Mathematical Theory of Consciousness

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Abstract

On the basis of this axioms we are proposing that consciousness is the result of a global workspace in the brain, which distributes information to the huge number of paralell unconscious processors forming a rest of the brain. Our theory is founded on the view that the brain is composed of many different paralell processors, or moduls, each capable of performing some task on the symbolic representations that it receives as input. The moduls are flexible in that they can combine to form new processors capable of performing novel tasks, and can decompose into a smaller component processors.

Axiom A: The consciousness is a mathematical structure, with neurobiological semantics.

Axiom B: The higher mental processes all correspond to well-defined but, at present, poorly understood information processing functions that are carried out by physical systems, our brains.

Keywords: Axiom A; Axiom B; Brain; Unimodal processors

Introduction

We are interpreting the brain as a large group of separable, very specialized systems that are unconscious much of the time that they operate. At least some of these processess can, one by one, become conscious, and the successive outputs of these processess contstitute conscious experience [1-4]. Significant, though, is the idea that only one process can be conscious at one instant of time. We understand consciousness as a serial phenomenon. Any respectable theory of consciousness will have to explain how it is a serial stream of thought emerges from the masses of unoconscious mental computations that are proceeding simultaneously in the brain, what it is about this stream of thought that makes it conscious, and what evolutionary advantages of such a system are.

Conscious Experience Means Context Sensitive

Conscious experiences means context sensitive, while representations processed unconsciously are not [1].

Global workspace system S consist from two modular subsystems:

1. The first is a processing network of the coputational space S_1, composed of a net of parallel, distributed and functionally specialised processors (from primary sensory processors-area V_1, unimodal processors-area V_4, heteromodal processors-the mirror neurons in area F5). They are processing categorical, semantic information.

2. The second is the projective space of a global wokspace S_2, consisting of a set of cortical neurons characterized by ability to receive from and send back to homologous neurons in other cortical areas horizontal projections through long-range excitatory axons. This populations of neurons are distributed among brain areas in variable proportions, originate from the pyramidal cells of layers 2 and 3, which give or receive the so called "association" efferents and afferents. These cortical neurons establish strong vertical and reciprocal connections, via layer 5 neurons, with corresponding thalamic nuclei [5-7].

Through their mutual projection the dynamics of workspace neuron activity is a constant flow of individual coherent episodes of variable duration.

Perceptual circuits give the workspace acces to the present state of the external world. Include the object-oriented ventral and lateral areas of the temporal lobes, and the temporal and inferior parietal areas involved in language comprehension (Wernicke area). The content of any attended object, or communication can accesss the global workspace [2].

The Evaluation Circuits Allow Representations in the Workspace

The evaluation circuits allow representations in the workspace to be associated with a positive or negative value. The main anatomical systems in this respect include the orbitofrontal cortex, anterior cingulate (AC), hypothalamus, amygdala, ventral striatum as well as the mesocortical catecholaminergic and cholinergic projections to prefrontal cortex. Reciprocal projections allow evaluation circuits to be internally activated by the current workspace content (auto-evaluation) and, conversely, to selectively maintain or change workspace activity according to wheter its value is predicted to be positive or negative [8,9].

Gating and processing inhibitory units are classical McCulloch-Pitts units whose activity level S_{INH}, ranging from 0 to 1, obeys the update rule $S^i_{INH} = sigmoid\ (\sum W^{i,j} s^j)$, where the sigmoid function is defined $(x) = 1/(1 + e^{-x})$, and the $w^{i,j}$ are the synaptic weights of neurons contacting inhibitory unit i. Now only the synaptic weights between two excitatory units are assumed to be modifiable according to a reward-modulated Hebbian rule $\Delta w^{post,pre} = \varepsilon R S^{pre} (2S^{post} - 1)$, where R is reward signal provided after each network response:

$(R = +1,\ correct,\ R = -1,\ incorrect)$, pre is the presynaptic unit and post the postsynaptic unit.

