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Modal Analysis of FGM Plates (Sus304/Al_2O_3) Using FEM

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Abstract

The present work aims to carry out modal analysis of a functionally graded material (FGM) plate to determine its natural frequencies and mode shapes by using Finite Element Method (FEM). For this purpose, a code was written in MATLAB and linked with ABAQUS. First, a simulation was performed in accordance to other researcher's model, and then after comparing the obtained results, the accuracy of the present study was verified. The obtained results for natural frequency and mode shapes indicate good performance of user-written subroutine as well as FEM model used in present study. After verification of obtained results, the effect of clamping condition and the material type (i.e., the parameter n) was investigated. In this respect, finite element analysis was carried out in fully clamped condition for different values of n. The results indicate that the natural frequency decreases with increase of n, since with increase of n, the amount of ceramic phase in FGM plate decreases, while the amount of metal phase increases, leading to decrease of the plate stiffness and hence, natural frequency, as the Young modulus of Al_2O_3 is equal to 380 GPa and the Young modulus of SUS304 is equal to 207 GPa.

Keywords: FGM plates; Finite element method; Modal analysis; Natural frequency

Introduction

Modal analysis is a technique used to determine structure's vibration characteristics: Natural frequencies and Mode shapes. These are most fundamental of all dynamic analysis types whereas technology development at an ever increasing rate, the need for advanced capability materials becomes a main concern in the modal analysis of engineering structures of more intricate and higher performance systems. These requirements can be seen in many fields in which engineers are exploring the applications of new engineered materials [11].

Functionally Graded Materials (FGMs) are a relatively new technology and are being studied for the use in components exposed to high temperature gradients. An FGM material property allows the designer to tailor material response to meet design criteria. An FGM composed of ceramic on the outside surface and metal on the inside surface eliminates the abrupt change between coefficients of thermal expansion, offers thermal protection, and provides load carrying capability. This is possible because the material constituents of an FGM changes gradually through-the-thickness; therefore, stress concentrations from abrupt changes in material properties are eliminated. The FGM plates has numerous applications in advanced engineering fields such as thin-walled structural components in space vehicles, nuclear reactors, and other high thermal application areas. The functionally graded material (FGM) can be intended for specific function and applications. The material properties such as young's modulus, density and poisons ratio values are varying continuously throughout the thickness direction according to the simple volume constituents defined by power law.

In the recent years functionally graded material plates modelling and analysis are carried out by [1]. They have presents the principal developments in functionally graded materials (FGMs) with an importance on the recent work published since 2000. Pendhari et al. [2] has developed the analytical and mixed semi-analytical solutions for a rectangular functionally graded plate. Singha et al. [3] has investigated the high precision plate bending finite element for nonlinear behaviour of functionally graded plates under transverse load. In their analysis based on the first order shear deformation theory considered the physical/exact neutral surface position. A rectangular functionally graded material plate with simply supported boundary conditions subjected to transverse loading has been investigated by Chi and Chung [4]. Their analysis carried out, bases on the classical plate theory and Fourier series expansion; the series solutions of power-law FGM (simply called P-FGM), sigmoid FGM (S-FGM), and exponential FGM (E-FGM) plates are obtained. Shahrjerdi et al. [5] recently estimates the natural frequency of functionally graded rectangular plate using second-order shear deformation theory (SSDT). The material properties of functionally graded rectangular plates, except the Poisson's ratio, are assumed to vary continuously through the thickness of the plate in accordance with the exponential law distribution. Talha and Singh [6] has studied the static and free vibration analysis of functionally graded material plates by using higher order shear deformation theory with a special modification in the transverse displacement in conjunction with finite element models. Vel and Batrab [7] has developed a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. The exact solution was valid for thick and thin plates, and for arbitrary difference of material properties in the thickness direction. Hosseini-Hashemi [8] carried out the new exact closed form method for free vibration analysis of functionally graded rectangular thick plates. Their analysis was based on the Reddy's third-order shear deformation plate theory [9].

