Magnetohydrodynamic (MHD) Stability of Oscillating Fluid Cylinder with Magnetic Field

Barakat HM*

Department of Mathematics, Faculty of Science and arts, Al Jouf University, Al Jouf, The Kingdom of Saudi Arabia

Abstract

The magnetohydrodynamic (MHD) stability of oscillating fluid with longitudinal magnetic field has been discussed. The problem is formulated and the (MHD) basic equations are solved. By using the computer procedure from different values of the acting magnetic field the stable and unstable regions are identified. This phenomenon is interest, academically and during the geological drilling in the crust of the earth as we have superposed gas-oil layer mixture fluids. A general eigenvalue relation is derived studied analytically and results are confirmed numerically. The oscillating liquid has stabilizing tendency, in the absence of the effect of the electromagnetic field in the liquid and gas cylinder region, so the model is only subject to the capillary force. It has been found that the model is unstable in the region $0 < x < 1$, while it is stable in the region $1 \leq x < \infty$ where x is the longitudinal dimensionless wave number. This means that the model is just unstable in small domains of axisymmetric perturbation but it stables in all domains. For very high intensity of magnetic field the model is completely stable for all values of wavelengths. The capillary force is destabilizing only in a small axisymmetric domain while it is stabilizing in all other axisymmetric perturbations. The stability behavior of the model comes after destabilizing behavior of the model when it be reduced and suppressed.

Keywords: Magnetohydrodynamic; Magnetic field; Electromagnetic field; Fluid cylinder

Introduction

The classical of the capillary instability of a gas cylinder submerged into a liquid are given for first time by Chandrasekhar[1] for axisymmetric perturbation. Hasan [2], Elazab et al. [3] and Drazin and Reid [4] gave the dispersion relation valid for all axisymmetric and non- axisymmetric modes. Cheng [5] discussed the instability of a gas jet in an incompressible liquid for all modes of perturbation. However, we have to mention here that the results given by Cheng [5]. Kindall [6] performed experiments with modern equipment to check the breaking up of that model. Moreover, he attracted the attention for the importance of the stability and discussions of that model for its application in many domains of science. Concerning the hydrodynamic stability of a hollow jet endowed with surface tension we may refer to Chauhan et al.[7], Abramowitz and Stegun [8], Chen and Lin [9], Cousin and Dumouchel [10], Lee and Wang [11,12], Mehring and Sirignano [13], Parthasarathy and Chiang [14], Shen and Li [15], Shi et al. [16], Shukudov and Sisoev [17], and Villermaux [18], Melachlan and Kelly [19,20]. Soon afterwards a lot of researchers treated with the magneto-dynamic stability of such model analytically [21-23] and numerically upon utilizing appropriate basic equation and boundary condition. Hamdy M Brakat and Khloud R Khater studied the Hydromagnetic self-gravitating stability of streaming fluid cylinder with longitudinal magnetic filed [24]. It all foregoing works the liquid may be at rest or unform streaming in the unperturbed state. Here we explain the hydromagnetic stability of oscillating fluid cylinder with longitudinal magnetic field.

Formulation of the problem

We consider a fluid cylinder (with negligible motion) of radius R_0 surrounded by an oscillating liquid cylinder with velocity,

$$\underline{U}_0 = (0, 0, U \sin \Omega t), \tag{1}$$

Where U and Ω are the amplitude and oscillation frequency of the velocity.

The interior cylinder is being a gas with constant pressure P_0^g and pervaded by the longitudinal magnetic field,

$$\underline{H}_0^g = (0, 0, \alpha H_0), \tag{2}$$

and the liquid is penetrated by magnetic field

$$\underline{H}_0 = (0, 0, H_0), \tag{3}$$

where H_0 is the intensity of the magnetic field and α is the parameter of H_0^g. The components of equations (1 - 3) are considered along the cylindrical polar coordinates (r, Φ, z) with the z-axis is coinciding with the axis of the hollow jet. The model is acted by the inertia, pressure gradient, capillary and electromagnetic forces. The hydro-magnetic fundamental equations appropriate for studying the stability of the fluid model under consideration are the combination of the pure hydrodynamic equations and those of Maxwell concerning the electromagnetic theory.