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Workspace neuron activity is under the influence of both vigilance and reward signals. The vigilance signal V is treated as having a descending modulatory influence on workspace neurons according to the above-described gating mechanism [10-12].

The reward signal R influences the stability of workspace activity through a short-term depression or potentiation of synaptic weights: $\Delta w^{post,pre} = 0.2(1 - w^{post,pre})$,where w´ is a short-term multiplier on the excitatory synaptic weight from unit pre to unit post.

A Plausible Molecular Implementation of the Chemical Hebb's Rule

A plausible molecular implementation of this rule has been proposed in terms of allosteric receptors. It postulates that the time coincidence of a diffuse reward signal and of a postsynaptic marker of recent neuronal activity transiently shifts the allosteric equilibrium either forward, or from, a desensitized refractory conformation. Through this "chemical Hebb rule", negative reward destabilizes the self-sustaining excitatory connections between currently active workspace neurons, thus initiating a change in workspace activity [13,14].

Brain-imaging experiments indicate that dorsolateral prefrontal cortex and AC (anterior cingulate) posses these properties during activities such as mental calculation. They also correlate with subjective conscious perception. Contribution of horizontal, long-distance connections in establishing a coherent workspace, a dense network of connections linking dorso-lateral prefrontal and inferior parietal areas to anterior and posterior cingulate, temporal cortices, and parahipocampal cortices has been identified. It may support the interconnection of the workspace to high-level perceptual, motor, memory, attentional, and evalutional circuits.

The workspace neurons are the specific targets of projections from neuronal structures that provide reward and vigilance inputs, presumably via specialize neurotransmitter pathways. Mesocortical dopaminergic neurons and cholinergic pathways in particular, are known to differentially target prefrontal cortex. The decoding of such signals by workspace neurons may be affected by specific subtypes of neurotransmitter receptors [15-17].

The Implementation of A Global Computational Workspace

The implementation of a global computational workspace operating under conditions of selection by reward is one possible description of a "conscious workspace". The inclusion of novelty detection mechanisms, presumably implemented in the hippocampus, which may serve as input to workspace units, and the connection to the workspace of self-representations that might allow the simulated organism to reflect on its own internal processes [2,18-26].

The emergence of high-frequency synchronous firing in the thalamocortical system depends critically on the dynamics of corticothalamic and corticocortical reentrant circuits (Edelman) and on the opening of voltage-dependent channels in the horizontal cortico-cortical connections [27-34]. A possible approach to measuring differences that make a difference within an integrated neural system is to consider it as its own "observer". This can be achived by dividing the system into two subsets (hemispheres, areas) reciprocally monitoring one another and then measuring their mutual information [35-41].

Consider a j th of k elements subset (X_j^k) taken from isolated neural system X, and its complement $X - X_j^k$ Interactions between the subset and the rest of the system introduce statistical dependence between the two. This is measured by their mutual information MI $(X_j^k, X - X_j^k) = H((X_j^k) + H(X - X_j^k) - H(X),$ which captures the extent to which the entropy of (X_j^k) is accounted for by the entropy of $X - X_j^k$ and vice versa (H indicates statistical entropy).

The value of mutual information $MI(X_j^k, X - X_j^k)$ between a j th subset X_j^k of the isolated system X and its complement $X - X_j^k$ will be high if two conditions are met. Both X_j^k and $X - X_j^k$ must have many states (their entropy must be relatively high, and the states of X_j^k and $X - X_j^k$ must be statistically dependent (the entropy of X_j^k must be accounted for by interactions with $X - X_j^k$ and vice versa. The expression MI (X_j^k , $X - X_j^k$) reflects how much, on average, changes in the states of $C_N(X) = 1/2\sum MI(X_j^k, X - X_j^k)$ make a difference to the state of X_j^k and vice versa [42-44]. To obtain an overall measure of how differentiated a sytem is, we can consider not just a single subset of its constituent elements, but all its possible subsets [45-53].

