

# Mathematical Modeling of Airflow on Respiratory Narrowing Due to Mumps Using Finite Element Method

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## Abstract

Mumps can cause narrowing of the airway respiratory when the swelling occurs toward the chest because it can suppress the airways (trachea). This research will be constructed a mathematical model airway on respiratory narrowing due to mumps and solve it by using finite element method. The main problem in this research, how the results of simulation analysis airflow velocity airway on respiratory narrowing due to mumps are influenced by the size of radius of respiratory tract and the initial velocity. After the simulated, will be searched a relative error to be compared with the value of tolerance. Based on the simulation result using MATLAB found that the first simulation influence the size of radius with an initial velocity 0.4 m/s during normal condition ( $r=0.025$  m) velocity in the narrowing airway 0.4010 m/s. If  $r=0.020$  m and  $r=0.015$  m the velocity will be increased being 1.0060 m/s and 3.2199 m/s with relative error value 0.005964. The second simulation an initial velocity and  $r=0.01631$  m and velocity are 0.2 m/s, 0.4 m/s and 0.6 m/s found that the velocity in the narrowing airways increase being 1.0024 m/s, 2.2982 m/s and 4.0929 m/s with relative error value 0.002394.

**Keywords:** Respiratory narrowing; Mumps; Finite element method; Mathematical modeling

## Introduction

Math is used as an important tool in many fields, including the natural sciences, economic sciences, even in medical science [1]. Medical science is a science which includes knowledge about the system of the human body, diseases and treatment and the application of such knowledge [2]. Many branches of the discipline of medicine that studies the structure and function of organs, one of them is about the thyroid gland [3].

The thyroid gland is located in front of the right in the bottom of the cricoid cartilage in the left and right side of the trachea [4]. In adults weighs approximately 18 grams [5,6]. Under normal conditions, the performance of the thyroid gland tends to not we realize as well as other internal organs [7-9]. But if there is a swelling of the thyroid gland, will form a lump in the neck. The usual conditions we know with mumps (goiter).

Reported in 2009 in the United States, cases of Goiter are found on a number of more than 250,000 patients. The disease occurs mainly in women than men. Generally 95% cases of mumps are benign (benigna), the remaining 5% of cases may be malignant (maligna). According to the WHO, Indonesia is a country that categorized endemic goiter incidents.

However, not all experience symptoms of mumps. Indication appearing is the existence of abnormal lumps on the neck with a lump the size varies for each patient. Small-sizes lump usually does not cause any symptoms, with a note is when the swelling that occurs against the chest because it could suppress the airway (trachea). This can cause constriction of the airway.

The development of the mumps can be categorized in several categories (Table 1).

On this research will be constructed a mathematical model that is constructed and analyzed based on the partial differential equations which presented the laws of physics like the momentum equation would then be resolved and simulated with MATLAB and FLUENT.

The momentum equation is formed under the laws of momentum viscosity derived through differential equations. The equations of the model of respiratory tract constriction due to mumps are shown in the following equation:

$$\frac{\partial}{\partial t} \rho u_j + \nabla \rho u_j u_j = -\nabla P + \rho \nabla g_i + \nabla \tau_{ij} \quad (1)$$

$$\text{with } Q = \frac{\pi Pr^4}{8\eta l} \quad (2)$$

because model of the fluid flow is one dimension that is aligned with the y-axis, then the voltage style substance use  $\tau_{yy}$ , where the value of the voltage is

$$\tau_{yy} = 2\mu \frac{\partial \phi}{\partial y} \quad (3)$$

where

$Q$ =volumetric flow ( $7.5\text{m}^3/\text{s}$ );  $\rho$ =density ( $1.34\text{ kg/m}^3$ );  $\eta$ =viscosity ( $1.725 \times 10^{-7}\text{Pa s}$ );  $\mu$ =friction coefficient ( $0.15 \times 10^{-5}$ );  $g$ =force of gravity

( $9.8\text{m/s}^2$ );  $l$ =length (m);  $v$ =velocity (m/s);  $P$ =pressure (Pa);  $r$ =radius (m)

After getting the momentum equation, then the equation is solved using finite element method. Finite element method is a numerical method an extremely popular in the resolution of problems mechanics continuum, voltage, strain, force and displacement on the structure that is linear and nonlinear [2].

