turmoil that extend throughout the globe. Volatility prediction is a critical task in asset valuation and risk management for investors and financial intermediaries. The paper will focus on Value-at-Risk (VaR) which is a standard model that has been forecasted using both non-parametric and parametric approaches and then Backtesting procedure had been applied to achieve the both outcome. One is to detect the underlying credit risk which is associated with the market as well as portfolio risk, and other is to perceive model which provide more accurate forecasting.

Keywords: VaR; Credit risk; Garch model; Filtered historical simulation; Volatility

JEL Classification: B26; G33; G21

Introduction

Econometric modelling for Value-at-Risk

Value at Risk (VaR) provide the maximum losses not exceeding a given probability defined as the confidence level, over a given period of time. VaR methodology forecasting risk is applied by investment banks to measure the market risk of their asset portfolios (market value at risk), however VaR has a wide spectrum for quantitative risk management for many types of risks.

Here we present the various parametric and non-parametric models including various econometric models that are used for forecasting VaR mostly focuses on market side risk and it is one of the best approaches to capture the market risk through volatility. The VaR entails the estimation of quintile of the distribution returns. VaR is usually computed separately for the left and right tails of the returns distribution of the risk managers. The VaR of a long position (left tail of the distribution function) over a given time horizon t and probability p, while p is one minus the VaR confidence level, is defined as:

$$VaR(x)=F^{-1}(p) \quad (1)$$

In the above equation F is the cumulative distribution function that describes the profit and loss distribution (P&L) of the risky financial position and F-1 denotes its inverse function. In order to estimate VaR for our research we will focus on some parametric and non-parametric models and then compare them to predict the best outcomes or most accurate models to estimate VaR.

GARCH model

According to Engle and Bollerslev the VaR is based upon the assumption that the standard deviation in returns does not change over time (homoskedasticity), Engle argues that we get much better estimates by using models that explicitly allow the standard deviation to change of time. Suggesting two variants – Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) – that provide better forecasts of variance and, by extension, better measures of Value at Risk.

Engle, Ng provide the ability to forecast the volatility on the basis of conditional mean and variance in the financial market with the proper selection of the financial assets to structure an investment portfolio. Merton shows that the expected returns on the market are related to the accurate volatility forecast. There is a relationship between risk and return when dealing the fair valuation of assets, Glosten, Jagannathan and Runkle pointed out in certain period of time; there is a higher yield from riskier asset, depended on particular strategy.

Nelson proposed the extended version of GARCH which unlike the ARCH and GARCH allows for the symmetry in the responsiveness to shocks, does not impose the nonnegative constraints on parameters and reduces the effect of outliers on the estimation results. EGARCH has been commonly used to examine interest rate, inflation rate of future markets, exchange rate and in the analysis of stock returns. The GARCH model that has been described is typically called the GARCH (p,q) model. The (1,1) in parentheses is a standard notation in which the first number refers to how many autoregressive lags, or ARCH terms, appear in the equation, while the second number refers to how many moving average lags are specified, which here is often called the number of GARCH terms. The GARCH models explicitly model the conditional volatility as a function of past conditional volatilities returns.

Data Methodology

Computing VaR forecast to calculate expected shortfall or expected value of the loss given that an event outside a given probability level has occurred out-of-sample for the period of 2001-to-2011 for banks which survived the crises, and forecast VaR for the period of 2006-to-2008 banks who were failed during the sub-prime from the data of daily historical price. The historical return series data have been taken for three banks (i.e. JP Morgan Chase & Co., Merril Lynch, and Bank of America) as per the methodology and these three banks have identical

***Corresponding author:** Rajeev Rana, APB Govt. PG College, Augustmuni, Rudparyag, Uttarakhand, India, Tel: 96532-34800; E-mail: rajeevjeet@gmail.com

Received March 18, 2018; **Accepted** March 30, 2018; **Published** April 06, 2018

Citation: Rana R (2018) Modelling Value-at-Risk in Investment Banks: "Empirical Evidence of JP Morgan, Merrill Lynch and Bank of America". Int J Econ Manag Sci 7: 515. doi: 10.4172/2162-6359.1000515

operations but different attributes. The JPM is one of the largest investment banks and does not offer general banking services but have been more stable during pre and post crises period, while ML Banks who also offered identical services but collapsed during the 2007-08 turmoil and lastly BofA, offering investment banking services as well general banking (i.e. accepting deposits and loans of customer's) also survived in the crises. To study of forecasted value-at-risk predicted volatility and capital requirement to overcome from the expected shortfall and number of Hit's when bank's losses exceeded then their daily return. We have taken bank's daily return data of historical price from the period of 1991-2011 except ML data has been analysed for the limited period of 2001-2008 due to non-availability of historical price data.

Further, the VaR has been computed using both non-parametric and parametric approaches. The non-parametric includes Historical Simulation (HS) approach while parametric approach includes GARCH, Exponential-GARCH (E-GARCH), and Threshold ARCH (TARCH) and another standard approach which combines both non-parametric and GARCH model well known as Filtered Historical Simulation (FHS).

The important assumptions for applying VaR is confidence level α on given time horizon at losses with amount that will exceed with a probability of 1-α on given time horizon. In our research the VaR computed on both 95% and 99% confidence level, α and 1-α represent significance level at 0.05% or 0.01% with the assumption that return is normally distributed with mean zero and standard deviation one.

So, VaR formula with the confidence level α and 1-α represent significance level:

$VaR_{\alpha}=\inf\{L:Prob[Loss>L]\leq 1-\alpha\}$.

Where: L is the lower threshold of loss with the probability of losing more than L on a particular time horizon is 1-α.

Descriptive Analysis

The return series has been generated from the daily stock price data from Bloomberg, to convert daily data into return series the normal log has been computed with the standard formula i.e. $\ln P_t/P_{t-1}$ taking from natural logarithm current information divided with previous information. Table 1 shows the descriptive statistics.

As the descriptive statistics shows that a mean of BOA: 0.00077, JPM: 0.000101 and ML: 0.000108. hence mean value is non-zero so we reject the our Null Hypothesis rejecting assumption of mean equal to zero, with median, and std. deviation of all three banks with the Jarque-Bear statistic shows that the null hypothesis of normality is rejected at any level of significance, as evidence by high excess kurtosis and negative skewness due to conditional volatility. The unconditional distributions are non-normal and have a long left tail relative to a symmetric distribution cause of higher volatility near future due to negative shock.

Further, another table display the descriptive results compute GARCH/E-GARCH and T-GARCH with the help of the statistical package in eview's the results have been shown in the Table 2.

From the Table 2, we could conclude that the volatility term in the mean equation is statistically significant as indicating that rather than being constant the mean return is dependent on the level of volatility.

The GARCH (p,q)-model has been computed using one lag i.e. p lags of the squared error. To capture the autocorrelation present in residual and diagnostic with Correlogram: Q-Statistics conduct in eview's up to twenty lags for the autocorrelation capture. For checking hetreoscadisticity the modified white test has been conducted. White test statistic is computed as the number of observations times of r-squared. It is asymptotically distributed with degrees of freedom. Table 3 shown below of F-statistics shows that almost it is non-significant as the F-test value is almost non-significant so our alternate hypothesis rejected that assumes presence of hetroscadisticy.

