

# Liouville's Theorem in Classical Mechanics and the Global Information Field

Solov'ev EA\*

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia

## Abstract

The connection between the concept of a 'Lagrangian manifold' and Liouville's theorem in classical mechanics and the concept of a global information field in quantum physics is discussed.

**Keywords:** Classical quantity; Quantum physics; Diffraction; Angular momentum

## Introduction

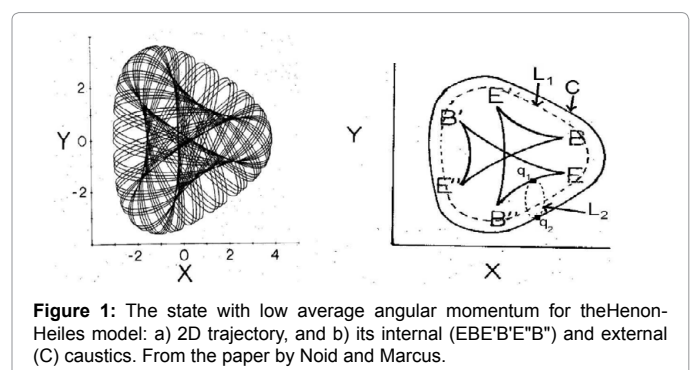
Quantization rules for multidimensional systems were proposed in 1917 by Einstein [1], who noted that a classical trajectory fills a torus in the phase space, which is the Lagrangian manifold, and that actions along topologically independent contours on the torus, according to Liouville's theorem, are invariant with respect to the deformation of these contours. The hypothesis that the action for one period is the main characteristic of a quantum state was justified by Ehrenfest [2]. His approach was based on the assumption that if the external field is changing in time very slowly, we can expect that transitions between quantum states are absent because the flux of energy from the external field is too weak to provide a sudden jump between well-separated discrete energy levels. What is the classical quantity which is conserved when the external interaction changes slowly in time? Or, what is the adiabatic invariant? Such adiabatic invariants should be responsible for the identification of quantum states. Ehrenfest has proved that this invariant is an action calculated during one period. The interpretation of quantum physics as a global information field theory has been formulated in [3,4]. In fact, the global information field arises already in classical electrodynamics. The study of the diffraction of light demonstrates that the distribution of photons on the screen is predicted by the Maxwell equations. The same phenomenon for propagation of electrons through double slits is described by the Schrödinger equation. In both cases the global information field is determined by the wave-type differential equations - the Maxwell equations in classical electrodynamics and the Schrödinger equation in quantum physics. Standard notations for the solutions of these equations are an electromagnetic field and a wave-function, respectively. However, a more adequate name for them is a global information field, which emphasizes that the material (classical) world is accompanied by a global information field on all levels. The existence of the global information field has been demonstrated by the Einstein-Podolsky-Rosen experiment e.g., for the radiative decay of a hydrogen atom from a Meta stable 2S-state. In this case two photons are emitted having spins with opposite directions, since the initial (2S) and final (1S) states of hydrogen have angular momentum equal to zero. However, the orientation of each spin is uncertain. If the measurement of the direction of the spin of the first photon is performed, the second photon takes the opposite direction of spin, according to the global information field, independently of the distance between the two photons. The fixation of the spin orientation of the second photon happens immediately, and it is an actual changing of state of the second photon without any contact. In the experiment [5] the distance between photons at the moment of measurement was 18.0 km. It was found that the propagation of

quantum information at the moment of measurement is at least  $10^4$  times faster than the speed of light  $c$ .

To demonstrate the connection between the global information field and the Lagrangian manifold in phase space let us quote the example from [4], "The global information field has no direct relation to time for closed systems. Namely, in the semi classical approach a classical trajectory forms the Lagrangian manifold in phase space. If a classical trajectory fills a torus, then in the  $N$  dimensional problem only  $N$  topologically independent quantization contours exist and the actions along these contours are, according to the Liouville theorem, invariant with respect to their deformation. Figure 1 shows one of the 'states' (i.e. a projection of a torus onto the coordinate plane) for the Hamiltonian of the Henon-Heiles model (Figure 1a).

$$H = \frac{1}{2}(P_x^2 + P_y^2 + x^2 + y^2) + \lambda x \left( y^2 - \frac{1}{3}x^2 \right).$$

In two dimensions it is a non-separable problem. This figure demonstrates that caustics are the non-local characteristic of states, which can be determined only if the whole trajectory is known. In (Figure 1b) two topologically independent contours in the quantization rule are indicated by dashed lines  $L_1$  and  $L_2$ . Thus, the topological theorems, being a very nice but quite useless tool in classical physics,



**Figure 1:** The state with low average angular momentum for the Henon-Heiles model: a) 2D trajectory, and b) its internal (EBE'B'E'B") and external (C) caustics. From the paper by Noid and Marcus.

\*Corresponding author: Solov'ev EA, Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia, Tel: +7 (49621) 65-059; Fax: +7 (495) 632-78-80; E-mail: [esolov@theor.jinr.ru](mailto:esolov@theor.jinr.ru)

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play a decisive role in quantum physics. Even on the semi classical level the global properties of trajectories, which are related rather to the information than to the local time dynamics, are most important. Moreover, the corresponding wave function does not contain the time variable. Interaction between the wave function (and the global information field) and our (classical) level is measurement. Obviously, this interaction is irreversible in time and determines the arrow of time. "At first sight, the concept of a 'Lagrangian manifold' and Liouville's theorem do not play a significant role in classical mechanics; they are rather abstract statements, which are not directly related to the calculation of the classical trajectory. But, what is astounding, they are the foundation stone for understanding of the global information field, which is outside of classical mechanics.

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