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$$E\{q\} + [E_t]\{\dot{q}\} = \{Q\} \tag{4}$$

with

$$[E] = 2\mu \int_0^l [B]^T [B] \delta y - 2\mu [N]^T [B] \Big|_0^l + \rho v \int_0^l [N]^T [B] \delta y$$

$$[E_t] = \rho \int_0^l [N]^T [N] \delta y$$

$$[Q] = -P \int_0^l [B] \delta y + \rho g \int_0^l [B] \delta y$$

where

$$[B] = \left[ \left( \frac{4y}{l} - \frac{3}{l} \right) \left( \frac{4}{l} - \frac{8y}{l^2} \right) \left( \frac{4y}{l^2} - \frac{1}{l} \right) \right]$$

$$[N]^T = \left[ \left( 1 - \frac{2y}{l} \right) \left( 1 - \frac{y}{l} \right) \frac{4y}{l} \left( 1 - \frac{y}{l} \right) - \frac{y}{l} \left( 1 - \frac{2y}{l} \right) \right]$$

Next will be completed in time, the time in the global matrix can be written:

$$[K]\{r\} + [K_t]\{\dot{r}\} = [R] \tag{5}$$

with

$[K], [K_t]$  = matrix assembly properties

$\{r\}$  = vector of magnitudes of an unknown node

$[R]$  = vector of assembly force parameter

Global matrix equations will be resolved over time, where it will be used in the completion of a different approach to Crank-Nicholson pattern. Global matrix constituent equation can be written in finite different form:

$$[K](\theta\{r\}_{t+\Delta t} + (1-\theta)\{r\}_t) + [K_t] \left\{ \theta \left\{ \frac{\partial\{r\}}{\partial t} \right\}_{t+\Delta t} + (1-\theta) \left\{ \frac{\partial\{r\}}{\partial t} \right\}_t \right\} = \{R\}_{t+\Delta t} \tag{6}$$

next taken  $\theta = \frac{1}{2}$ , so to be obtained pattern Crank-Nicholson with a pattern approaches:

$$\frac{1}{2} \left\{ \left\{ \frac{\partial\{r\}}{\partial t} \right\}_{t+\Delta t} + \left\{ \frac{\partial\{r\}}{\partial t} \right\}_t \right\} \cong \frac{\{r\}_{t+\Delta t} - \{r\}_t}{\Delta t} \tag{7}$$

will the global matrix equation obtained over time :

$$\left( [K] + \frac{2}{\Delta t} [K_t] \right) \{r\}_{t+\Delta t} = 2\{R\}_{t+\Delta t} - \left( [K] - \frac{2}{\Delta t} [K_t] \right) \{r\}_t \tag{8}$$