These equations may be given as follows in the liquid region.

$$\rho(\frac{\partial}{\partial t} + (\underline{u}.\nabla))\underline{u} = -\nabla P + \mu(\underline{J} \times \underline{H}), \tag{4}$$

$$\underline{J} = \nabla \times \underline{H}, \tag{5}$$

$$\nabla \times \underline{u} = 0, \tag{6}$$

$$\nabla \times \underline{H} = 0. \tag{7}$$

*Corresponding author: Barakat HM, Department of Mathematics, Faculty of Science and arts, Al Jouf University, Al Jouf, The Kingdom of Saudi Arabia, Tel: +96646257869; E-mail: brakat.hamdy@yahoo.com

$$\frac{\partial \underline{H}}{\partial t} = \nabla \times (\underline{u} \times \underline{H}) \qquad (8)$$

In the region surrounding the gas cylinder, we have

$$\nabla \times \underline{H}^g = 0, \qquad (9)$$

$$\nabla \cdot \underline{H}^g = 0,. \qquad (10)$$

and along the gas- liquid interface the curvature pressure is,

$$P_s = -T(\nabla.\underline{N}), \qquad (11)$$

where $\underline{H}$ and $\underline{H}^g$ are the magnetic field intensities in the gas and liquid regions, ρ is the mass density, $\underline{u}$ velocity vector, P kinetic pressure, and P_s the curvature pressure due to the capillary force, T the surface tension coefficient, $\underline{N}$ is the unite outward normal vector to the gas-liquid interface. In the unperturbed state equation (4) reduces to

$$\rho(\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u}) - \mu(\underline{H}.\nabla)\underline{H} = -\nabla\Pi, \qquad (12)$$

$$\Pi = P + \frac{\mu}{2}(\underline{H}.\underline{H}), \qquad (13)$$

where Π is the total hydromagnetics pressure which is the sum of the liquid kinetic pressure and magnetic pressure, and equation (8) can be rewritten as

$$\frac{\partial \underline{H}}{\partial t} + (\underline{u}.\nabla)\underline{H} = (\underline{H}.\nabla)\underline{u}, \qquad (14)$$

using the equations (4) and (14) we get,

$$P_0 + \frac{\mu H_0^2}{2} = constant, \qquad (15)$$

$$P_{0s} = \frac{-T}{R_0}. \qquad (16)$$

The continuity of the total stress tensor across the gas-liquid interface at ($r = R_0$) yields, the unperturbed pressure P_0 of the liquid as

$$P_0 = \frac{-T}{R_0} + \frac{\mu H_0^2}{2}(\alpha^2 - 1) + P_0^g. \qquad (17)$$

Here P_0^g is the gas constant pressure in the initial state, $(\frac{-T}{R_0})$ is the contribution of capillary force, while $(\frac{\mu H_0^2}{2}(\alpha^2 - 1))$ is the net magnetic pressure due to the effect of electromagnetic force acting in the gas and liquid regions,

Perturbation analysis

For small departures from the equilibrium state due to an infinitesimal perturbation every perturbed quantity Q(r, q, z, t) can be expanded as;

$$Q(r,\varphi,z,t) = Q_0(r) + \varepsilon(t)Q_1(r,\varphi,z) \qquad (18)$$

where the quantities with subscript 0 are those of equilibrium while those with 1 are small increments due to the perturbation. The Q(r, ϕ, z, t) stands for $\underline{H}, \underline{H}^{(g)}$, p and $\underline{U}$, P_s and N and the perturbed radial distance of the fluid cylinder.

The amplitude ε of the perturbation is given by

$$\varepsilon(t) = \varepsilon_0 exp(\sigma t). \qquad (19)$$

Where ε_0 is the initial amplitude at (t=0) and σ is the temporal amplification of instability. Consider a sinusoidal wave along the gas-liquid interface, using a single fourier term, the perturbed cylinder radial distance of the gas jet is described by

$$r = R_0 + R_1, \qquad (20)$$

where

$$R_1 = \varepsilon_0 exp(\sigma t + ikz). \qquad (21)$$

Is the elevation of the surface wave measured from unperturbed position. Here K (any real number) is the longitudinal wave number, and m an integer is the azimuthal wave number. In view of the foregoing expansions (18 - 21), the relevant perturbation equations are given as follows. For the interior of the cylindrical jet

$$\rho(\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0.\nabla)\underline{u}_1) - \mu(H_0.\nabla)\underline{H}_1 = -\nabla\Pi, \qquad (22)$$

where

$$\Pi = P_1 + \mu\nabla(H_0.\underline{H}_1), \qquad (23)$$

and by using equations (6), (7) and (8) we get,

$$\nabla \cdot \underline{u}_1 = 0 \qquad (24)$$

$$\nabla \cdot \underline{H}_1 = 0 \qquad (25)$$

$$\frac{\partial \underline{H}_1}{\partial t} = (H_0.\nabla)\underline{u}_1 - (u_0.\nabla)\underline{H}_1 \qquad (26)$$

