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Modeling Open Channel Fluid Flow with Trapezoidal Cross Section and a Segment Base

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Abstract

This study investigates the suitability of trapezoidal cross-section with segment base in drainage system design. The study has considered steady uniform open channel flow. The saint-Venant partial differential equations of continuity and momentum governing free surface flow in open channels have been solved using finite difference approximation method. We investigate the effects of the channel radius, area of the cross section, the flow depth and the manning coefficient on the flow velocity. The flow variables are velocity and the flow depth while the flow parameters are cross section area of flow, channel radius, slope of the channel and manning coefficient. The study has established that increase in cross section area of flow leads to a decrease in flow velocity. Further, increase in channel radius and cross section area of flow leads to a decrease in flow velocity and increase in roughness coefficient cause flow velocity to decrease. Additionally, increase in flow depth increases velocity. The physical conditions of the flow channel have been applied to conservation equations to arrive at specific governing equations. The results of the study have been presented graphically.

Keywords: Velocity; Slope; Channel radius; Drainage systems

Nomenclature

Roman symbols

v: Mean velocity of flow (m/s)

L: Length of the channel (m)

g: Acceleration due to gravity (ms^{-2})

Q: Discharge (m^3s^{-1})

A: Cross-section area of flow (m^2)

n: The manning coefficient of roughness ($Sm^{-1/3}$)

S_o: Slope of the channel bottom

S_f: Friction slope = $\frac{n^2V^2}{R^{\frac{4}{3}}}$

P: Wetted perimeter of the channel cross-section (m)

T: Top width of the free surface (m)

Y: depth of flow (m)

T: time (s)

q: Lateral uniform inflow/outflow ($m^2\ s^{-1}$)

R: hydraulic radius (m)

Fr: Froude number (dimensionless) = $\frac{V}{\sqrt{gL}}$

Re: Reynolds number (dimensionless) = $\frac{\rho uL}{\mu}$

x: Distance along the main flow direction (m)

D: Hydraulic depth (m)

r: channel radius (m)

$\frac{\partial A}{\partial t}$: Rate of change in area of flow with time (m^2/s)

$\frac{\partial v}{\partial x}$: Rate of change of mean velocity of the flow with distance (ms^{-1})

$\frac{\partial y}{\partial x}$: Rate of change of depth of the flow with the distance (ms^{-2})

$\frac{\partial Q}{\partial x}$: Rate of change in discharge with distance (m^2s^{-1})

Greek symbols

μ: Kinematic viscosity (m^2/s)

ρ: Density (kgm^{-3})

τ: Shear stress (nm^{-2})

Introduction

We are examining the suitability of a trapezoidal channel with a segment base in drainage design system. Water on the street if not properly drained can create hydroplaning. This affect severely traffic flow and safety of road users which is a common feature in many roads in Kenya. This is caused by poor drainage in the existing road network systems. Thus, the provision for adequate drainage is of paramount importance in road design and cannot be over emphasized. Road construction will affect the natural surface and sub-surface drainage system pattern of a water shed or individual slope. The study of free surface water flow in open channels has many important applications. These include the reduction of energy generated by flowing water and the destructive power of flowing water. This destructive power of water increases exponentially as its velocity increases. Therefore,