The main objective of this work is to propose a finite element approach for modal analysis of rectangular FGM plates based on the Kirchhoff plate theory or classical plate theory. The material properties are assumed to be graded through the thickness in accordance with a simple power-law distribution. Using the finite element formulation

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derived for the FGM plate element, a finite element program has been developed. Some examples are solved, and the results are compared with those available in the literature. In addition, the effects of power law index on the FGM plate natural frequency and mode shapes with different boundary conditions are studied.

Modal Analysis of FGM Plate

The first step in modal analysis of FGM plate is geometric modeling of the plate and in this research, in order to be able to compare the results with the results of other researchers, the geometric dimensions of the plate were selected like as reference [10] that the plate had length and width of 1 m and 0 thickness of 0.01 m (Figure 1).

Mechanical properties of a FGM plate were defined after the geometric modeling of plates. As mentioned earlier, it was assumed that Poisson's ratio is fixed in the thickness direction but the Young's modulus and density are variable and a power function was used to define these variable properties, according to equation (1). Where V_c is volume fraction of ceramic material, V_m is volume fraction of steel material, h is thickness of the plate, E and ρ are the Young's modulus and density respectively, E_c and ρ_c are Young's modulus and density of ceramic material, E_c and ρ_c are Young's modulus and density of steel material, n is the power of power function and y is the distance from the geometric center of the plate (Figure 2).

$$V_c(z_m) = \left(\frac{2z_m + h}{2h}\right)^n, V_m(z_m) = 1 - V_c(z_m) \tag{1}$$

$$V_c(z_n) = \left(\frac{2z_n + h + 2d}{2h}\right)^n$$

$$d = \frac{\int_{-\frac{h}{2}}^{+\frac{h}{2}} E(z_m) z_m dz_m}{\int_{-\frac{h}{2}}^{+\frac{h}{2}} E(z_m) dz_m}$$

The values of the mechanical properties of the ingredients for FGM plate (SUS304 stainless steel and Al_2O_3 ceramic) has been selected from reference [10] (Table 1).

In addition, the type of analysis has been chosen and for the purpose, option of frequency analysis was selected from the linear chaos in Step modules of software and it was determined the number of modes of vibration and the maximum frequency. Boundary condition and the type of supports of the piece was defined that the type of clamped (CCCC) mode were selected in this study. Equation below shows boundary condition for this type of supports with respect to the coordinate system shown in Figure 1.

Clamped mode (CCCC):

$$u = v = w = \theta_x = \theta_y = \theta_z = 0 \; where \; 0 \le x, z \le 1$$

Where u, v and w are the displacements in the direction of x, y and z axes respectively. θ_x, θ_y and θ_z are also rotation around the x, y and z axes respectively.

FGM plate was meshed after defining the boundary conditions. In this regard, the shape and type of elements was determined at first, so that the shape of elements was cubic and the type of elements was continuum 3D8 nodded reduced integration (C3D8R). Finally, after the meshing of the work piece, modal analysis was performed to determine the natural frequencies and mode shapes. In order to define the variable elastic properties, a sub-program was written in MATLAB programming language and was linked with ABAQUS software. It is worth noting that summon of the sub-program written in MATLAB programming language and linking it with ABAQUS software took place in this field.

Finite Element Formulations

The displacement field of the classical plate theory is:

$$U = u(x,y) - z\frac{\partial w(x,y)}{\partial x} \tag{2}$$

$$V = v(x,y) - z\frac{\partial w(x,y)}{\partial y}$$

$$W = w(x,y)$$

The strain-displacement relationship are given as

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{3}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