In the gas region,

$$\nabla \times \underline{H}_1^g = 0, \qquad (27)$$

$$\nabla \cdot \underline{H}_1^g = 0 \qquad (28)$$

as along the gas-liquid interface,

$$P_{1s} = \frac{-T}{R_0^2}(R_1 + R_0^2 \frac{\partial^2}{\partial z^2})R_1. \qquad (29)$$

Based on the equations (18 - 21) every fluctuating quantity Q_1(r, 0, z, t) could be written as

$$Q_1(r,0,z,t) = Q_1(r)\exp(\sigma t + ikz). \qquad (30)$$

By the aid of this expansion, equations (22) and (24) are combined to give magnetic field intensity in the form,

$$\underline{H}_1 = ikH_0(\sigma + ikU \sin \Omega t)^{-1}\underline{u}_1. \qquad (31)$$

Since the liquid is assumed to be non-dissipative and irrotational, the velocity $\underline{u}_1$ could be derived from a scalar function Φ_1(r, 0, z, t) such that,

$$\underline{u}_1 = \nabla\Phi_1. \qquad (32)$$

Combining equations (24) and (32) we get,

$$\nabla^2\Phi_1 = 0, \qquad (33)$$

by using equation (28) means the magnetic field $\underline{H}_1^g$ could be derive from a scalar functions say Ψ_1^g, such that

$$\underline{H}_1^g = \nabla\Psi_1^g \qquad (34)$$

Combining equations (28) and (34) we get,

$$\nabla^2\Psi_1^g = 0, \qquad (35)$$

by using equations (22) and (31) we find,

$$\underline{u}_1 = \frac{(\sigma + ikU \sin \Omega t)}{\rho(\sigma + ikU \sin \Omega t)^2 + L^2} \cdot \nabla\Pi_1, \qquad (36)$$

where $L = \sqrt{\frac{\mu k^2 H_0^2}{\rho}}$. So by using equations (24) we get,

$$\nabla \Pi = 0. \tag{37}$$

Now, we may see that the system of perturbed equations (22) and (24 -28) could be solved, as Laplace,s equations (33), (35) and (37) are solved for this task, we may write the expansion

$$Q_1(r,0,z,t) = Q_1(r)\varepsilon_0 \exp(\sigma t + ikz), \tag{38}$$

for $\Phi_1(r, 0, z, t)$, $\Psi_1(r, 0, z, t)$ and $_1(r, 0, z, t)$. Substituting equation (38) into equations (33), (35) and (37), we obtain the total second order differential equation of Bessel. For the problem under consideration, the nonsingular solutions of (33), (35) and (37),are given by

$$\Phi_1(r,0,z,t) = C_1\varepsilon_0 K_0(kr)\exp(ikz), \tag{39}$$

$$\Psi_1^g(r,0,z,t) = C_2\varepsilon_0 I_0(kr)\exp(ikz), \tag{40}$$

$$\Pi_1(r,0,z,t) = C_3\varepsilon_0 K_0(kr)\exp(ikz), \tag{41}$$

Where C_1, C_2 and C_3 are constant of integration to be determined, while K_0 and I_0 are Bessel functions of the first and second kind order m = 0. Along the gas-liquid cylinder fluid interface, the surface pressure in the perturbed state due to the capillary force, in view of equations (22) and (29), given by

$$P_{1s} = \frac{T}{R_0^2}(1-x^2)\varepsilon_0 \exp(ikz), \tag{42}$$

Where (x = kR_0) is the longitudinal dimensionless wave number.