	Bank of America	JP Morgan	Merrill Lynch
Mean	-0.00077	-0.000101	-0.000108
Median	0	-0.00023	0.304661
Maximum	0.302096	0.223917	0.304661
Minimum	-0.691688	-0.232278	-0.486431
Std. Dev.	0.037104	0.029075	0.030468
Skewness	-2.586215	0.29134	-0.703725
Kurtosis	63.20616	14.53012	40.28772
Jarque-Bera	420839.6	15360.89	135580.6
Probability	0	0	0
Sum	-2.129216	-0.280135	-0.252996
Sum Sq. Dev.	3.806696	2.337411	2.168547
Observations	2766	2766	2337

Table 1: Descriptive Statistics of Banks.

		RETURN			
	Variable	Coefficient	Std. Error	t-Statistic	Prob.
BOA GARCH	C	-0.001	0.000	-1.657	0.098
	RETURN(-1)	0.014	0.02	0.697	0.486
JPM GARCH	C	0.000	0.000	1.137	0.256
	RETURN(-1)	-0.025	0.016	-1.533	0.125
ML GARCH	C	0.001		2.164	0.03
	MERRIL_RETURN(-1)	0.029	0.022	1.326	0.185
BOA T-GARCH	C	-0.001	0.000	-1.056	0.291
	RETURN(-1)	0.012	0.02	0.622	0.534
JPM T-GARCH	C	0.001	0.000	2.989	0.003
	RETURN(-1)	0.002	0.017	0.109	0.914
ML T-GARCH	C	0.000	0.000	-0.198	0.843
	MERRIL_RETURN(-1)	0.015	0.02	0.721	0.471
BOA E-GARCH	C	-0.003	0.000	-8.436	0.000
	RETURN(-1)	-0.049	0.012	-4.083	0.000
JPM E-GARCH	C	0.000	0.000	-0.024	0.981
	RETURN(-1)	-0.01	0.016	-0.61	0.542
ML E-GARCH	C	0.001	0.000	1.845	0.065
	MERRIL_RETURN(-1)	0.046	0.015	3.008	0.003

Table 2: Statistical package in eview's results.

		F-statistic	Obs*R squared
BOA	GARCH	0.041951	0.083948
JPM	GARCH	0.237014	0.474254
ML	GARCH	0.262013	0.524583
BOA	T-GARCH	0.053126	0.10631
JPM	T-GARCH	0.107497	0.215108
ML	T-GARCH	0.047166	0.09445
BOA	E-GARCH	0.178872	0.357923
JPM	E-GARCH	0.223761	0.447738
ML	E-GARCH	0.039138	0.078373

Table 3: Heteroskedasticity White Test.

Empirical Analysis

Historical simulation (HS)

The first and the most commonly used method is referred to calculating value-at-risk were called historical simulation or historical VaR. A contemporaneous description of historical simulation is provided by Linsmeier and Pearson [1]. The main idea behind the HS is assumption that a historical distribution of returns will remain the same over the next periods (i.e. assumption of that price changes behaviour repeats itself over time). As a result, the VaR based on HS is simply the empirical quintile of the distribution associated with the desired likelihood level.

$$VaR_{t+1,\ p} = Quantile\{[X]_{t=1}^{n}\} \quad (2)$$

Where quantile have been calculated with confidence at 95% (i.e. at the significant of 5%) to furcate for the given period from the daily return data.

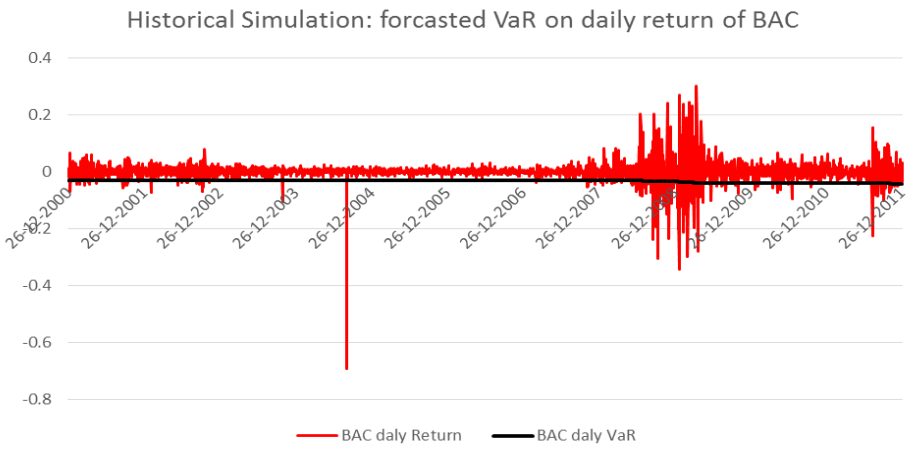
The Figures 1-3 represents, the computed Historical Simulation of forecasted value-at-risk (VaR) for the Bank of America, JP Morgan Chase & Co., and Merrill Lynch

Filtered historical simulation

To overcome with the shortcoming of the non-parametric approach of historical simulation the another approach introduce by Hull, White and Barone-Adesi et al. combines with the Historical simulation and the GARCH model known as Filtered Historical Simulation (FHS). The importance of the model is that not making any distributional assumption for the standardized return during forecasting variance in the volatility model. The FHS model is assumed to be more superior then historical simulation. FHS combine the GARCH-model for return series and use Historical Simulation to infer the distribution of the residuals through applying the quantiles of the standardized residuals (i.e. residual divide by conditional standard deviation) and the conditional mean forecasts from a volatility model.

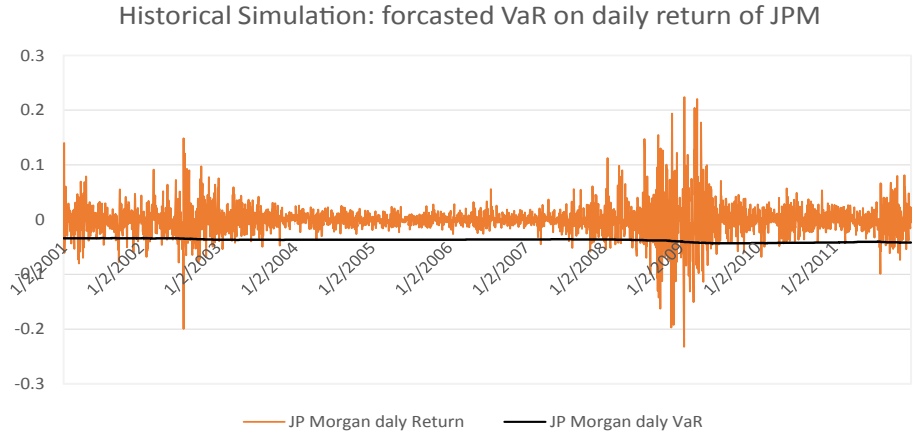
$$VaR_{t+1,\ p} = \mu_{t+1} + \sigma_{t+1} Quantile\{[X]_{t=1}^{n}\} \quad (3)$$

Where, μ_{t+1} conditional mean, and σ_{t+1} is conditional standard deviation.