The corresponding measure, neural complexity, is given by $C_N(X) = 1/2\sum MI(X_j^k, X - X_j^k)$ where the sum is taken over all k subset sizes and the average is taken over all jth combinations of k elements. Complexity is thus a function of the states of a system that result from interactions among its elements [20-22].

If V is a physical system in interaction with another system W. The standard state vector $\Gamma(t)$ of the two interacting systems has a set of polar decompositions: $\Gamma = \Sigma kqk\varphi k \otimes \phi k$, with the qk complex. These are parametrized by the right toroid T of amplitudes $q=(qk)k$ and comprise a singular bundle over S, the enlarged state space of U=V+W. The right toroids arise as the amplitudes in polar decompositions, the fibers of the polar synaptical bundle. The main diagonal circle (or subgroup) plays a central role. For any t ($p_i p_2\ldots$), we call a translate t. $D \subset T(r)$ of a diagonal [23-25].

The evolution of q is determined via the connection on this bundle. Each fieber T (axons, dendrites) has a unique natural convex partition $(p_i p_2\ldots)$ yielding the correct probabilities, since the circle of unit vectors which generate the ray corresponding to the state of Γ intersects p_j in an arc of length $|q_j|^2$. In this interpretation, rays in Hilbert space correspond to ensembles, and unit vectors in a ray correspond to individual member plane waves of such ensemble [26-28].

We start from a total system S and consider a subsystem S_1 together with its complement S_2 symetrically. The state of a composite system $S=S_1+S_2$ consist of an element of the fieber P_1 above vector Γ in the unit sphere S of H. This fieber can be thought of as the set all the possible complex polar docomposition of Γ:

$$\Gamma = \frac{\Sigma}{k} qk\Gamma k$$

where the $qk \in C$ and the Γk are bi-orthonormal. Thus each Γk is of the form $\varphi k \otimes \varphi k$ where the φk and the φk are orthonormal [29,30].

The polar decompositions of Γ forms a right toroid T (r,Γ) isometric to $\Pi k^{s^1}(r_k)$, the product of circles with radii $r_k := |q_k|$. The additional information they carry about the state of S beyond Γ is a compounding of the extra phase data utilized in the first step of refining the ray a unit vector Γ in the ray. The polar bundle P can be regarded as the natural amalgamation of the polar decomposition with the Hopf bundle S→P. The state represented by an element of P is a state of the composition $S=S_1+S_2$, it depends on all three of S, S_1, S_2, although any two of them

determine the other. The contextuality of global workspace refer to these states as polar states of $S=S_1+S_2$ [31-33].

The Mathematical Theory of Consciousness

In our mathematical theory of consciousness are ensembles idealized objects which can be realized in good approximation, by weak beams of correlations. The irrefinable ensembles or pure states S of S correspond to the set P (H) of rays in H, with the transition propbability given by $|\langle \alpha, \beta \rangle|^2$ for unit vectors α, representing pure states. We can also extend the previous pure version where the rays in Hcorrespond to the 1-dimensional othogonal projections of correlations on them [34-36].

The paper of Kochen [49] was the first to define a pure state of an interacting system as a spectral ray $[\varphi k]$ of the polar decomposition. Our main goal is to find consistent models of the individual systems such as single atoms. The mathematical formalism must allow modeling of individuals as well as statistical ensembles. In this term $[qk]^2$ is the probability that an individial is in the state $[\varphi k]$. In applications, it is interpretted, as usual, in terms of the relative frequency of an ensemble of systems all in state $[\varphi]$. This subjective interpretation of probability is consistent. The heart of the problem is to consistently attribute a pure state to an individual system, including one such as S_1 which is a subsystem of S interacting withS_2, and then to mathematically model these pure states [37-40].