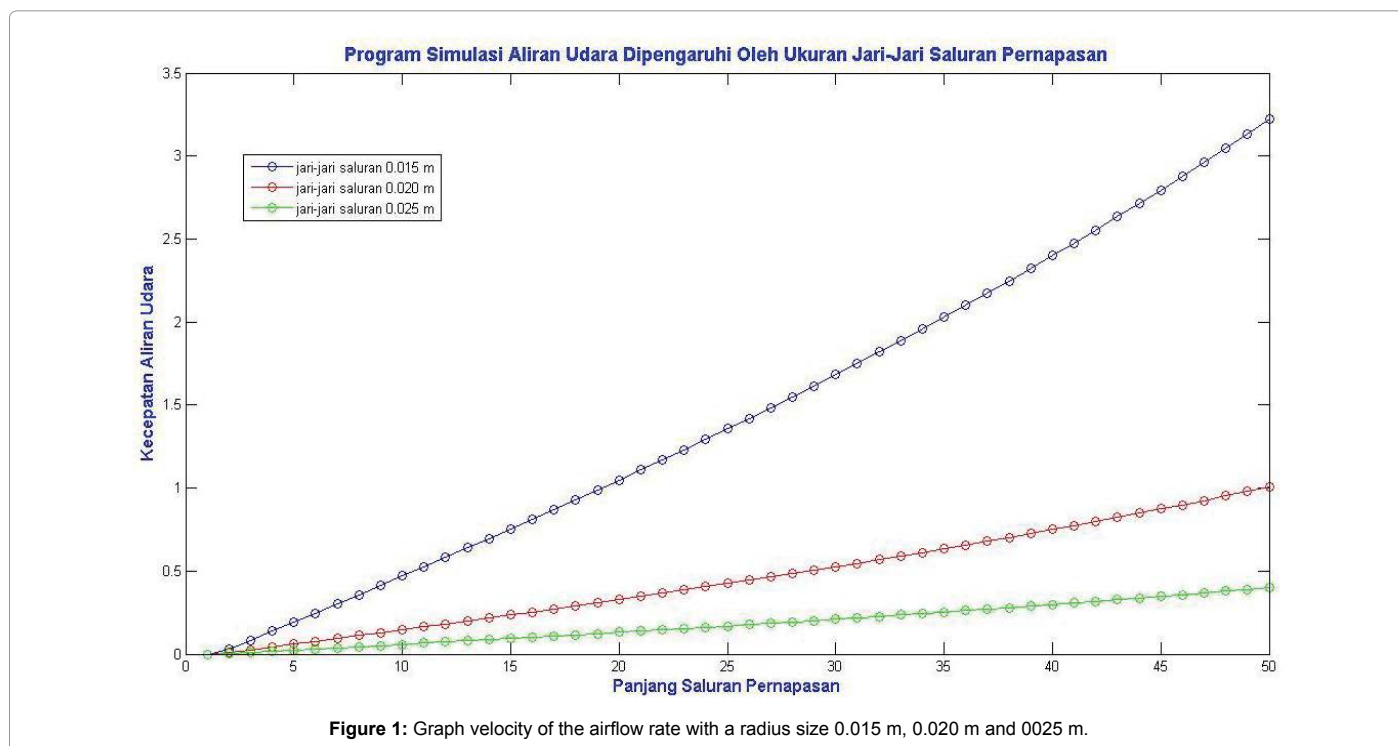
### Research Method

A research procedure is a series of research activities conducted up to gained an outcome or data that will be analyzed to reach a conclusion. Procedures performed on this research are resolved mathematical model on respiratory narrowing due to mumps using finite element method which will then be resolved by MATLAB program. After the simulated will then look for error values are compared with the values of the tolerances (limits of error). Tolerance is used to find out the effectiveness of the method element to complete mathematical model. After get the result of numerical of mathematical model next will be analyze the results, and then do a simulation of FLUENT. Before doing the simulation of FLUENT, first make the design geometry and boundary conditions determine the flow using GAMBIT and FLUENT simulation results analyzing.

### Results and Discussion

After get the result of numerical of mathematical model on respiratory narrowing due to mumps using finite element method will then be resolved by MATLAB program with substitution the coefficients are known. The results obtained in the form of graphs and tables of the airflow velocity in the followings.