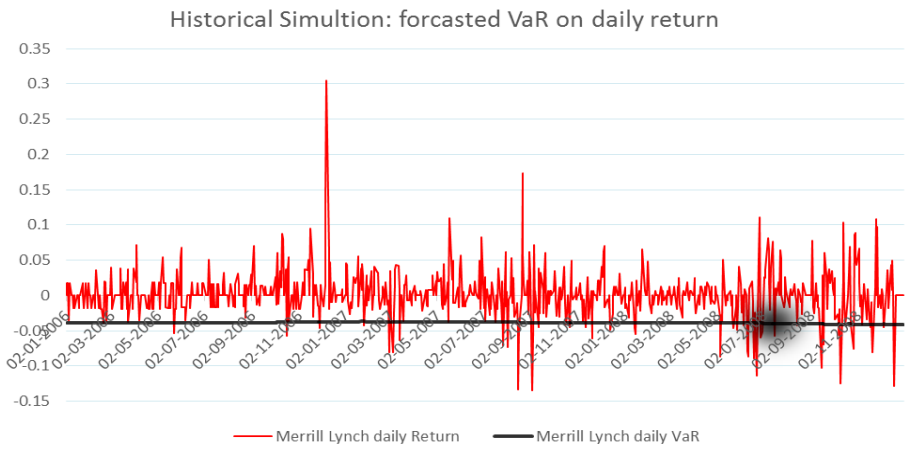
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 1: Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America.



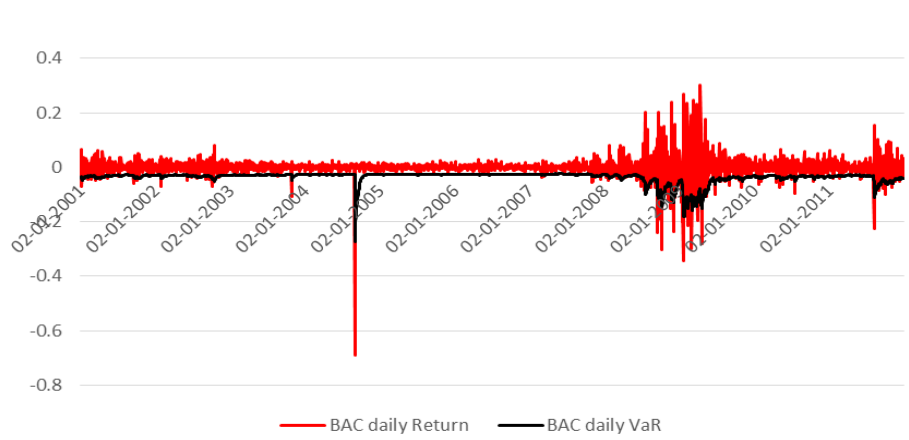
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 2: Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of JP Morgan Chase & Co.



Source: Bloomberg daily data have been taken for the period of 2001-2008 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2006-to-2008.

Figure 3: Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2006 to 2008 from the daily Historical Price of Merrill Lynch Bank.



Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 4: Filtered Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America.

The Figures 4-6 representing the computed FHS, calculated with confidence at 95% (i.e. at the significant of 5%) to furcate for the given period from the daily return data of forecasted value-at-risk (VaR) for the Bank of America, JP Morgan Chase & Co., and Merrill Lynch.

GARCH: GARCH extends ARCH process to include past squared return and past variance in the model. So, in the Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-model the conditional variance is defined as a linear function of lagged conditional variance, and squared past returns with the assumption of *iid* and zero-mean.

$$X_t=\mu_t+Z_t\sigma_t \quad (4)$$

Where, μ_t conditional mean, and σ_t is conditional variance process.

The model has following assumption:

$Z_t \sim iidf(E(Z_t))=0.V(Z_t)=1$

$_t = (\Omega_t)\cdot$

Where, $\sigma_t^2 = E(X_t I\Omega_{t-1}) = \mu_t$ conditional mean with given the information set of available at time t-1, $\Omega_{t-1}, \{\varepsilon\}_t^n = 0$ is the innovation process with conditional variance $V(X_t I\Omega_{t-1}) = \sigma_t^2$, f (.) is the density function of $\{Z\}_t^n = 0$ and g is the functional form of conditional volatility. To computing VaR in GARCH model assumed normal distribution with the following equation of VaR.

$VaR_{t+1,p}=\mu_{t+1}=\mu_t+Z_t\sigma_{t+1}$

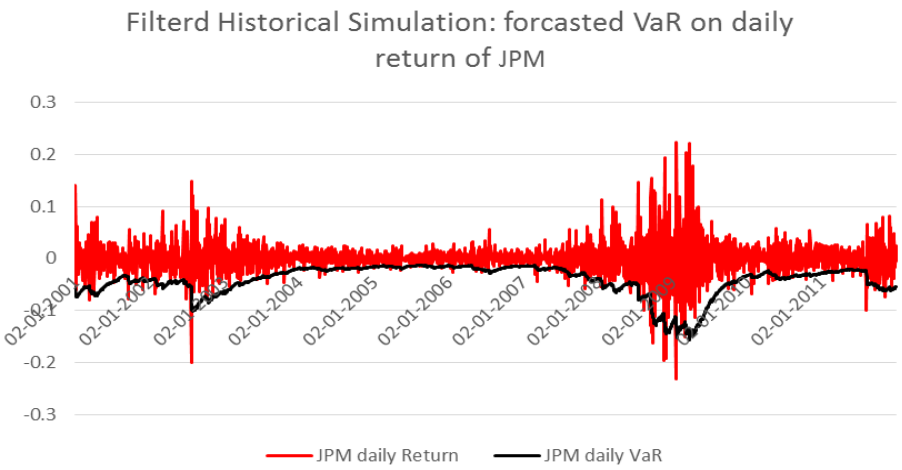
Where, μ_{t+1} and σ_{t+1} are the conditional forecasts of the mean and the standard deviation at time t+1, given the information at time t.

In the AR (1)-GARCH (1,1) model for value of p and q result of a specification search in terms of AIC an BIC criteria.

$\mu_t=\alpha_0+\alpha_1 X_{t-1}$

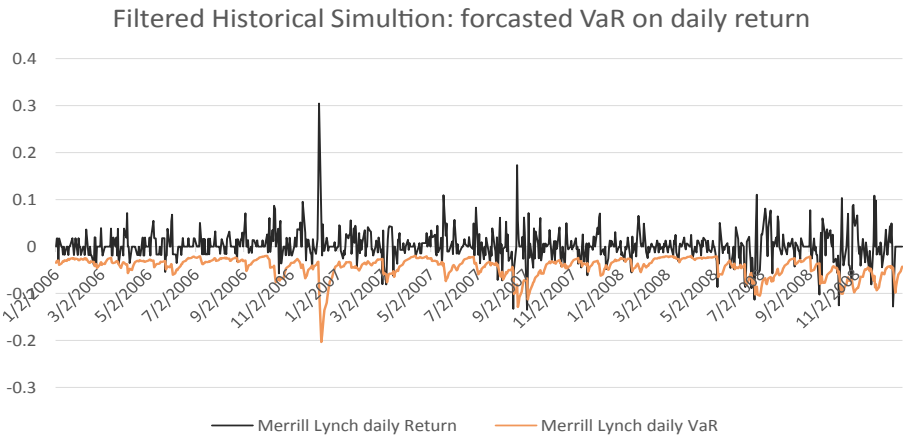
$\sigma_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2$

This model is fitted to data series using a pseudo maximum



Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 5: Filtered Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America and JP Morgan Chase & Co.



Source: Bloomberg daily data have been taken for the period of 2001-2008 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2006-to-2008.

Figure 6: Filtered Historical Simulation: Value-at-risk (VaR) forecasted for the period of 2006 to 2008 from the daily Historical Price of Merrill Lynch Bank.

likelihood estimation assuming normal distributed innovation to estimate parameters θ and standardized residuals $\left(\frac{X_t-\mu_t}{\sigma_t}\right)$.