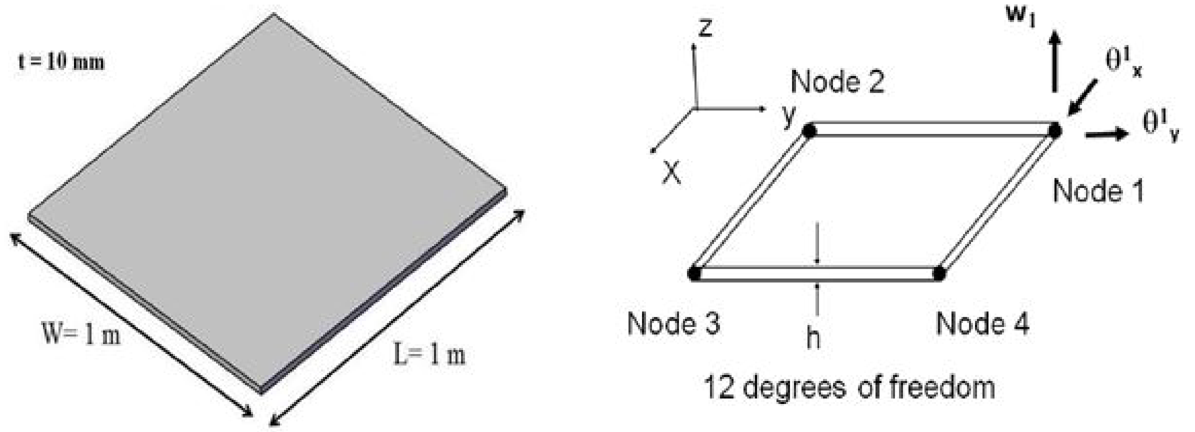

Figure 1: Geometric dimensions of FGM plate.

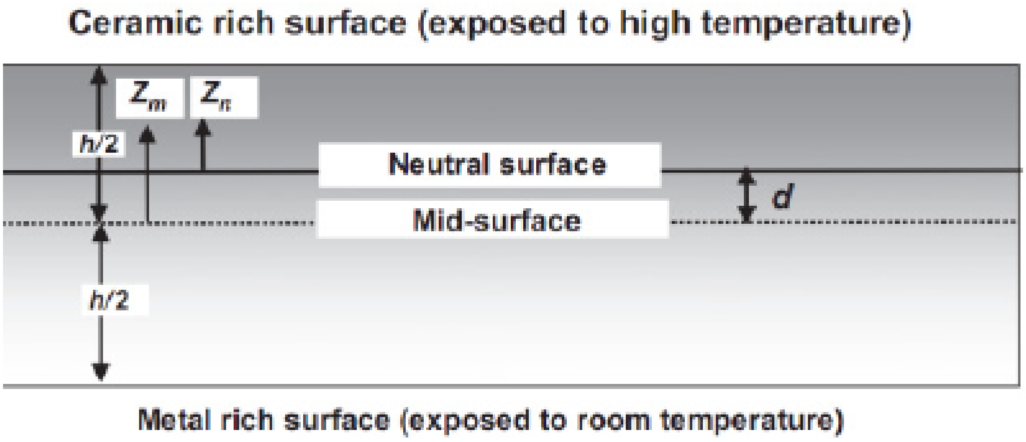

Figure 2: Schematic representation of FGM plate.

Materials	Properties		
	Young's modulus (N/m^2)	Poisson's ratio	Density (kg/m³)
SUS304 stainless steel	207 × 10^9	0.3177	8166
Al_2O_3 ceramic	380 × 10^9	0.3	2707

Table 1: Mechanical properties of ingredients of FGM plate.

The stress-strain relationships of the functionally graded plate in the global x-y-z coordinates system can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{4}$$

Where:

$$Q_{11} = \frac{E(z)}{1-\nu^2}, Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2}, Q_{66} = \frac{E(z)}{2(1+\nu)}$$

The total strain energy can be expressed as

$$U_e = \int\int\int \sigma_{ij}\varepsilon_{ij}\ \delta\ dx\ dy\ dz \tag{5}$$

The kinetic energy of the plate can be expressed as follows

$$T_e = \frac{1}{2}\iint \rho h w^2\ dx\ dy \tag{6}$$

The FGM plate is model using four node rectangular elements. Four node rectangular elements are having four nodes at each corner as shown in Figure 1. There are three degrees of freedom at each node, the displacement component along the thickness (w), and two rotations about x and y directions in terms of the (x, y) coordinates respectively. The each element consists of four nodes 1, 2, 3 and 4 with w is the transverse displacement and θ_x, θ_y represents the rotations about x and y axis respectively.