Figure 1 is the graph of numerical calculation of results the speed



of air flow on the constriction of the airway due to mumps that is affected by the size of the radius of the respiratory tract by using the MATLAB program. On Table 1 and Figure 1 the input entered with radius size 0.015 m, 0.020 m and 0.025 m with initial speed of 0.4 m/s. It can be seen graph velocity of airflow with radius 0.025 m i.e., at a time when normal conditions or the presence of airway constriction of the airflow velocity tends to be constant i.e., 0.4010 m/s. Whilst on the tract narrowed so that the radius of the tract be 0.020 m and 0.015 m speed of the airflow is increased successively became 1.0060 m/s and 3.2199 m/s. On the graph visible distinction flow velocities in three sizes of fingers that line, the smaller the size of the radius of respiratory airflow speed then the greater when approaching an area constriction. With the level of tolerance of relative error value obtained 0.01 is 0.005964.

Next is the results of numerical calculation of the speed of the airflow at the constriction of the airway due to mumps that is affected of the initial velocity using the MATLAB program.

In Figure 2 and Table 2 the input entered radius of tract 0.01631 m there is a difference between velocity of the flow. With an initial

Size of Radius of The Respiratory Tract	Velocity of The Airflow	
	Initial (m/s)	Area of Narrowing (m/s)
0.025 m	0.4	0.4010
0.020 m	0.4	1.0060
0.015 m	0.4	3.2199

Table 1: The velocity of the airflow at the constriction of the airway due to mumps which affected the size of the radius of the respiratory tract.

Size of Radius of The Respiratory Tract	Velocity of The Airflow	
	Initial (m/s)	Area of Narrowing (m/s)
0.01631 m	0.2	1.0024
0.01631 m	0.4	2.2982
0.01631 m	0.6	4.0929

Table 2: The velocity of the airflow at the constriction of the airway due to goiter that is affected by the velocity of the initial.

velocity of 0.2 m/s flow velocity of air is increased to 1.0024. Whereas with an initial speed of 0.4 m/s velocity also increased until 2.2982 m/s. similar things also occur in the initial speed 0.5 m/s, increased his speed getting bigger until 4.0929 m/s that is, with the increasing speed of the start is given then the flow rate on the narrowing areas will also be increased. With the level of tolerance of relative error value obtained 0.01 is 0.002394.

On simulation with FLUENT simulations will be performed twice. The first simulation, the simulation is done using three GAMBIT that has different radius, i.e., at the time of the respiratory tract under normal circumstances with a radius 0.025 m, narrowed breathing passages with 0.020 m and 0.015 m and with an initial speed of 0.4 m/s.

In Figure 3a is a simulation of the speed of the airflow at the respiratory tract under normal circumstances with fingers channel 0.025 m. on the simulation of the image can be seen that the flow velocity at each point of the tract tends to a constant incoming flow rate means the same as the speed of the exit. It can be shown with blue almost every parts of the respiratory tract. While in Figure 3b and 3c images are simulations of flow velocity on the tract is narrowed by the size of the radius of the tract in a row is 0.020 m and 0.015 m, it can be seen that the velocity of flow increases when heading to the area of narrowing and speed back on the conditions of entry away from the area of narrowing. In Figure 3b from blue to green even in the area of narrowing of the yellow color of approaching turns in Figure 3c in the area of constriction turned into an orange and red. Change of speed can be seen on the numbers printed on the indicator tint that is on the left side of the image.

The second simulation, i.e., to know the speed of the airflow at the respiratory tract narrowed if affected early speed. The radius of the respiratory tract in this simulation is 0.01631 m and an initial velocity to be used is 0.2 m/s 0.4 m/s and 0.6 m/s.

