Methods for Solving a System of Linear Equations

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Abstract

The content of the system of linear equations makes it an interesting matter that can also be further expanded and deepened. The three methods most commonly used to solve systems of equations are substitution, elimination, and extended matrices. Replacement and elimination are simple methods that can efficiently solve most systems of two equations in a few direct steps. The extended matrix method requires several steps, but its application extends to a number of different systems. The elimination method is useful when the coefficient of one of the variables is equal (or its negative equivalent) in all equations.

Keywords: Direct methods • Factorization methods • Business • Management

Introduction

Solving a system of linear equations is one of the basic tasks of linear algebra. There are exact algorithms for solving systems, however when it comes to systems of larger dimensions (e.g. over 10,000 equations), most of these algorithms are not applicable in practice. One of the algorithms is Kramem's rule or determinant method, but this algorithm is interesting only from a theoretical point of view. It is enough, for example, to prove that the determinant of the system is different from zero to know that the system has exactly one solution, i.e., in homogeneous systems that the determinant is equal to zero shows that the system also has non-trivial solutions. In practice, for a system of dimension n it is necessary to perform O(n^3) arithmetic operations, which speaks enough about the inadmissibility of this method for solving systems of large dimensions. A somewhat more acceptable method for solving the system of equations is the Gaussian elimination method (which will be implemented in this paper for comparison), the complexity of this method is O(n^3).

Description

The solution of a system of n linear equations with n unknowns is n numerical values, such that when they are replaced in the equations of the system, all n equations are satisfied. A system can have a single solution, an infinite number of solutions and no solution (in the other two cases the systems are singular). There are two types of numerical techniques for solving systems of linear equations:

- direct methods—in the final number of steps we come to the correct solution.
- Iterative methods—we get an approximate solution with a certain accuracy.

From the direct methods we have Gaussian method of elimination and its variants: with the selection of the main element and with the scaled selection of the main element in business management.

In computational mathematics, an iterative method is a mathematical procedure that uses an initial value to generate a series of improvements of approximate solutions for a class of problems, in which the n-th approximation is derived from the previous ones. The specific application of the iterative method, including the termination criteria, is the iterative method algorithm. An iterative method is called convergent if the corresponding string converges for the given initial approximations. Mathematically rigorous analysis of the convergence of the iterative method is usually performed; however, iterative methods based on heuristics are also common. In contrast, direct methods try to solve the problem with a final series of operations. In the absence of rounding errors, direct methods would provide an accurate solution (such as solving a linear system of equations). By direct method we mean methods that theoretically give the correct solution a system in a finite number of steps such as Gaussian elimination. In practice, of course the resulting solution will be contaminated with a rounding error associated with the arithmetic is used in business.

Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.


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