The Figures 7-9 represents the computed GARCH, calculated with confidence at 95% (i.e. at the significant of 5%) to furcate for the given period from the daily return data for forecasted value-at-risk (VaR) for the Bank of America, JP Morgan Chase & Co., and Merrill Lynch.

T-GARCH: The appropriate method of measuring is Threshold GARCH proposed by Zakoian. The threshold GARCH model in the contest of conditionally heteroscedastic time series have been found appropriate to analysing asymmetric volatilities. While the effect of current volatility on the future volatility decreases to zero at an exponential rate for standard-threshold-GARCH (TGARCH) processes, here we introduce a class of T-GARCH as shown in Figures 10-12 processes exhibiting persistent (in volatilities) properties when current volatility constantly remains for long time in future volatilities for all-step ahead forecasts Nelson [2-5].

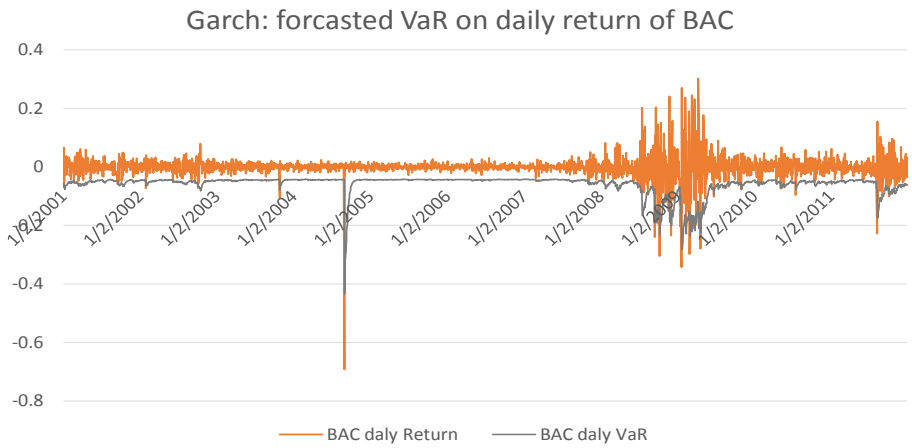
A general class of Threshold-GARCH (T GARCH) model can be derived as:

$$\varepsilon_t = \sqrt{h_t e_t} \tag{5}$$

Where, h_t volatility depends on whether the past squared value that is positive or negative asymmetric GARCH via 'threshold'. e_t is a sequence of iid random variable with mean zero and variance unity.

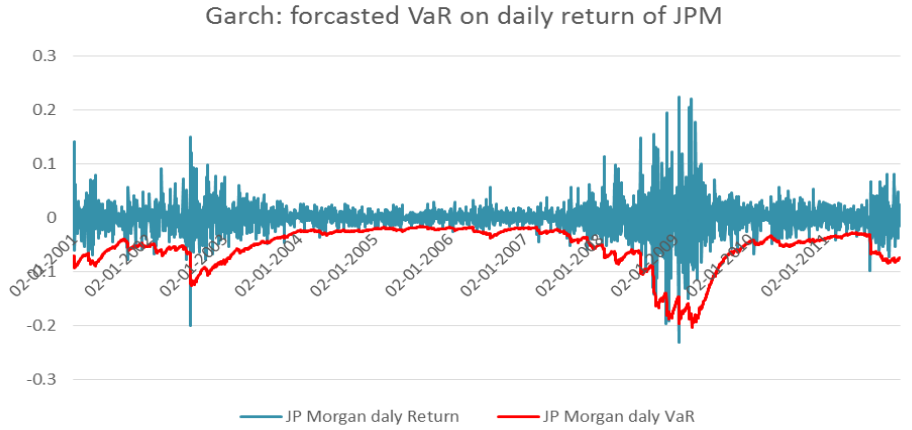
The time series $\{\varepsilon_t\}$ governed by (1,1) will be referred to as T-GARCH (p,q).

Figure 6 representing the computed GARCH, calculated with confidence at 95% (i.e. at the significant of 5%) to furcate for the given period from the daily return data for forecasted value-at-risk (VaR) for the Bank of America, JP Morgan Chase & Co., and Merrill Lynch.



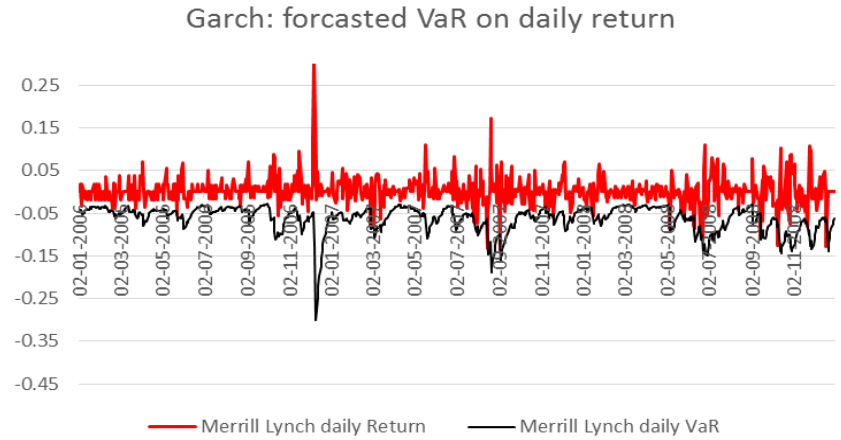
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 7: GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America.



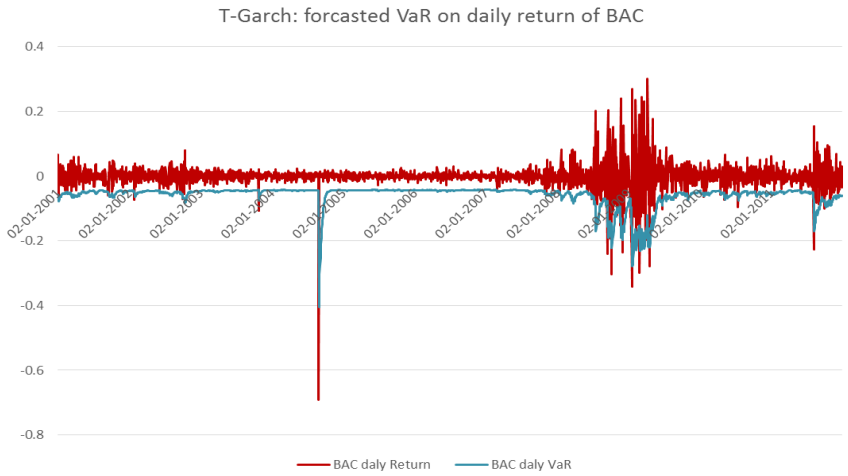
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 8: GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America and JP Morgan Chase & Co.