In Figure 4a at the time of entering the tract is marked with blue color and then changed to green to orange when it is in the constriction (Table 3). In Figure 4b flow velocities greater than that of previous

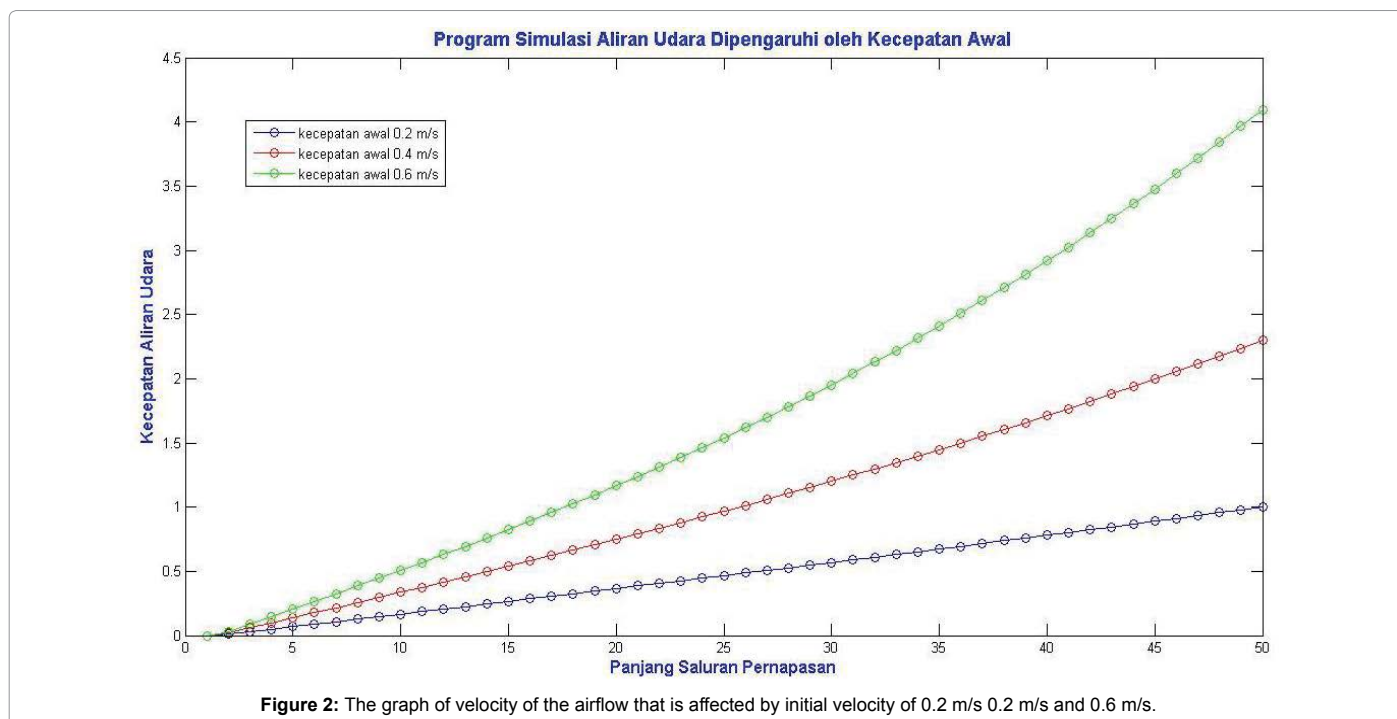


Figure 2: The graph of velocity of the airflow that is affected by initial velocity of 0.2 m/s 0.2 m/s and 0.6 m/s.