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water should not be allowed to develop sufficient volume or velocity to cause excessive water along ditches, below culverts or along exposed running surfaces, cuts or fills. When flow velocities exceed a certain threshold, damage to the channel by scourging may result, or siltation of suspended material may occur. A good drainage system design should allow for a minimum of disturbance of natural drainage system, in addition, it must drain surface and sub-surface water away from roadway and dissipate it in a way that prevents excessive collection of water in unstable areas and subsequent downstream destruction. In Kenya and the world at large, engineers have tried to direct water to desired areas such as drainage systems and dams for electricity generation among others. Most roads networks in Kenya both rural and urban lack efficient drainage systems, hence repeated occurrence of road destruction, loss of life and economic degradation especially when it rains. This impact negatively on achieving Kenya vision 2030 that seeks to create a globally competitive and prosperous nation with high quality of life by the year 2030. The vision 2030 has three pillars on which the government wants to achieve the vision. These are economic, political and social pillars. These three pillars are related to our study due to the fact that poor drainage affects the economy of the people directly. For instance, when it rains and road are cut off by run off, transport is hampered and this affects movement of goods and services. In addition, substantial amount of money is used to repair roads, sewers, airports and playing fields. People have also gone on strike due to blockage of sewers and roads and this affect smooth operation of businesses. Additionally, poor drainage is a threat to health of the population due to outbreak of disease and other related health problems. Thus, our study seeks to find the solutions to these problems brought about by drainage to help in achievement of vision 2030. The study will focus on suitability of trapezoidal cross-section with segment base to help mitigate the problem of drainage channel blockages which is a common feature in drainage systems. We hope that the findings of this study will go a long way in providing reference point for the design of drainage systems for road construction, sewage construction, street drainage and airports construction in Kenya and world at large. In practical, channel hydrodynamics where conservation of resources is of prime importance, it is crucial to investigate the issue of most hydraulically optimum shape. This means the channel shape that allows maximum discharge for a fixed area of flow, surface roughness and bed slope. Channel design must optimize dimensions and shape which both minimizes cost, maximize discharge in normal seasons and regulate the discharge to minimize velocity fluctuations during overflow. The applicability of trapezoidal channel and segment base in drainage design will reduce the effects of destruction caused by poor drainage and this will help Kenya achieve vision 2030, through construction of roads and drainage systems that are not easily destroyed by rain water, easy to maintain and reducing wastage of time in traffic jams. The Figure 1 below shows the trapezoidal cross section and segment base. Region a_1 is the trapezoidal part and a_2 is the segment part. y represents the depth of flow of the trapezoidal cross section, while h is the depth of flow of segment part called sagitta. A segment represents a section of the arc of a circular cross section bounded by an arc.

The flow of water in open channels has been extensively researched and a substantial volume of literature has been accumulated. Open channels have been studied for long time with Chezy equation as one of earliest empirical uniform flow equations developed for calculating average velocity of uniform flow. In 1768 French Engineer Henderson developed the Chezy equation. It resolves forces and hydraulic head gradients on water in an open channel to estimate average velocity. Swiss engineers Gausuillet and Kutter came up with better results by showing that Chezy's roughness coefficient, depended on hydraulic radius R,

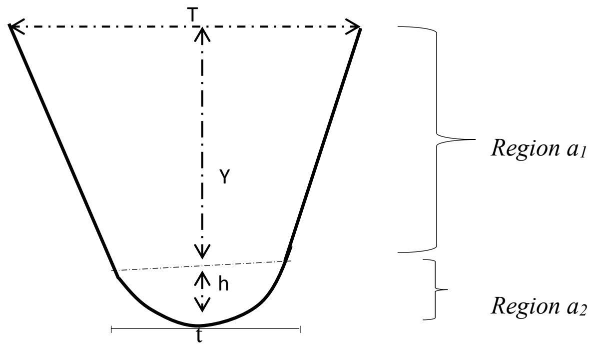

Figure 1: A trapezoidal cross-section with a segment base.