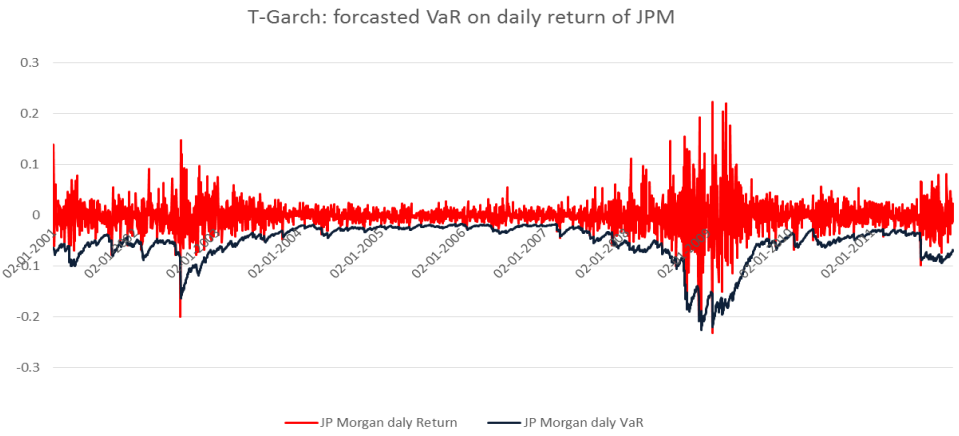
Source: Bloomberg daily data have been taken for the period of 2001-2008 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2006-to-2008.

Figure 9: GARCH: Value-at-risk (VaR) forecasted for the period of 2006 to 2008 from the daily Historical Price of Merrill Lynch Bank.



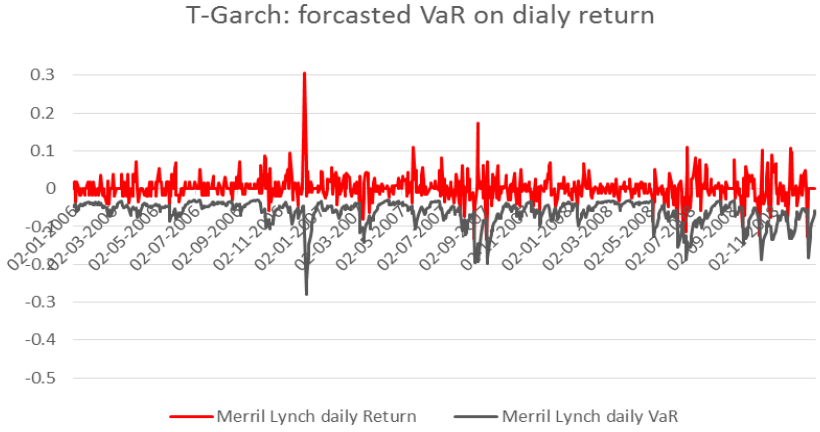
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 10: T-GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America.



Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 11: T-GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of JP Morgan Chase & Co.



Source: Bloomberg daily data have been taken for the period of 2001-2008 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2006-to-2008.

Figure 12: T-GARCH: Value-at-risk (VaR) forecasted for the period of 2006 to 2008 from the daily Historical Price of Merrill Lynch Bank.

E-GARCH: Exponential GARCH-model considers the best model and basically computed logarithm of conditional variance and address that a negative return affect more than positive. As it variance calculates in logarithm and predicted more accurate as squared of value become positive. In the GARCH model, it's assumed that only the magnitude of unanticipated excess returns determines σ_t^2. Otherwise it would be argued that not only the magnitude but also the direction of the returns affects volatility i.e. negative shocks (event/news) tend to impact volatility more than positive shocks. The limitation is that the how does a shock linger in the volatility estimate [6-10]. This two limitations are the main factor for developing E-GARCH model as shown in Figures 13-15.

$$g(Z_t) = \theta Z_t + \lambda([Z_t] - E[Z_t])) \quad (6)$$

and $g(Z_t) = \theta Z_t + \lambda([Z_t] - E[Z_t]))$

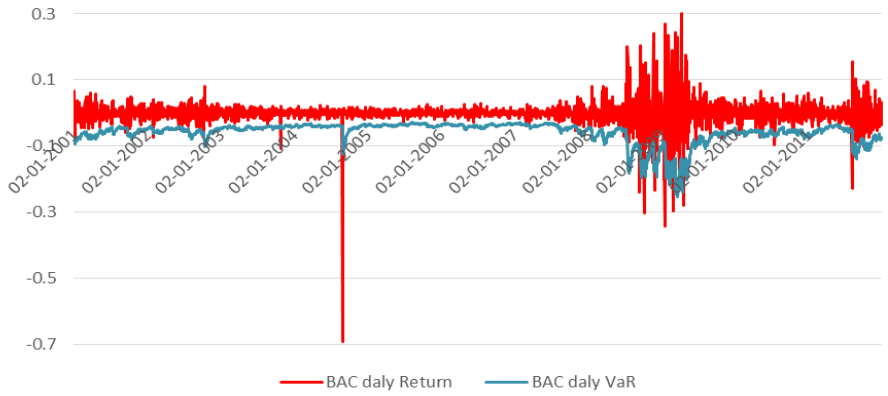
where ω, β, α, θ, and λ are coefficients, and Z_t comes from a generalised error distribution.

Using E-GARCH we can drive better estimate for the volatility for assets return then classic GARCH model.

Representing the computed GARCH, calculated with confidence at 95% (i.e. at the significant of 5%) to furcate for the given period from the daily return data for forecasted value-at-risk (VaR) for the Bank of America, JP Morgan Chase & Co., and Merrill Lynch.

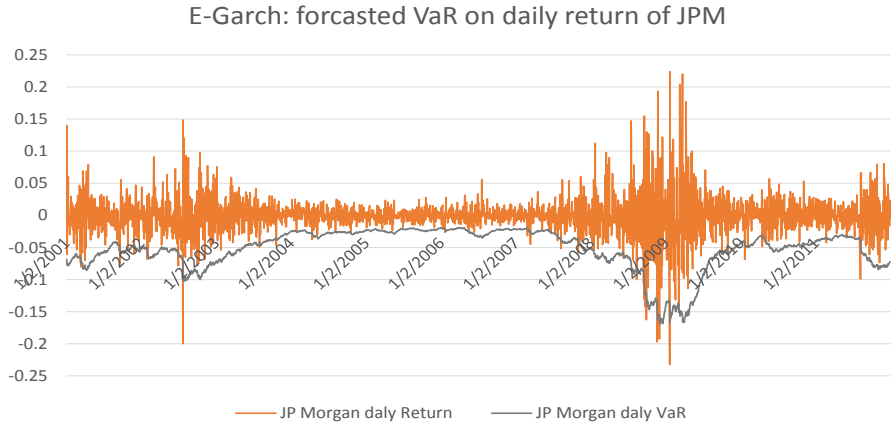
Summary - Violation of VaR: Form the computed Value-at-risk now we have to calculate the total number of Hit's made during the forecasted periods which shows the probability of losses exceeded then predicted as shown the table below. This has been calculated from 2001-to-2011for BOA & JPM, while 2006-to-2008 for ML Bank's daily return time-series data for the 2767 days VaR forecasted and total number of violation has been calculated at those points where daily return exceeded from VaR forecasted.

The high volatility is captured by VaR during the period of global turmoil which proved that bad event has more weight in volatility then good news, all model capture huge volatility in the crises period.