The brain seems to have internal theories about what the world is like. It then uses sensory input which to be patchy and disorganized to choose between these. In some sensory situations, different theories (unit vectors, plane waves) come into conflict, sending our perceptions awry. In a case of motion-induced blindness, catches the brain ignoring or discarding information. This may be one of the brains useful tricks. The illusions results from a tussle for supremacy between the left and right halves of the brain. The left hemisphere seems to supress sensory information that conflicts (collision operators) with its idea of what the world should be like, the right sees the world how it really is. The brains theories about what the world should be like seem to emanate from a region of the parietal lobe. Between brain´s mathematical theories is a internal perceptual rivalry in Darwinian sense [41-43].

Kurt Gödel has pointed out, that we do have something like a perception of the objects of set theory. The axioms force themselves upon us being true. This kind of perception is mathematical intuition, which in sense of perception induces to build up theories. Even our ideas referring to physical objects contain constituents qualitatively different from sensations, which is the idea of object itself. The given underlying mathematics is closely related to the abstract elements containd in our empirical ideas. The data of this second kind, are something purely subjective, and they too may represent an aspect of objective reality (Cabala-as mode of biodigital perception, Tikkun-as quantum error correction). Their presence in us may be due to another kind of relationship between ourselves and reality [51,52].

System of Consciousness can be Described by the Polar Decomposition

Any quantum system we can describe due to the polar decomposition as a whole system given by two subsystems, from which each one is observing another. During this observation they are becoming to interaction by changing their mutual information. After that process of observation reentry they could reech a level of minimal complexity allowing becoming of informational, i.e.

mathematical consciousness. The mode of perception is based on mathematical structure of mutual observing between two subsystems, realizing an interchange of the dynamical flows of correlations in the meaning of the consciousness. The flows of correlations in prefrontal cortex and thalamocortical, etc. structures are networking the human consiousness. The consciousness is a mathematical structure, with neurobiological semantics. The consciousness is a mathematical structure, arrising from the input of the mathematics as a mode of perception between two (or more) subsystems of the system. The core function of what is called „consciousness" is the mutual information carried by the complex flows of correlations through given neuronal structures [44-47].

In Erwin Schrödinger´s book What Is Life ? [43] the entropy production $d_i S$ is compensated by entropy flow $d_i S + d_e S = 0$, or $d_e S = d_i S < 0$. Life, concludes Schrödinger, feeds on a „negative entropy flow". Far from equilibrium they become mechanism dependent. Far from equilibrium the correlations acquires new properties, achives long-range character. New processes set in and increase the production of entropy. More closely we see a critical effect of fluctuations, far from equilibrium they play a central role. The system chooses one from the possible branches available far from equilibrium. But nothing in the macroscopic equations nothing justifies the preference for any solution. This introduce an irreducible probabilistic element, for example the so-called „pitchfork bifurcation". It is the fluctuations decide which branch will be selected. At the bifurcation point, the termodynamic branch becomes unstable, and the two new solutions b_1 and b_2 emerge. There are still many states available to the system among which it chooses. To describe this approach to consciousness, we need the idea of correlation. Consider a probability distributions $p(x_1, x_2)$ depending on variables x_1, x_2. If x_1 and x_2 are independent, we have the factorization $p(x_1, x_2) = p_i(x_i)P_2(x_2)$.

The Flow of Correlations in the Brain as a Flow of Global Workspace Communication

The probability $p_i(x_i)P_2(x_2)$ is the product of two probabilities. If $p(x_1, x_2)$ cannot be factorized, x_1 and x_2 are correlated. The collisions between elements produce correlations. By collisions a binnary correlations are then transformed into ternary, and so on. We now have a flow of correlations. But in the opposite to Peter Lynds interesting article on time flow (54), we see the exact solution for each individual element in the mental flow of correlations. The flow of correlations in the brain is realised (as in society) as flow of global workspace´s communication, which is subsequently modify the brain´s conscious thinking. If the associations, mental image´s waves, etc., interact with the obstacle, the collisons varies the velocities and creates correlations between the particles and the obstacle. The opposite process is caused by the effect of a velocity inversion, as a result of the inverted collison. The correlations with the obstacle are destroyed, and the initial velocity is recovered. In the thermodynamical description is important the role of Poincaré resonances, and the description will be a dynamics of correlations leading to the equilibrium distribution as the actual self-consciousness [48-50].