Boundary Condition

The solution of the unperturbed and perturbed sates equations (39),(41) must satisfy appropriate boundary condition across the fluid interface at (r = R_0), these conditions are given as follows

Kinematic condition

The first condition states that "the normal component of the velocity $\underline{u}$ of the liquid must be compatible with the perturbed boundary gas-liquid at (r = R_0). This condition is given,

$$\frac{\partial F}{\partial t} + (\underline{u}_0 \cdot \nabla)F = 0, \tag{43}$$

so we can get

$$\underline{u}_{1r} = (\sigma + ikU\sin\Omega t)\varepsilon_0 \exp(ikz + \sigma t), \tag{44}$$

by using equation (39) and (32) we have,

$$\underline{u}_{1r} = \frac{\partial \Phi_1}{\partial r} = C_1 k\varepsilon_0 k_0'(kr)\exp(ikz + \sigma t), \tag{45}$$

then,

$$C_1 = \frac{(\sigma + ikU\sin\Omega t)}{kk'(kr)_0}. \tag{46}$$

The second boundary condition could be described and utilized as follows, by using equations (24) and (31) we have

$$\rho(\frac{\partial u_{1r}}{\partial t} + (ikU\sin\Omega t)u_{1r}) + \frac{\mu H_0^2 k^2 u_{1r}}{(\sigma + ikU\sin\Omega t)} = \frac{-\partial \Pi_1}{\partial r}, \tag{47}$$

where,

$$\frac{-\partial \Pi_1}{\partial r} = -C_3\varepsilon_0 KK'(kr)_0 \exp(ikz + \sigma t), \tag{48}$$

by using equation(48) we obtain,

$$C_3 = \frac{-\rho}{KK_0'(KR)}(\sigma^2 + 2ikU\sigma\sin\Omega t + ikU\cos\Omega t - k^2U^2\sin^2\Omega t + \frac{\mu H_0^2 x^2}{\rho R_0^2}) \tag{49}$$

The magneto-dynamic condition

This condition is being "the normal component of the magnetic field must be continuous across the gas-liquid interface at (r = R_0)",mathematically this condition is given by

$$\underline{N}_0 \cdot \underline{H}_1 + \underline{N}_1 \cdot \underline{H}_0 = \underline{N}_0 \cdot \underline{H}_1^g + \underline{N}_1 \cdot \underline{H}_0^g \tag{50}$$

with $\underline{N}_0 = (1,0,0)$ and $\underline{N}_1 = (0,0,-ik)\exp(\sigma t + ikz)$,// if we put $\underline{N}_0, \underline{N}_1$ in equation (50) we have,

$$\underline{H}_{1r} - ikH_0\exp(\sigma t + ikz) = \underline{H}_{1r}^g - ik\alpha H_0\exp(\sigma t + ikz), \tag{51}$$

where

$$\underline{H}_{1r} = ikH_0(\sigma + ikU\sin\Omega t)^{-1}\underline{u}_{1r}, \tag{52}$$

by using equations (46), (40) and (34) we get,

$$\underline{H}_{1r}^g = \Psi_{1r}^g = C_2 k\varepsilon_0 I_0'(kr)\exp(ikz + \sigma t) \tag{53}$$

substituting in equation (51) we get the value of

$$C_2 = \frac{i\alpha H_0}{I_0'(kr)} \tag{54}$$

Stresses condition

This condition is being" The jump of the normal component of the stresses in the gas and liquid regions must be discontinuous by the surface pressure P_{1s} across the cylindrical gas-liquid interface at (r = R_0) ". This condition is being

$$\Pi_1 + \exp(ikx + \sigma t)\frac{\partial \Pi_0}{\partial r} - \mu(\underline{H}_0^g \cdot \underline{H}_1^g) - \frac{\mu}{2}\exp(ikz + \sigma t)\frac{\partial}{\partial r}(2H_0^2) = p_{1S}. \tag{55}$$

From this equation we get the dispersion relation

$$\sigma^2 + 2ikU\sigma\sin\Omega t + ikU\Omega\cos\Omega t - k^2U^2\sin^2\Omega t + \frac{\mu H_0^2 x^2}{\rho R_0^2} =$$

$$\frac{-T}{\rho R_0^3}(1-x^2)(\frac{xK_0'(X)}{K_0(X)}) + \frac{\mu H_0^2}{\rho R_0^2}(-x^2 + \alpha^2 x^2 + \frac{I_0(x)K_0'(x)}{I_0'(x)K_0(x)}) \tag{56}$$

