Perspective
Volume 16:6, 2022

Generalized Lie Theory and Applications

ISSN: 1736-4337 Open Access

Lie Symmetry Analysis and Soliton Solutions

Alice Fialowski*

Department of Mathmetics, Eotvos Lorand University, Budapest, Hungary

Introduction

Electric and magnetic fields spread perpendicular to one another and the direction of propagation. Radio waves spread through a variety of modalities and methods. The categories are ionospheric, tropospheric, and surface waves. Nonlinear physical properties have been connected to nonlinear equations to produce significant discoveries and applications in mechanics, wave propagation, fluid dynamics, fluid flow, nonlinear networks, optical fibre, and soil stability. The continuous equations that represent the conservation of energy, mass, electric charge, or momentum in engineering and the applied sciences are known as partial differential equations (PDEs) [1]. These equations cover gas dynamics, plasma physics, nonlinear optics, quantum physics, magnetohydrodynamics, and fluid mechanics. The topics covered include magnetohydrodynamics, fluid mechanics, nonlinear optics, quantum physics, and particle, gas dynamics, and plasma physics.

Description

In soliton theory, fluid dynamics, nonlinear optics, condensed matter physics, mathematical physics, plasma physics, biology, and many other areas may all be greatly impacted by the dynamics of solitary wave solutions. The primary goal of this article is to use the Lie symmetry analysis method to obtain symmetry reductions and novel explicit precise solutions to the (2+1)-dimensional Sharma-Tasso-Olver (STO) equation. The Lie group invariance criterion was met in order to obtain the infinitesimals for the STO equation [2]. Then, with the aid of an ideal system, the two stages of the governing equation's symmetry reductions are derived. The STO equation will be transformed into new partial differential equations (PDEs) using the Lie symmetry technique that have fewer independent variables. The majority of physical issues are by their very nature nonlinear. Due to their physical applications in fields including engineering, biology, theoretical physics, plasma physics, and condensed matter physics, these kinds of nonlinear issues are of great interest to many researchers, including mathematicians, physicists, engineers, and others. Exact solutions of these nonlinear PDEs are crucial for forecasting potential physical phenomenon behaviour. Analytical and numerical techniques such as the Lie symmetry method, homogeneous balance method, tanh function method, Hirota bilinear method, Backlund transformation method, Darboux transformation method, truncated Painleve expansion, and F-expansion method were used to investigate the various types of rational solutions [3].

We would like to investigate how to construct the explicit exact-soliton solutions using the Lie symmetry technique, which is motivated by the aforementioned references and studies. Additionally, for the equation under consideration, we obtain a number of Lie symmetry reductions and localised solitary wave solutions. There are many scientific applications, and wave and dispersive equations are frequently involved. The pursuit of such equations' lone solutions is of utmost significance in mathematical physics [4]. A soliton is a self-reinforcing solitary wave that is widely known to be produced by a delicate balance between nonlinear and dispersive effects in the medium. A growing number of physics problems, including those involving fluid mechanics, biology, viscoelasticity, engineering, etc., are being modelled using dimensional differential equations. With the aid of appropriate similarity transformations, the technique simplifies the challenging short pulse equation to a set of ordinary differential equations. In order to find exact answers, these systems of nonlinear ordinary differential equations under each subalgebra are solved. Furthermore, soliton solutions of the complex short pulse equation are produced that take the shape of hyperbolic and trigonometric functions using similarity variables, similarity solutions, and exact solutions of the nonlinear ordinary differential equation.

Differential equations (DEs) aid in the evaluation, comprehension, and resolution of real-world phenomena and circumstances. The engineering sciences, as well as mathematics, physics, biology, chemistry, astronomy, and economics, are all examples of real-world occurrences. The analysis is increasingly influenced by differential equations, which raises challenging mathematical problems. As a result, there is a vast body of literature on discovering their solutions, conservation laws, and Lie-point symmetries. A key area of research is finding exact solutions to DE. For the purpose of calculating natural solutions, academics have devised a variety of approaches. These approaches are known for the Lie group analysis method, developed by Sophus Lie. He introduced the group properties acknowledged by the DEs, which produced the precise solutions, instigated by the Galois Theory. This theory can be used to find the symmetry group analysis of DEs, including ODEs and PDEs. These symmetries are used to calculate the number of independent variables, conservation laws, linearization of NLPDEs, similarity solution, and order of equations. conservation laws. Additionally, they are used to assess the solution's stability and overall behaviour. Conservation restrictions serve as a useful tool in this examination of DEs. Plasma physics, ferromagnetic chains, water wave tanks, meta-materials, nonlinear wave propagation, and nonlinear optical fibres all make use of nonlinear evolution equations [5].

***Address for Correspondence:** Alice Fialowski, Department of Mathmetics, Eotvos Lorand University, Budapest, Hungary, E-mail: fialowskialice@gmail.com*

Date of Submission: 02 June, 2021; Manuscript No. GLTA-22-78560; **Editor Assigned:** 04 June, 2022, PreQC No. P-78560; **Reviewed:** 06 June, 2022, QC No. Q-78560; **Revised:** 15 June, 2022, Manuscript No. R-78560; **Published:** 22 June, 2022, DOI: 10.37421/1736-4337.2022.16.342

In the literature, many approaches are employed to estimate the conservation laws of DEs. The conservation laws can also be extracted using Noether's method, but it is only applicable to the variational problem and requires the availability of standard Lagrangian. The direct construction method is an algorithmic manner of finding these laws with any number of dependent and independent variables. This approach requires no variational concept; hence it is easy to apply. Several articles about conservation laws are released using the multiplier approach. Another computer package was prepared by Cheviakov to determine conservation laws based on the multiplier method. The partial Noether approach is an alternative method of finding the conservation laws for those problems for which standard Lagrangian does not succeed in existing. Using this method, researchers extracted conservation laws of many mathematical models. The remaining text is organised in the manner listed below. Equation modelling makes predictions. In "Lie analysis," we determine the Lie symmetries of the described model [6]. In "Symmetry reduction," we use Lie symmetries to transform the given problem into an ionic wave. The New Extended Direct Algebraic Approach to the Equation is then used to calculate numerous families of soltons solutions. Using similarity variables, we reduce the equation in "Symmetry reduction." Using the direct algebraic method, we compute the travelling wave solution for the current model in traveling wave solutions. To make the elements of the discovered solutions easier to understand, we interpreted them graphically in "Comparison Plots" utilising pertinent 2D and 3D graphics.

Conclusion

Finding accurate solutions to nonlinear fractional partial differential equations (NLFPDEs) is a crucial tool for describing nonlinear physical events, as is well known. Using the notion of derivatives and integrals of fractional order, a physical phenomenon may depend not only on the time instant but also on the time history. The subject of fractional calculus is as old as the calculus of differentiation and integration and dates back to the time when Leibniz, Newton, and Gauss developed this type of calculation. Fractional calculus is the generalisation of ordinary differentiation and integration to noninteger (arbitrary) order. Due to its applications in modelling physical processes related to their historical states (nonlocal property), which can be effectively described by using the theory of derivatives and integrals of fractional order, it is also regarded as one of the most interesting topics in a variety of fields, particularly mathematics and physics. This is because models described by fractional order are more realistic than those described by integer order.

Conflict of Interest

None.

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How to cite this article: Fialowski, Alice. "Lie Symmetry Analysis and Soliton Solutions." *J Generalized Lie Theory App* 16 (2022): 342.