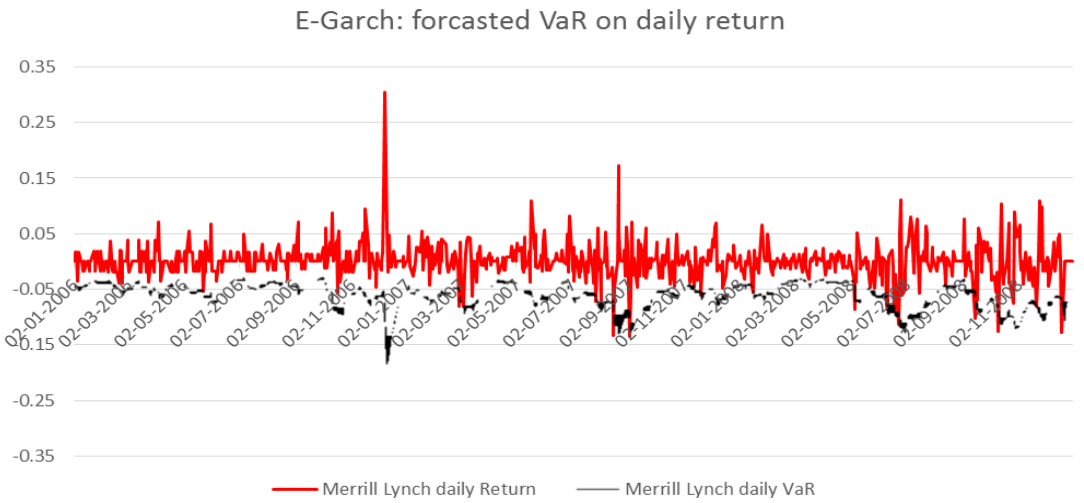
Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 13: E-GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of Bank of America.



Source: Bloomberg daily data have been taken for the period of 1991-2011 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2001-to-2011.

Figure 14: E-GARCH: Value-at-risk (VaR) forecasted for the period of 2001 to 2011 from the daily Historical Price of JP Morgan Chase & Co.



Source: Bloomberg daily data have been taken for the period of 2001-2008 for the purpose of VaR forecast to calculated expected shortfall or expected value of the loss given that an event outside a given probability level has occurred for the period of 2006-to-2008.

Figure 15: E-GARCH: Value-at-risk (VaR) forecasted for the period of 2006 to 2008 from the daily Historical Price of Merrill Lynch Bank.

However, the VaR of Bank of America is seems to be more stable as the gaph's shows less volatility captured during the defined period.

The violation occurs when a realized return is greater than the predicted VaR. the violation ration is defined as the total number of violation, divided by the total number of one-period forecast. If the violation ration at the p^{th} -quantile is greater than α percent, then it's implies excessive underestimation of the realized return. If the violation ration remains less than α percent at the p^{th}-quantile, there is excessive overestimation of the realized return by the underlying model. For example, if the violation ration is 4% at the 95th quantile, the realized return is only 4% of the time greater than what the model predicts. Sometimes it argued by the experts or researcher that if violation is too low i.e. not-significant that mean bank's maintain enough capital to cover their expected losses but missed out the profit opportunities, as there is trade-off between risk and return, but it's important that banks should maintain ample capital bad event.

The Tables 1 and 2 shows violation at 95% and 99% confidence level at long tail (buy side) with the number of violation and percentage of violation under different model. We will check the efficiency and relevance of the model when comparing predicted VaR with actual returns and further back-testing.

Statistical parametric backtesting: The purpose of statistical backtesting methodology is adopted by banks for their internal risk measurement comparison of daily profits and losses with model-generated risk measures to assess the quality of and accuracy for their risk measurement systems. This whole process is called backtesting, and known a technique for evaluating banks risk measurement with a selection of more accurate model. At present bank select different model for the purpose of backtesting with an aim of selecting more accurate and inaccurate models.

The out-of-sample problem has been reviewed by many expert; i.e. Kupiec, Berkowitz and O'Brien, Engle and Manganelli, to name but a few. These references have assumed the correct specification of the VaR model in forecast evaluations. Therefore, prior to the forecasting stage, the risk manager has to decide, using the available information, i.e. all the sample, which econometric model is most adequate for the conditional VaR process. This preliminary stage involves model selection and validation, hence the importance of quantile specification tests associated to VaR.

The backtesting framework[1]: The underlying ideas of applying backtesting are basically adopting strategy for internal risk measurement through best models. Originally the backtesting consist a periodic comparison of the bank's daily value-at-risk measures with the daily return series. Initially to measure the market base and operation risk side of credit risk of the portfolio of the banks. The comparison of risk measure with the daily return series and marked the number of times when risk measure were larger than the daily return series of the bank called Hit's.

Then this outcome of hit's actually compared with the assumed level of coverage to assess the performance of bank's risk model. For comparison of those hits a large number of statistical tests could be applied. As the VaR framework provide the risk assessment for an estimate of the amount that could be lost in a day due to change in market movement over a given holding period with a pre-determined confidence level. The standard procedure of applying Backtesting is to compare observed percentage of outcomes of covered risk with pre-determine i.e. 95% or 99% level of confidence. This implies that banks 95th or 99th percentile risk measures truly covered 95% or 99% of the bank's actual outcome.

The another approach of applying backtesting with specifying the appropriate risk measures and actual outcomes arises due to sensitivity of a static portfolio with adverse volatility in price, or price shocks. In the VaR and its backtesting procedure end-of-day return series are consider as an input for the risk measurement model. And this series to be applied for the possible change in the value of static portfolio due to high volatility in the price of return series during the holding period.

The Basel framework of backtesting had suggested the use of risk measures calibrated to a one-day holding period. As it is appropriate to employ one day actual outcome for the purpose of benchmark in backtesting. While other suggested that the actual trading outcomes

[1]The backtesting framework suggested by Basel Committee, to implement banks for their risk measurement control.

experience by the banks to be consider most important also, relevant for risk management purpose, which should be benchmarked against the reality.

To be more precisely the backtesting procedure is viewed purely a statistical test of the quality of the value-at-risk measures. As suggested that it should employ daily results that allows for an "uncontaminated" test. Backtesting using actual outcomes of profit and losses become also important as it can uncover cases where the risk measurement technique are accurately capturing daily volatility in spite of being measuring with integrity.

The framework adopted by Basel Committee for backtesting purpose is most straightforward procedure for comparing the risk measures with the daily outcomes. This is a simple method of calculating the number of hit's (i.e. exceptions) that is and not covered by daily out come by the risk measures.

Statistical Backtesting methods: An accurate VaR model must qualify unconditional coverage test. In a good VaR model the number of violation should be as per the selected confidence level. Which always may not be true? In mathematical terms, VaR to be considered for a portfolio's value-at-risk is defined to be α quantile of the portfolio's profit and loss distribution:

$$VaR_t(\alpha) = -F^{-1}(\alpha|\Omega_t)$$

Where, F-1($\alpha|\Omega_t$) to be consider the quantile function of the profit and loss distribution which varies over time with a market conditions and the portfolio's composition, as embodied in Ω_t change. This can be drawn with the help of example as if 5% VaR, i.e. VaR at (0.05) is Rs 100,000 then it could be said that at 5% time we should expect to observe a loss on this portfolio in excess of Rs 100,00. Since bank's adhere to risk-based capital requirements by using their own internal risk models to determine their 5% or 1% value-at-risk i.e. VaR at (0.05) or (0.01). Therefore, it is important to have a means of examining whether or not reported VaR represents an accurate measure of a bank's actual level of risk.

Backtesting implementation process – Methodology: The following is the standard procedure conducting backtesting is shown in Figure 16:

The exception of hit's (violations): The Table 1 summarize the violation (i.e. exceptions) are occurred at the 95th and 99th percent confidence level under different model's applied for predicting VaR on daily basis for the data from 2001-to-2011 for the BOFA and JP Morgan Chase while Merrill Lynch the VaR has been forecasted for the period of 2006-to-2008 due to limited data availability [11-15]. The VaR predicted mostly to considering the period of highly vulnerable for banks and most of banks failed during the period of crises which includes Merrill Lynch, which later on merged with Bank of America. So, the study of ML data includes pre-merger data i.e. till 2008.