slope of the channel, S_0 and manning number, n. The commonly used formula in open channel fluid flows is manning formula, similar to Chezy's equation which was developed empirically have studied non-uniform and unsteady open channel flow with a circular cross section which is partially full. The flow variables were velocity and depth of flow while they considered flow parameters such as cross section area, channel radius and slope of the channel. They concluded that increase in slope increases the flow velocity. In addition, increase in cross section area leads to increase in wetted perimeter that leads to increase in shear stress between the sides and bottom of the channel with the particles which results to decrease in the velocity. The study established that increase in cross section area of the flow decreases the flow velocity and also increase in channel radius decreases the flow velocity. Further, a decrease in lateral inflow per unit length of the channel increases the velocity. The manning formula has proved to be very reliable in practice. An Irish Engineer, Chow [1] studied open channel flows and developed many relationships such as velocity formula for open channel flows. Kwanza et al. [2] analyzed effects of channel width, slope of the channel and lateral discharge on fluid velocity and channel discharge for both rectangular and trapezoidal channels. They concluded that the discharges for both rectangular and trapezoidal increases as the specified parameters are varied upwards. Chagas and Souza studied rivers by use of saint-Venant equation to come up with equations that govern the propagation of a flood wave. They showed that hydraulic parameters play important role in propagation of flood wave. Tuitoek and Hicks modeled unsteady flow in compound channels with the aim of controlling floods. They used St. Venant equation to come up with the model. They included mass and momentum transfer terms. Tsombe et al. [3] has also studied fluid modeling in an open channel with circular cross-section. Their study focused on open channel flow in a closed conduit with circular cross-section. Their study considered unsteady non-uniform flow in a closed conduit with circular cross section. They investigated the effects of flow depth, cross section area of flow, channel radius, slope of the channel, roughness coefficient and energy coefficient on the flow velocity as well as the depth which flow velocity is maximum. The study established that for a given flow area, the velocity increases with increasing depth and the velocity is maximum slightly below the free surface. In addition they also established that increase in slope of channel and energy coefficient leads to an increase in flow velocity whereas increase in roughness coefficient, flow depth, radius of the conduit and area of flow leads to a decrease in flow velocity. Morton and Erdeyi tried to design a computer programme to calculate common open channel flows for different cross-sections. They used the manning equation for steady uniform flowing fluids. Yen [4] studied open channel flow resistance differences between momentum and energy resistant point,

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cross-section area and resistance coefficient were analyzed. They also analyzed the alluvial channel resistance. They also discussed the issue of linear separation approach versus non-linear approach to alluvial channel resistance. Junke and Piene Jullen studied shear stress in smooth rectangular open channels flows. They established that shear stress is a function of three components, gravitational force, secondary flow and interfacial shear stress. Khan [5] studied open channel flow over dry bed with aim of understanding flow over islands during flood stage, flow downstream of the hydraulic structures, during intermittent release of water and flood ware either due to natural causes or sudden failure of hydraulic structures dry bed. Bates [6] in the of design of fish passage in a culvert or water ways have recommended use of trapezoidal cross-section because they do not hinder their movements in addition to few alluvial deposits and for low channel flow especially culvert construction. They also recommend use of trapezoidal channel where there is limited cover and where less obstruction of water way is desirable. Adepoju studied resistance to flow in smooth channels of circular cross-section. The results of the test showed that measured function factors are larger than those for a pipe of equivalent diameter. He showed that the shape effects are better studied when depth of flow is compared to wetted perimeter(Y/P) instead of using a single parameter like the hydraulic mean radius. Norman [7] noted that discharge through a culvert is controlled by either inlet or outlet conditions. Inlet control means that flow through a culvert is limited by culvert entrance characteristic. Outlet means that flow through a culvert is limited by friction between the flowing water and culvert barrel. Zeng studied the effects of friction and viscosity in trapezoidal cross-section. They sub-divided the cross section into several sub-sections and the friction factor in each sub-section was calculated using manning formula by adopting local parameters. The results showed that the friction factor is the major parameter affecting accuracy of the analytical solutions. Hassan [8] studied the flow characteristics of partially vegetated trapezoidal channels cross-section with flexible vegetation. He considered flow in 2D velocity distribution and modeled the effects of vegetation on sides and bed of trapezoidal channels. Sinha investigated the development of laminar flow of a viscous incompressible fluid in partially full circular pipe. They observed that velocity increases more rapidly during initial development of flow in comparison to further downstream. This is because the boundary layer is thinner compared to the downstream. Warren proved mathematically that the maximum flow by manning formula in open channel of trapezoidal cross-section of fixed side slopes and with fixed free board, occurs $r=\left(s/2\right)^{fx}/q$, in which r is the hydraulic radius of the wetted section, x is the bottom width channel section $q=2l-w$, the difference between the total length side slope $2l$, and the top width w of excavated section and f called free board which is the vertical distance between the present or undisturbed ground surface and the proposed water surface in the finished channel. It is assumed to be below the original ground surface. In the road drainage manual, department of transport, Queens land government has provided guidance in relation to the planning, design, construction, maintenance and operation of road drainage structures in urban and rural environments for main roads. Javid [9] in the international journal of engineering have come up with mathematical software package for calculation of boundary shear stress in trapezoidal channels. Crossley [10] has come up with accurate and efficient numerical solutions for Saint Venant equations of open channel flows.