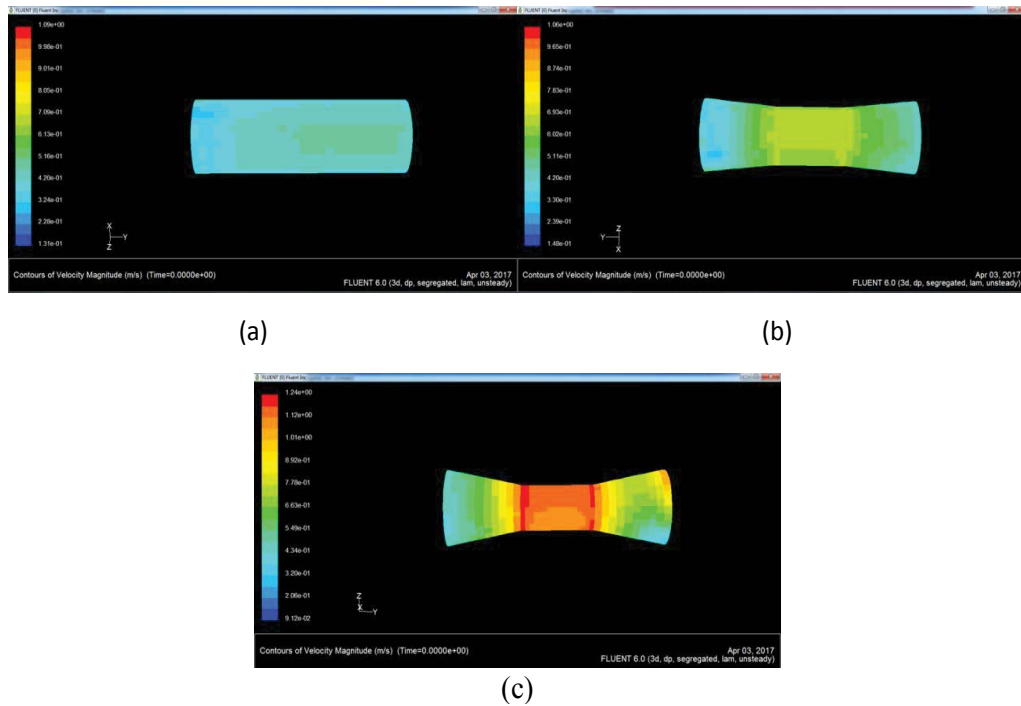


Figure 3: Simulation of velocity air flow with radius tract are (a) 0.025 m (b) 0.020 m dan (c) 0.015 m.

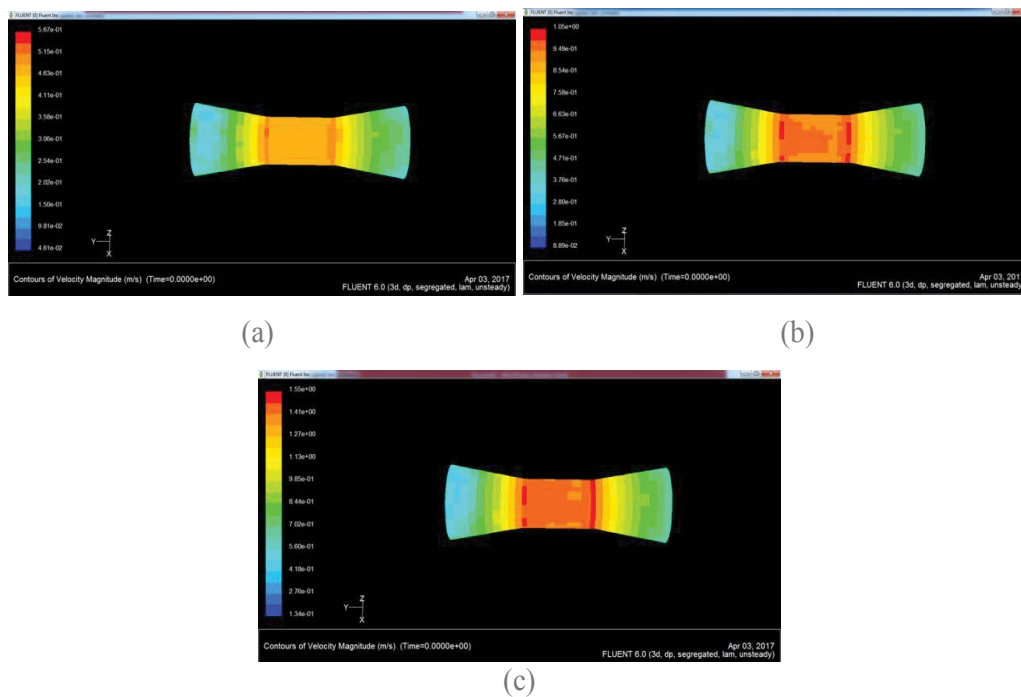


Figure 4: Simulation of velocity air flow with initial velocity are (a) 0.2m/s (b) 0.4m/s dan (c) 0.6 m/s.