General Discussion

The relation (56) is the dispersion equation of a fluid cylinder acting upon the electromagnetic force. It relates the growth rate σ with the modified Bessel function $I_0(x)$ and $K_0(x)$ and their derivative, the wave number m and x, the amplitude U of the streaming velocity, Ω the oscillation frequency of the oscillating streaming, α the parameter of the magnetic field in the gas cylinder and with the parameters T, ρ, R_0, μ and H_0 of the problem. One has to mention here that the relation (56) contains $(\frac{T}{\rho R_0^3})$ as well as $(\frac{\mu H_0^2}{\rho R_0^3})$ as unit of (time)^2 and this fact is vital in formulating (56) in dimensionless form. Relation (56) is a simple linear combination of eigenvalue relations of gas-liquid cylinder subject to the electromagnetic force only.It is remarkable that in the axisymmetric mode at m = 0 for a fluid cylindrical jet pervaded by constant axial field, subject to its own attraction and lorentz force. This character of a simple linear combination is also true if the acting force are the capillary and electromagnetic forces. Whether the model are a full liquid-gas cylinder. The physical interpretation of such phenomenon is given explicitly in this paper reference. Since the relation (56) is somewhat more general, several stability criteria can be obtained as limiting cases from the eigenvalue of relation (60). The discussion of this equation reveals that the uniform streaming of the liquid has destabilizing effect, and that effect is valid not only in the axisymmetric mode m = 0.

If we assume that U = 0, then the eigenvalues relation(56) will be

$$\sigma^2 = \frac{-T}{\rho R_0^3}(1-x^2)(\frac{xK_0'(X)}{K_0(X)}) + \frac{\mu H_0^2}{\rho R_0^2}(-x^2 + \alpha^2 x^2 + \frac{I_0(x)K_0'(x)}{I_0'(x)K_0(x)}) \qquad (57)$$

In order to examine the effects of the capillary and magneto-dynamic forces on the stability of the present model, we have to write down about some properties of the modified Bessel functions, consider the recurrence relation

$$I_m'(x) = 0.5(I_{m-1}(x) + I_{m+1}(x)), \qquad (58)$$

$$K_m'(x) = 0.5(-K_{m-1}(x) - K_{m+1}(x)), \qquad (59)$$

by using the Wronskian relation,

$$W_0(I_0(x), k_0(x)) = I_0(x)K_0'(x) - K_0(x)I_0'(x) = -x^{-1} \qquad (60)$$

$$I_0'(x) = I_1(x), \quad K_0'(x) = -K_1(x). \qquad (61)$$

Then the dispersion relation will be

$$\sigma^2 + 2ikU\sigma \sin\Omega t + ikU\Omega \cos\Omega t - k^2U^2\sin^2\Omega t =$$

$$\frac{-T}{\rho R_0^3}(1-x^2)(\frac{xK_1(X)}{K_0(X)}) - \frac{\mu H_0^2}{\rho R_0^2}(-x^2 + \alpha^2 x^2 + \frac{I_0(x)K_1(x)}{I_1(x)K_0(x)}) \qquad (62)$$

so, at U = 0, Ω = 0, H_0 = 0, the relation (62) reduces to

$$\sigma^2 = \frac{-T}{\rho R_0^3}(1-x^2)(\frac{xK_1(X)}{K_0(X)}) \qquad (63)$$

This relation has been given by Drazin and Reid [12], which is valid for all axisymmetric and non-axismmetric modes (m = 0, m ≥ 1) of perturbation.

The discussion of the dispersion relation (63) reveals that

$$\frac{(\frac{\sigma^2}{T})}{\rho R_0^3} > 0 \quad \text{as } (0 < x < 1) \text{ when } (m = 0)$$

This,means that the hollow gas jet is capillary unstable only in the axisymmetric mode (m = 0) in the small domain (0 < x < 1).

As at (U = 0, T_0 = 0, and Ω = 0) we have from equation (62)

$$\sigma^2 = -\frac{\mu H_0^2}{\rho R_0^2}(-x^2 + \alpha^2 x^2 + \frac{I_0(x)K_1(x)}{I_1(x)K_0(x)}). \qquad (64)$$

The axial magnetic field pervaded in the liquid is represented by the term $(\frac{\mu H_0^2}{\rho R_0^2}(-x^2))$. It has strong stabilizing effect and that effect is independent of the perturbed modes (m = 0) and (m = 1). The effect of the magnetic field pervaded in the gas cylinder is represented by the term $[\alpha^2 x^2 + \frac{I_0(x)K_1(x)}{I_1(x)K_0(x)}]$ followed by $(\frac{\mu H_0^2}{\rho R_0^2})$. It has strong stabilizing effect and this effect is valid for all axisymmetric mode m = 0 and non axisymmetric modes m = 1. So we see that the present model is purely stabilizing under the acting electromagnetic forces in gas and liquid regions.