Mathematical Formulation

Continuity equation (conservation of mass)

This is a differential equation describing the transport of conserved quantity. A conserved quantity cannot decrease or increase, it can only move from place to place. By Tsombe [3], the equation is;

$$\frac{\partial y}{\partial x}vT + A\frac{\partial v}{\partial x} + T\frac{\partial y}{\partial t} - q = 0 \qquad (1)$$

Equation of conservation of momentum

The equation describes the motion of fluid substances. It is derived from Newton Second law of motion. The law states that the acceleration of an object depends upon two variables, the net force acting upon it and the mass of the object. That is; the time rate of change of the linear momentum of the system is equal to the sum of the external force acting on the system [11-14].

$$\frac{g}{\beta}\left(S_0 - S_f\right) - \frac{qv}{A} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial t} + \frac{g\partial y}{\beta\partial x} \qquad (2)$$

The specific continuity and the momentum equations governing our flow problem are given as;

$$(by+2xy)\left(v\frac{\partial y}{\partial x}+\frac{\partial y}{\partial t}\right)+\left\{by+xy^2+(h-H)\sqrt{h(2H-h)}+H^2\left[\sin^{-1}(\frac{h-H}{H})+\frac{\pi}{2}\right]\right\}\frac{\partial v}{\partial x}-q=o \qquad (3)$$

$$\frac{g}{\beta}\left(S_o-S_f\right)-\frac{qv}{bx+xy^2+((h-H)\sqrt{h(2H-h)}+H^2\left[\sin^{-1}(\frac{h-H}{H})+\frac{\pi}{2}\right])}=\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{g}{\beta}\frac{\partial y}{\partial x} \qquad (4)$$

Solution procedure

The governing equations; Equations 3 and 4 are solved using a diffusing scheme as proposed by Viessman, these equations are given as;

$$y_{i,j+1} = 0.5(y_{i-1,j} + y_{i+1,j} - \Delta t\left\{\frac{1}{T}(V_{i,j}\frac{y_{i1,j}-y_{i-1,j}}{2\Delta x} + A\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x}+q)\right\}\cdot$$

Substituting A and T, the equation yields;

$$y_{i,j+1}=0.5(y_{i-1,j}+y_{i+1,j}-\Delta t\left\{\frac{1}{b+2xy}(V_{i,j}\frac{y_{i1,j}-y_{i-1,j}}{2\Delta x}+\mathrm{bx}+2\mathrm{x}y^2+(h-H)\sqrt{h(2H-h)}+H^2\left[\sin^{-1}(\frac{h-H}{H})+\frac{\pi}{2}\right]\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x}+q)\right\} \qquad (5)$$

The momentum equation is also written in finite difference form as follows;

$$v_{i,j+1}=0.5\left\{V_{i-1,j}+V_{i1,j}\right\}-\Delta t\left\{V_{i,j}\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta}+g\frac{y_{i+1j}-y_{i-1,j}}{2\Delta x}-g\left(s_0-\frac{S_{f\,i-1,j}+S_{f\,i+1,j}}{2}\right)-\frac{qv}{A}\right\}$$

The friction slope S_f can be estimated from manning equation as [15-18].

$S_f = \frac{n^2V^2}{R^{\frac{4}{3}}}$ and substituting A, we get;

$$v_{i,j+1}=0.5\left\{V_{i-1,j}+V_{i1,j}\right\}-\Delta t\left\{V_{i,j}\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta}+g\frac{y_{i+1j}-y_{i-1,j}}{2\Delta x}+g\left(s_0-\frac{S_{f\,i-1,j}+S_{f\,i+1,j}}{2}\right)-\frac{qV}{\mathrm{bx}+2\mathrm{x}y^2+(h-H)\sqrt{h\left(2H-h\right)}+H^2\left[\sin^{-1}(\frac{h-H}{H})+\frac{\pi}{2}\right])}\right\} \qquad (6)$$

Equations 5 and 6 are solved subject to initial and boundary conditions;

v (x, 0) = 20, y (x, 0) = 0.5, for all X > 0

v (0, t) = 20, y (0, t) = 0.5, for all t > 0

v (x, t) = 20, y (x, t) = 0.5, for all t > 0

Results and Discussion

The values for velocity against those of depth were plotted giving the values in the Figure 2. Additionally, by varying the specified parameters; the curves in Figures 3-5 were obtained.