No.	Grade	Description
1.	0 (Normal)	Mumps is palpable or invisible
2.	1	Mumps is palpable but invisible when the neck in the normal (invisible enlarged thyroid)
3.	2	Enlargement in the neck which is clearly visible on the neck in its normal position and on palpation indeed enlarged (the thyroid gland is considered large when enlarged lateral lobe of each more than the volume of the patient's thumb phalanges)

Table 3: Development of the Mumps.

simulation it can be seen when entering the channel marked with the blue color which then turns into the color of green, yellow and even red. Whereas, in Figure 4c on the results of a simulation of the change of the color more dominant while in the constriction. That is, with the increasing speed of the start is given then the flow rate on the narrowing areas will also be increased. Change of speed can be seen on the numbers printed on the indicator tint that is on the left side of the image.

## Conclusions and Suggestions

Based on the results of the research that's been done, then the conclusions obtained that the first simulation influence the size of radius with an initial velocity 0.4 m/s during normal condition ( $r=0.025$  m) velocity in the narrowing airway 0.4010 m/s. If  $r=0.020$  m and  $r=0.015$ m the velocity will be increased being 1.0060 m/s and 3.2199 m/s with relative error value 0.005964. The second simulation an initial velocity and  $r=0.01631$  m and velocity are 0.2 m/s, 0.4 m/s and 0.6 m/s found that the velocity in the narrowing airways increase being 1.0024 m/s, 2.2982 m/s and 4.0929 m/s with relative error value 0.002394.

The advice that can be given by researchers after doing research about numerical analysis on air flow constriction of the airway due to mumps is developed by analyzing the research of other factors can also

affect the velocity of the flow for example pressure due to enlargement of the mumps and others.

## References

1. Yustini A (2010) Hubungan Penyakit Gondok dengan Kadar Yodium dalam Urin Murid Madrasah Ibtidaiyah Negeri (MIN) Korong Gadang Kecamatan Kuranji Kota Padang. *Majalah Kedokteran Andalas* 34: 184-192.
2. Anwar D, Hasan H (2010) Metode Elemen Hingga dengan MATLAB dan Aplikasi SAP 2000 untuk Analisis Struktur Cangkang. *SMARTek Jurnal* 8: 71-80.
3. Arif F (2010) Pemodelan dan Penyelesaian Numerik dari Permasalahan Korosi Besi yang Didasarkan pada Sifat Kimia Larutan. *KadikMa* 2: 71-80
4. Arif F (2013) Analisis Penyebaran Aliran Panas Pada Oven Surya Berbantuan Software Fluent. *KadikMa* 4: 1-8.
5. Arif F (2013) Pemodelan dan Penyelesaian Numerik dari Permasalahan Penyebaran Asap Menggunakan Metode Volume Hingga. *Saintifika* 15: 1-7.
6. Dafik D, Arif F, Hardiyanti SA (2015) Analisis Kecepatan Aliran Hidrogen Peroksida ( $H_2O_2$ ) pada Sterilisasi Saluran Akar Gigi Menggunakan Metode Numerik Volume Hingga. *KadikMa* 6: 13-26.
7. Kusminarto (2007) Fisika: Penerapannya dalam Bidang Medis. Yogyakarta: UGM.
8. Drolia M, Mohamed MS, Laghrouche O, Seaid M, Trevelyan J (2017) Enriched finite elements for initial-value problem of transverse electromagnetic waves in time domain. *Elsevier Inc.: Computers and Structure* 82: 354-367.
9. Frank WM (1998) Mekanika Fluida. Jakarta: Erlangga.