Where:

$$w = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2y & xy^2 & y^3 & x^3y & xy^3 \end{bmatrix}\{\beta\} \tag{7}$$

$$\{w^e\} = [A]\beta \to \beta = [A]^{-1}\{w^e\}$$

$$\{w^e\}^T = \begin{bmatrix} w_i & \theta_{xi} & \theta_{yi} \end{bmatrix} \quad i = 1:4$$

$$w = [poly(x,y)][A]^{-1}\{w^e\} = [N]\{w^e\}$$

$$N_i^T = \begin{bmatrix} \frac{1}{8}(1+x_i x)(1+y_i y)(2+x_i x+y_i y-x^2-y^2) \\ \frac{1}{8}(1+x_i x)(y+y_i)(y^2-1) \\ \frac{1}{8}(x+x_i)(1+y_i y)(x^2-1) \end{bmatrix}$$

where i=1, 2, 3 and 4.

The element stiffness matrix and mass matrices are derived on the basis on principle of minimum potential energy and kinetic energy.

The element stiffness matrix is:

$$[K_e] = \int [B]^T[D][B]dv \tag{8}$$

Where:

$$[B]^T = z\begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial y^2} & 2\frac{\partial^2}{\partial x\,\partial y} \end{bmatrix}[N]^T$$

The element mass matrix is:

$$[M_e] = \int [N]^T[\rho][N]dv \tag{9}$$

The equation of motion for a plate element is obtained by using Hamilton's principle.

$$\int_{t_1}^{t_2}(U_e - T_e + W_e)dt = 0 \tag{10}$$

The element stiffness and mass matrices are assembled in to get global matrices. The equation of motion of the plate can be written as

$$([K] - \omega^2[M])\{w\} = 0 \tag{11}$$

Results

One of the most efficient methods to verify the accuracy of results of a finite element analysis is to compare its results with the results of other researchers. The results of reference Ramu, Mohanty was used

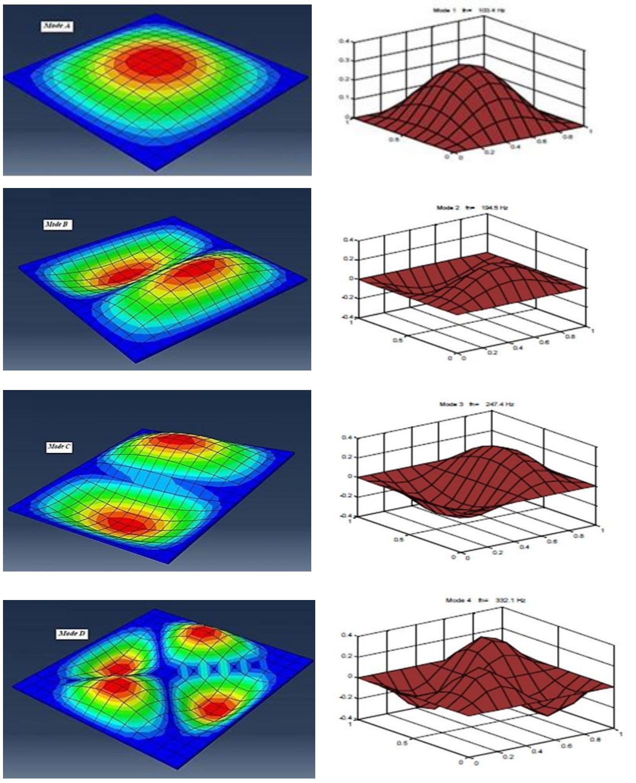

Figure 3: CCCC square FGM plate mode shapes 1, 2, 3 and 4 with index value n=1.