R =1, A =1.0275, P = 3.2351, n = 0.012, s = 0.0004

Graph of velocity versus depth

The velocity increases with depth up to a maximum of 12 m.

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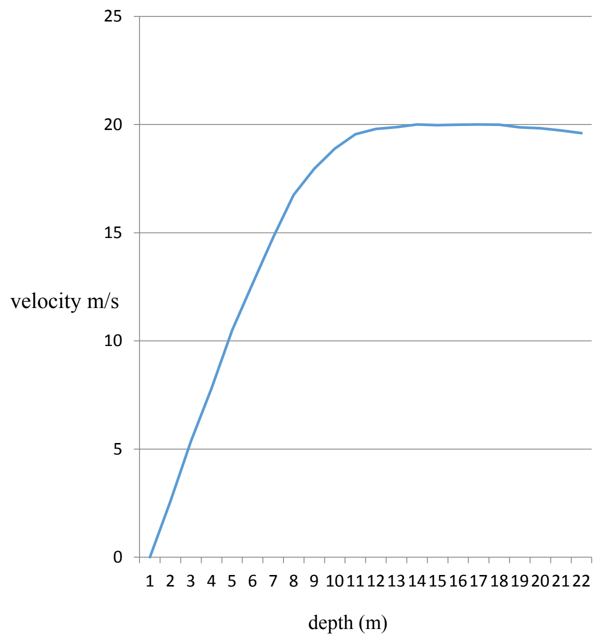

Figure 2: Velocity profiles with change in depth.

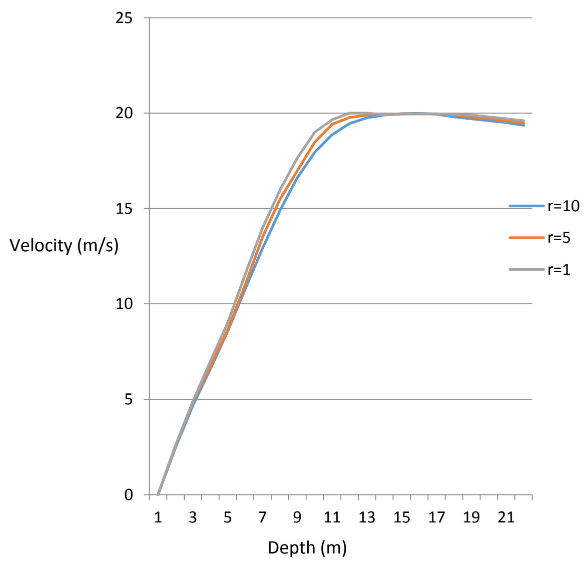

Figure 3: Velocity profiles with changing channel radius.

Further increase in depth, the velocity remains constant. The free surface occurs at a depth of 20.5 m and the velocity of the fluid is 20 m/s. In addition, maximum velocity occurs at a depth of 9 m and the velocity begins to decrease as it approaches the free surface. The free surface affects the velocity of the fluid due to friction resistance especially when the wind blows over the free surface in the opposite direction of the main flow direction. An increase in depth will lead to an increase in wetted perimeter which will result to an increase in shear stress, hence resulting to decrease in flow velocity.

n = 0.012, s = 0.0004

Effect of change of channel radius on the velocity

As shown in Figure 3 above, as the radius of the channel increases from 1-10 and depth increases, the velocity decreases. An increase in channel radius result to an increase in wetted perimeter as a result of the liquid spreading more in the conduit leading to a large cross-sectional area. A large wetted perimeter results to a large shear stress at the sides and bottom of the channel leading to reduction of flow velocity. A decrease in velocity result to a corresponding decrease in discharge since discharge is a function of cross section area of flow and velocity.