Conclusion

From the foregoing discussions, we may conclude the following:

(1) The axial magnetic field pervaded interior the fluid cylinder has stabilizing effect.

(2) The stability behavior of the model comes after the destabilizing behavior of the model when it be reduced and suppressed.

(3) The capillary force is destabilizing only in a small axisymmetric domains and all domains of non-axisymmetric perturbations.

(4) The present model is purely stabilizing under the acting electromagnetic in gas and liquid regions.

(5) For very high intensity of magnetic field the destabilizing character of the model could be suppressed completely for all value of wave lengths.

References

1. Chandeasekhar S (1961) Hydrodynamic and Hydromagnetics stability. Dover Publ, New York.

2. Hasan AA, Abdelkhalek RA (2013) Magnetogravitodynamic stability of streaming fluid cylinder under the effect of Capillary force. Boundary value problems.

3. Elazab SS, Rahman SA, Hasan AA, Zidan NA (2011) Hydromagnetic Stability of Oscillating Hollow jet. Appl Math Sci 5: 1391-1400.

4. Drazin PG, Reid WH (1980) Hydromagnetics stability. Cambridge University Press, London.

5. Cheng LY (1985) Instability of a gas jet in liquid. Phys Fluids 28: 2614.

6. Kendall JM (1986) Experiments on annular liquid jet instability and on the formation of liquid shells. Phys Fluids 29: 2086.

7. Chauhan A, Maldarwlli C, Papageorgiou DT, Rumschitzki DS (2000) Temporal instability of compound jets and threads. J Fluid Mech 420: 1-25.

8. Abramowitz, Stegun I (1970) Handbook of Mathematical functions. Dover puble, New York.

9. Chen J, Lin S (2002) Instability of an annular jet surrounded by a viscous gas in a pipe. J Fluid Mech 450: 235-258.

10. Cousin J, Dumouchel C (1996) Effect of Viscosity on the Linear Instability of a Flat Liquid Sheet. Atomization and Sptays 6: 563.

11. Lee C, Wang T (1989) Revisited Phys Fluids AI pp: 967.

12. Lee C, Wang T (1986) Phys Fluids 29: 2076.

13. Mehring C, Sirignano W (2000) Axisymmetric capillary waves on thin annular liquid sheets. I. Temporal stability. Phys Fluids 12: 1417-1439.

14. Parthasarathy R, Chiang K (1998) Temporal instability of gas jets injected in viscous liquids to three-dimensional disturbances. Phys Fluid 10: 2105.

15. Shen J, Li X (1996) Instability of an annular viscous liquid jet. Acta Mechanica 114: 167-183.

16. Shi S, Xi D, Qin J, Liu N, Shu G (1999) Unstable asymmetric modes of a Liquid Jet. J Fluid Engng 121: 379.

17. Shukudov V, Sisoev G (1996) Int J Multiphase Flow 22: 363.

18. Villermaux E (1998) Mixing and Spray Formation in Coaxial Jets. J Propul Power 14: 807.

19. Melachlan NW (1947) Theory and Application of Mathieu Function. Oxford University Press, New York.

20. Kelly RE (1965) The stability of an unsteady Kelvin-Helmholtz flow. J Fluid Mech 22: 547-560.

21. Radwan AE, Hasan AA (2009) Magneto hydrodynamic stability of self-gravitational fluid cylinder. J Appl Mathematical modelling 33: 2121.

22. Radwan AE (2005) J Appl Math Comput 160: 213.

23. Radwan AE (1998) Hydromagnetic instability of a streaming compressible cylindrical hollow jet. Int J Engng Sci 36: 87.

24. Brakat HM, Khater KR, Essawy AH (2014) Hydromagnetic Self-gravitating Stability of Streaming Fluid Cylinder with Longitudinal Magnetic Filed. Appl Math Sci 8: 1969-1978.