Materials	Results	1st mode	2nd mode	3rd mode	4th mode
Ceramic	Ramu I (2014)	3.50	7.10	7.10	10.50
	Percent	3.55	7.02	7.02	10.45
n=1	Ramu I (2014)	2.20	4.70	4.70	7.00
	Percent	2.27	4.50	4.50	6.68
n=5	Ramu I (2014)	2.00	3.90	3.90	5.80
	Percent	1.99	3.97	3.97	5.91
n=10	Ramu I (2014)	1.80	3.60	3.60	5.10
	Percent	1.78	3.53	3.53	5.25
Metal	Ramu I (2014)	1.63	3.37	3.37	4.89
n→∞	Percent	1.61	3.30	3.30	4.81

Table 2: Variation of the frequency parameter ($\bar{\omega}$) with the volume fraction index n for (CCCC) square (Al_2O_3) FGM plates.

for validation of the results in this thesis because of the impossibility of making FGM plate and experimental modal analysis; for this purpose, modal analysis was carried out in accordance with the parameters of reference Ramu and Mohanty and the results were compared. For this purpose, a FGM plate composed of (SUS304/Al_2O_3) in dimensions of 1 m × 1 m × 0.01 m was selected and has been analyzed in the cases of boundary condition clamped (CCCC) and it was obtained natural frequencies of the first four modes with the mode shapes. Figure 3 also show first four vibrational modes obtained from this study in clamped mode. As mentioned in the introduction, the vibration mode shape shows the shape of the piece in the maximum amount of deformation and as a result, vibrations of different points of piece per resonance frequency (natural frequency).

According to the above results (Figure 3), it can be seen a good agreement between the results and the results of reference [10], therefore, finite element analysis was performed in this study had sufficient accuracy for modal analysis of FGM plate and prediction of natural frequencies and mode shapes.

The Effect of "N" Parameter

The effect of power of the power function (n) on the natural frequency was evaluated that according to equation (1), n parameter specifies the influence of material, so that for n = 0, FGM plate is entirely of ceramic but whatever the value of n increases, the share of ceramics has been reduced and the share of steel material increases. The results of modal analysis for various n in boundary condition of CCCC at are summarized in the Table 2 and Figure 4. The result is ϖ dimensionless frequency parameter that this parameter is usually used to express the natural frequency in the literature. Equation (12) shows how to calculate $\bar{\omega}$:

$$\bar{\omega} = \omega\sqrt{\frac{12\left(1-\nu^2\right)\rho_c L^2 w^2}{\pi^2 E_c h^2}} \tag{12}$$

Where ω is the natural frequency, L is the length of plate; W is width and h is its thickness. Other parameters have been described previously.

About the impact of n parameter which shows the effect of material on the natural frequency, it can be said that the natural frequency decreases with respect to the results shown in Figure 4 with increasing n; because as stated previously, by increasing in the share of n, the share of ceramic phase in FGM plate decreases and share of steel phase increases leading to a reduction in stiffness of FGM plates and thereby reduce in the natural frequency. Because the Young's modulus of Al_2O_3 ceramic was 380 GPa and Young's modulus of SUS304 steel was 207 GPa.

Conclusion

Due to the adverse effects of resonance phenomenon which led to large tensions and displacement resulting in disintegration of the components of a mechanical structure, modal analysis is necessary to calculate the mode shapes and values of natural frequencies and to prevent the occurrence of resonance phenomenon. In this thesis, modal analysis of a FGM plate consisting of Al_2O_3 ceramic phase and 304 stainless steel metallic phases was performed by ABAQUS software with the assumption that the behavior of the material is elastic and mechanical properties (Young's modulus and density) are variable in the thickness direction. Therefore, a sub-program was written in MATLAB programming language and was linked with ABAQUS software. For modal analysis, a finite element analysis was carried out