As per the Table 4 the violation defines the relative performance of each model is calculated in terms of violation ratio. The violation for each banks under different model have been calculated with the percentage of violation occurred (the percentage of violation have been as calculated as per the period in which total points of portfolio loss and divided by the number of days VaR estimated) [16-19]. These empirical results are prepared for the purpose of backtesting which actually explain the best model for bank's for more accurate forecasting.

In the Table 5, Root mean squared error and mean absolute error have been computed through Conditional forecasting and not a high realized return frequency mode which is consider true. Eview's forecasting of VaR is based on r-squared conditional frequency with an assumption that is true. The selection of model is based on least error in the term of root mean squared error and mean absolute error, as the table suggested for BoA, the T-GARCH have least error in both case to be consider as more accurate. In the same method for the JP Morgan GARCH to be consider good model and for ML, T-GARCH to be consider appropriate model but subject to backtesting procedure as merely a least error can't predict the validity of accurate model.

Test of unconditional coverage: The unconditional coverage test is suggested one of the good, and suitable method for the purpose of backtesting of a value-at-risk (VaR) model to record the failure, which gives us the numbers that VaR is exceeded in a given observation.

Unconditional coverage refers to the fact that the fraction of overshooting's (ex post loss exceeds ex ante forecasted VaR) observed should be as per determined confidence level of the VaR. Failure of unconditional coverage means that the calculated VaR does not measure the risk accurately, while passing the unconditional coverage test mean model is more accurate to capture the underlying risk.

Kupiec unconditional coverage test: A likelihood ration test proposed by kupiec in 1995. To examine that the failure rate is

Figure 16: SOP for backtesting.

Models	HS		FHS		GARCH		E-GARCH		T-GARCH	
	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
Merrill Lynch Bank	86	21	47	10	18	8	18	8	18	9
Percentage of violation	7.101569	1.734104	6.01023	1.278772	2.304738	1.024328	2.048656	1.023018		
JP Morgan and Chase	171	53	108	21	45	17	42	18	44	14
Percentage of violation	6.179978	1.915432	3.903144	0.758995	1.62631	0.614384	1.517889	0.650524	1.59017	0.505963
Bank of America	184	56	145	31	39	19	41	21	39	19
Percentage of violation	6.649801	2.022391	5.240332	1.120347	1.409469	0.686664	1.481749	0.758945	1.40947	0.686664

Table 4: Summary of Hit's.VaR value-at-risk calculated at the 95% and 99% confidence level.

statistically equal to the expected one. Let **α** be the expected failure rate i.e. (**α**=1-p), where p is the confidence level for the VaR. Once the total number of such trials denoted by T, then the number of failure denoted with N can be modelled with a binomial distribution with the probability of occurrence equal to **α**.

In such a case the correct Null and Alternative Hypothesis could be drawn as:

$$H_0 : \frac{N}{T} = \boldsymbol{\alpha}, \textbf{and} H_1 : \frac{N}{T} \neq \boldsymbol{\alpha},$$

The appropriate likelihood ratio statistic is:

Let $N = \sum_{t=1}^{T} l_{t+1}$ is define the number of days over a T period in which portfolio loss was larger than the VaR forecasted, l_t+1 simply the sequence of VaR violation with the following assumption:

$$\text{Left tail: } l_{t+1} = \begin{cases} 1, & if\ X_{t+1} \langle VaR_{t+1} | t \\ 0, & if\ X_{t+1} \geq VaR_{t+1} | t \end{cases}$$

The appropriate likelihood ratio statistic is given below:

$$LR_{UC} = 2\left[\log((N/T)^N (1-N/T)^{T-N}) - \log(\alpha^N (1-\alpha)^{T-N}\right]$$

Where, $LR_{UC \to d} \varkappa^2(1)$ under H_o of good specification. This backtesting procedure is two side and rejected mode if it generates too many or low violation, but on the base of risk expert, can accept model that generate dependent exceptions [21-23].

According to the χ^2 (Chi-squared) distribution table provided the critical value at the 95% and 99% confidence level are 3.841 at 5% significance level and 6.635 at 1% significance level. The kupice unconditional coverage test provides the value given in the Table 6. The "green" marked showing these model are provided accurate forecast estimate as the value of this model is near to the critical value of χ^2 distribution. Further, they also put for testing to select the more accurate model [24,25].

Loss function: The underlying idea of using loss function is provided by Lopez propose another way to control for the magnitude of the exceedance in the violation. The basic approach of loss function is reflecting with a negative orientation. They provide higher score when failure takes place. Only that model which minimises the loss is preferred over the other models. VaR model selected on the base of comparison, and least loss model is selected with the assumption of it provide accurate estimate [26,27].

The following quadratic loss function suggested by Lopez with the magnitudes of violation.

$$\psi_{t+1} = \begin{cases} 1 + (X_{t+1} - VaR_{t+1})^2, & if\ violation\ occurs \\ 0, & otherwise \end{cases}$$

Where, $(X_{t+1} - VaR_{t+1})$ is a magnitude of difference between value-at risk and return series. Thus a score of one is imposed at exception occurs, with this numerical score increase with the magnitude of the exception.

According to the Lopez, loss function, a mode is preferred over other which minimizes the total loss i.e. $\psi^{loss} = \sum_{t=i}^{T} \psi_t$. The Table 7 have been showing computed value of loss function as define by Lopez, under both 95% and 99% confidence level. The loss function is applied only those model which passed the kupice unconditional coverage test (i.e. those model not pass under LR_{uc} not qualify for the loss function for further testing purpose).

		Root mean Squared Error	Mean Absolute Error
BoA	GARCH	0.03711	0.01835
BoA	T-GARCH	0.03711	0.01834
BoA	E-GARCH	0.03715	0.01884
JPM	GARCH	0.02901	0.01801
JPM	T-GARCH	0.02909	0.01806
JPM	E-GARCH	0.02941	0.01801
ML	GARCH	0.03237	0.02025
ML	T-GARCH	0.03225	0.02032
ML	E-GARCH	0.03227	0.02047

Table 5: Result of forecasting VaR of daily return.

LR_{uc} Model	HS		FHS		GARCH		E-GARCH		T-GARCH	
Banks	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
Merrill Lynch Bank	44.7999	15.3549	1.582671	0.564155	14.8667	0.00415	18.31787	0.0042	14.8667	0.17152
JP Morgan and Chase	7.56911	18.4689	7.555492	1.771521	88.8954	4.8189	96.05138	3.8949	91.2342	8.33194
Bank of America	14.4323	22.5943	0.331478	0.389631	103.643	3.08284	98.53298	1.7715	103.643	3.08284

Note: Green Box passing the kupiec as per calculation and test suggested by kupiec

Table 6: Results of Unconditonal coverage test at 95% and 99% confidence level.

Models	HS		FHS		GARCH		E-GARCH		T-GARCH	
	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
Merrill Lynch Bank			47.04529	10.0093						
Percentage of violation			6.016022	1.27996						
JP Morgan and Chase		171.272	108.0708	21.0189		17.017		18.0164		14.0094
Percentage of violation		6.18982	3.905919	0.75963		0.615		0.65112		0.5063
Bank of America			145.8214	31.5567		19.492		21.5262		19.4944
Percentage of violation			5.270017	1.14047		0.7645		0.77796		0.70453

Note 1: Loss funcion calculated only for those models which qualify the kupiec Unconditional coverage test.