The Perron-Frobenius operator admits a trajectory description $\delta(x - x_{n+1}) = \cup(x - x_n)$, which allows new solutions applicable only to statistical ensembles. In the systems with Poincaré resonances the flow of correlations is an essential element in the new solutions for the probability distributions. The Perron-Frobenius operator U transforms the probability distribution $p_n^{(x)}$ into $p_{n+1}^{(x)}$ concluding that there exist new solutions. Hilbert space can be described as a dynamical group broken into two semigroups [54,55].

The theory of Fourier integrals shows that a function of the coordinate x as $f(x)$ can be expressed as a superposition of periodic functions corresponding to wave vectors k, or as a superposition of plane waves e^{ikx}. In this superposition, each plane wave is multiplied by an amplitude $\phi(k)$, which is a function of k. The function $\varphi(k)$ is the Fourier transform of $f(x)$ The function $p(x_1, x_2)$ of coordinate x means a description $\phi(k)$ in wave vectors k. There is a kind of duality between $f(x)$ and $\phi(k)$ [56-58].

Solution of the Equation of Motion for the Free Association-Wave

The eigenfunction exp (ikx) is a periodic function, or plane wave, since exp $(ikx)=cos\ kx+i\ sin\ kx$. The solution of the equation of motion for the free association-wave, is due the statistical description through a superposition of plane waves. Using the theory of Fourier transform, we can reconstruct the trajectory starting with plane waves. Beacause the trajectory is at one point, we must superpose plane waves over the entire length of spectral interval (54). For $q=q_0$, the amplitudes of plane waves increase through constructive interference, and for $q \neq q_0$, they vanish through destructive interference [54,55].

In the statistical description is important to replace the coordinates, which depend on the distribution functions through their Fourier transform, with wave vectors as they appear in the spectral decomposition of the Liouville operator. The momenta p of interacting waves are modified (modification of the binary correlation p_2 between waves j and n. In the propagation diagram a dynamical event corresponding to the interaction of two waves leads from wave vectors k_j, k_n to k'_j, k'_n.

Poincaré Resonances between Frequencies Lead to Divergences With Small Denominators

The frequency of a wave of momentum p is kp/m, where k is a wave vector. Poincaré resonances couple the creation and destruction of correlations, and lead to diffusion [38].

Population dynamics is using an ensemble approach. As pointed by G. M. Edelman, the population thinking is the essence of Darwinism. An ensemble is represented by a cloud of points in phase space (Peter Lynds). The cloud in brain map is described by a function $p(q,p,t)$, which is the probability of finding at a time t, a point in the region of phase space around the point q,p A trajectory corresponds p vanishing everywhere except at the point q_0, p_0. Function that have the property of vanishing everywhere except a single point, are so called Dirac delta functions. The function $\partial(x-x_0)$ is vanishing for all points $x \neq x_0$. The distribution function p get the form: $p = \partial(q-q_0)\partial(q-q_0)$.

Poincaré resonances between frequencies lead to divergences with small denominators. The frequency of a wave of a momentum p is kp/m, where k is the wave vector. If k is a continuos variable, we can avoid the divergences and express the resonances in terms of ∂-functions. This involves a branch of mathematics associated with analytical continuation (Borel-sets, and Peter Lynds). For the two body process, the argument of the ∂-function is $k/m(p_1-p_2)$, leading to contributions whenewer the frequencies kp_1/m and kp_2/m are equal, and otherwise vanishing. A vanishing wave vector k corresponds to an infinite wavelength, and thus to a process which is delocalized in space. There are playing a role the new elements called collision operators. They act on the global workspace´s distribution functions in the brain.

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