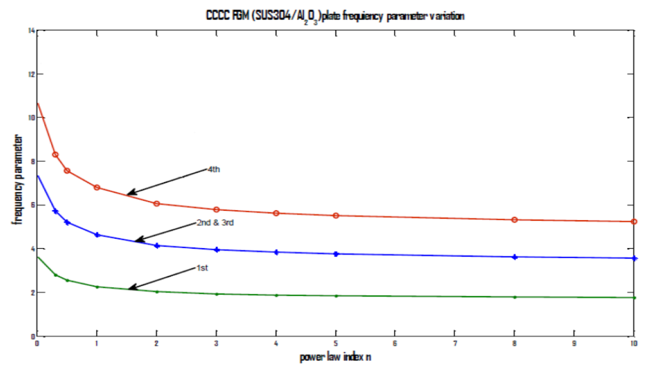

Figure 4: Variation of frequency parameter with index value at CCCC boundary conditions.

similar to other researchers at first and the accuracy of results was evaluated after comparison of results. Then it was evaluated the effect of material (n parameter) on the natural frequency. In this regard, finite element analysis was conducted for different values of n [(Ceramic), 1, 5, 10, (Metal)] in clamped modes. A summary of the results of this research is the following:

• Comparison of natural frequencies and mode shapes of the study with the results of other researchers indicated conformity and resulted in high performance of program written in MATLAB and high accuracy of finite element model used in this research.

• o compare the results, a dimensionless frequency parameter ($\bar{\omega}$) was defined as a function of the mechanical properties of FGM plate (density, Young's modulus and Poisson's ratio), geometric dimensions of the plate (length, width and height) and its natural frequency by software.

• The effect of n parameter that indicates the effect of material on the natural frequency, it can be said that the natural frequency decreases as n increases, because by increasing n, the share of ceramic phase in FGM plate has decreased and the share of steel phase decreased leading to reduce the stiffness of FGM plate and thereby reduce in the natural frequency. Because the Young's modulus of Al_2O_3 ceramic was 380 GPa and Young's modulus of SUS304 steel was 207 GPa.

References

1. Birman V, Byrd LW (2007) Modeling and analysis of functionally graded materials and structures. Appl Mech Rev 60: 195.
2. Pendhari SS, Kant T, Desai YM, Subbaiah CV (2010) Static solutions for functionally graded simply supported plates. Int J Mech Mater Des 8: 51–69
3. Singha MK, Prakash T, Ganapathi M (2011) Finite element analysis of functionally graded plates under transverse load. Finite Elem Anal Des 47: 453–460.
4. Chi SH, Chung YL (2006) Mechanical behavior of functionally graded material plates under transverse loads—Part I: AnalysisInt J Solids Struct 43: 3657–3674.
5. Shahrjerdi S, Mustapha F, Bayat M, Sapuan SM, Zahari R, et al. (2011) Natural frequency of F.G. rectangular plate by shear deformation theory. IOP Conf Series: Mater Sci Eng 17: 012008.
6. Talha M, Singh BN (2010) Static response and free vibration analysis of FGM plates using higher order shear deformation theory. Appl Math Model 34: 3991–4011.
7. Vel SS, Batrab RC (2004) Three-dimensional exact solution for the vibration of functionally graded rectangular plates. JSV 272: 703–730.
8. Hosseini-Hashemi S, Fadaee M, Atashipour SR (2011) Study on the free vibration of thick functionally graded rectangular plates according to a new exact closed-form procedure Compos Struct 93: 722–735.
9. Kumar JS, Reddy BS, Reddy CE, Reddy KV (2011) Higher order theory for free vibration analysis of functionally graded material plates. ARPN-JEAS 6: 10
10. Ramu I, Mohanty SC (2014) Modal analysis of functionally graded material plates using finite element method. Procedia Mater Sci 6: 460-467.
11. Hosseini M, Beiranvand P, Kalantari L (2017) Investigation of influence the pressure and stress on the water in buried concrete pipes in the different soil under blast loading. JCES 1: 52-56.