r = 1, A = 1.0275, P = 3.2351, n = 0.012,

Effect of slope on velocity

As slope increases from 0.004-0.02, the flow velocity also increases. Increase in slope causes the center of gravity to move up causing instability in the fluid molecules. This in turn causes increase in discharge. Increase in cross section area leads to decrease in velocity. This is due to increase in wetted perimeter that leads to increase in shear stress between the sides and bottom of channel with the fluid particles which results to decrease in flow velocity.

R = 1 A = 1.0275, P = 3.2351, s = 0.0004

Effects of change of manning constant with velocity

An increase in roughness coefficient from 0.012-0.035 leads to a decrease in velocity as can be seen in Figure 5. An increase in roughness coefficient results to large shear stress at the sides and bottom of the channel leading to a reduction in velocity. This is due to the fact that the motion of the fluid particles near the surface of the conduit will be reduced. The velocity of the neighboring molecules will also be lowered due to constant bombardment with the flow moving molecules leading to an overall reduction of flow velocity.

Conclusion

A trapezoidal cross section with a segment base has been developed with the resulting partial differential equations solved to obtain the velocity profiles having the velocity and depth as the flow variables while the flow parameters were cross section area of the channel, channel radius and channel slope. The results of our study have established that increase in radius of the circle forming the segment leads to decrease

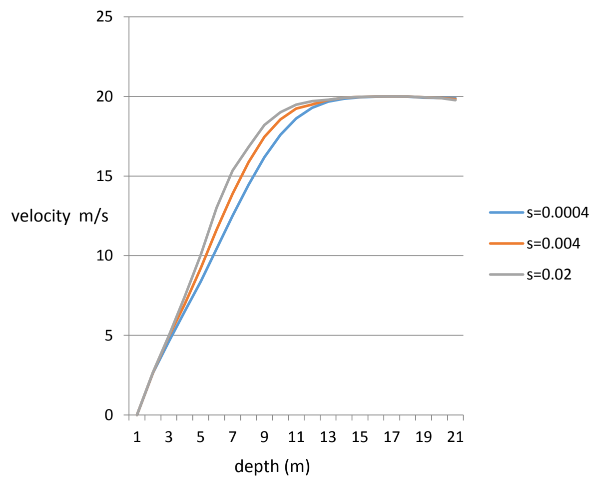

Figure 4: Velocity profiles with changing slope.

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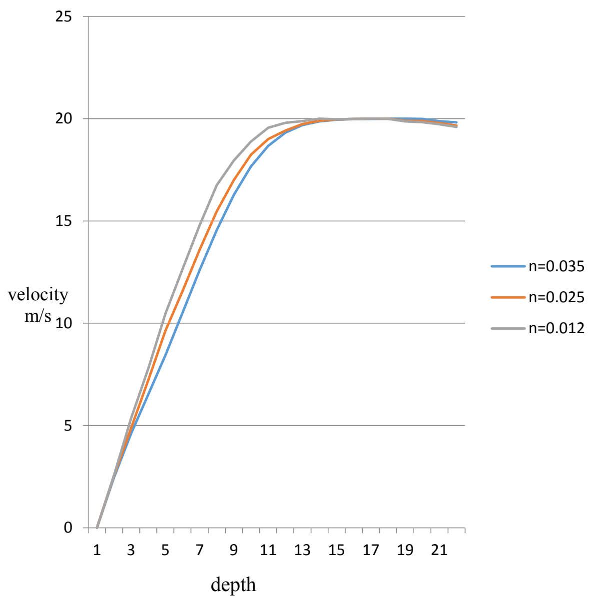

Figure 5: Velocity profiles with changing manning coefficient.

in flow velocity. Further, flow velocity increases with increase in depth. Increase in area of flow results to decrease in flow velocity. Additionally, increase in roughness coefficient result to decrease in flow velocity. The results of our study agree with earlier researches done on related work of different open channel cross section.

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