Note 2: More accurate model is selected based on the models that get least value form amongst the models.

Table 7: Loss function: Lopeze Test at the 95% and 99% confidence level.

Banks	Total No of Hit's occurred under different Model									
Models	HS		FHS		GARCH		E-GARCH		T-GARCH	
	95%	99%	95%	99%	95%	99%	95%	99%	95%	99%
Merrill Lynch Bank	86	21	47	10	18	8	18	8	18	9
Percentage of violation	7.101569	1.734104	6.01023	1.278772	2.304738	1.024328	2.048656	1.023018	2.30179	1.150895
JP Morgan and Chase	171	53	108	21	45	17	42	18	44	14
Percentage of violation	6.179978	1.915432	3.903144	0.758995	0.614384	0.614384	1.517889	0.650524	1.59017	0.505963
Bank of America	184	56	145	31	39	19	41	21	39	19
Percentage of violation	6.649801	2.022391	5.240332	1.120347	1.409469	0.686664	1.481749	0.758945	1.409469	0.686664

Note: Hit's are calculated based on violation occurred under different models, for 2767 days for JPM, BOFA, and 782 days for ML Banks.

Table 8: Summary of Traffic Light Test with 95% and 99% Confidence falling within Three Zone.

The previous Table 8 of Kupiec test Shows that in 95% confidence level only FHS model qualified for loss function, while at 99% confidence level all model qualified except Historical Simulation (HS). After applying loss function to select more accurate model is one which has least value or least loss model. Here, for Merrill Lynch FHS consider the accurate model while for JP Morgan and Bank of America, T-GARCH consider to be more accurate model as per loss function results suggested.

Backtesting Conclusion and Outcome

The pioneer exercising of Model validation in the form of backtesting provides very important results for the accuracy of VaR models at the end user of VaR forecasting. The emphasized should be given to the VaR quantitative parameters for backtesting. For the selection of more accurate model the experts should avoid high confidence level for backtesting as it found that the selection of high confidence level decrease the effectiveness of the statistical test as mention by Jorion [18].

The results shown that the Merrill Lynch Bank passed almost all test including showing very low exception. Which researcher found the bias, and that is only due to the difference in sample size, as the sample size are too low at comparison of the other banks. If these violation proportions take with the other banks sample size then it would be under more threat of banks performance.

The most appropriate and simple model for the purpose of backtesting is proposed by kupiec's POF-Test. The underlying issue with the model is if statistically too many or too few exceptions are observed, the model is rejected. As mention in the research of kupiec on conditional coverage test. The good model are those which value's lies near the critical value of the χ^2 distribution and model which generate few and too exception are rejected. The other suitable model for the purpose of bank's risk assessment seems to be Basel suggested traffic light test which suggest good model only those which generate few exception so banks have enough capital to mitigate the risk.

T-GARCH and Filtered Historical Simulation (FHS) model found to be the good model for all banks, almost predicted more accurate results as per loss function given. FHS has good property that becomes best model for forecasting VaR for all three banks. According to the model all banks having hit's which could become the case of distress in entire banking if banks underestimate VaR forecast and do not maintain enough capital to cover daily loss.

Presently, the VaR is become the most appropriate model for estimation of risks by not only banks but other financial institution and regulators also. However, the problem of implementing VaR aspect and interpretation are different for this agent. i.e., regulators view to accurate model are those who generate few exception but banks and financial institution preferred model which generate violation (exception) which results are near to the critical value of pre-estimate.

References

1. Linsmeier, Thomas J, Neil D (1996) Risk Measurement: An Introduction to Value at Risk, Risk Measurement: An Introduction to Value at Risk.
2. Nelson DB (1991) Conditional heteroskedasticity in assets returns: a new approach. Econometrica 59: 347-370
3. Fatima, Miguel (2013) Forecasting the variance and return of Mexican financial series with symmetric GARCH models. Theoretical and Applied Economics 20: 61-82.
4. Engle R (2001) Garch 101: the use of arch/garch in applied econometrics. J Econ Perspect 15: 157-168.
5. Pérignon, Christophe, Daniel R. Smith (2010) The level and quality of Value-at-Risk disclosure by commercial banks. J Bank Financ 34: 362-377.
6. Fabozzi J, Focardi T, Rachev, Bala G (2014) The Basics of Financial Econometrics: Tools, Concepts, and Asset Management Applications, John Wiley & Sons, Inc. USA.
7. Ghahramania M, Thavaneswaran A (2007) A note on GARCH model identification. Comput Math Appl 55: 2469-2475.
8. Park J, Baek J, Hwang S (2008) Persistent-threshold-GARCH processes: Model and application. Stat Probab Lett 79: 907-914.
9. Bo Sjo (2011) Estimation and testing for arch and GARCH. Modelling the volatility of the Electrolux stock.
10. Rupper D (2011) Statistics and Data Analysis for Financial Engineering, Springer Texts in Statistics.
11. Piroozfar G (2009) Forecasting Value at Risk with Historical and Filtered Historical Simulation Methods, Project Report, Uppsala University.
12. Robert E (2001) GARCH 101: The use of ARCH/GARCH models in Applied Econometrics. J Econ Perspect 15: 157-168.
13. Jing W (2011) Threshold GARCH Model: Theory and Application. The University of Western Ontario.
14. Velayoudoum M, Bechir R, Abdelwahed T (2009) Extreme Value Theory and Value at Risk: Application to oil market. Energ Econ 31: 519-530.
15. Ghulam A (2013) EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH and APARCH Models for Pathogens at Marine Recreational Sites. Journal of Statistical and Econometric Methods 2: 57-73.
16. Engle R, Christian BB, Kelly (2012) A practical guide to volatility forecasting through calm and storm. J Risk Res 14: 3-22.
17. Bollerslev T (1990) Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Modelî. Rev Econ Stat 72: 498-505.
18. Jorion P (2001) Value at Risk: The New Benchmark for Controlling Market Risk, 2nd edition, McGraw-Hill, New York.
19. Engle R, (2002) Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH. J Bus Econ Stat 20: 339-350.
20. Johan W, Rikard T (2010) Estimation periods in risk parameter computations, Uppsala University pp: 5-40.
21. Krzysztof Piontek. The Analysis of Power for Some Chosen VaRBacktesting

Procedures - Simulation Approach. Advances in Data Analysis, Data Handling and Business Intelligence pp: 481-490.

22. Marc G, Ronald R (2009) Shortcomings of a parametric VaR approach and nonparametric improvements based on a non-stationary return series model, working paper.
23. Campbell D (2005) A Review of Backtesting and Backtesting Procedures, Finance and Economics Discussion Series, Divisions of Research & Statistics and Monetary Affairs. Federal Reserve Board, Washington DC, USA.
24. Dirk T (2003) A traffic lights approach to PD validation, Germany, Working paper.
25. Basel (1996) Supervisory Framework for the use of "Backtesting" in Conjunction with the Internal Models Approach to Market Risk Capital Requirements, Basle Committee on Banking Supervision.
26. Basel (1999) Best Practices for “Credit Risk Disclosure”, Consultative paper issued by the Basel Committee on Banking Supervision.
27. Basel (1998) Sound Practices for Loan Accounting, Credit Risk Disclosure and Related Matters, Basle Committee on Banking Supervision.