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Left Multiplicative Generalized Jordan Derivations of Semiprime Rings

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Abstract

In this paper we prove that left multiplicative generalized Jordan derivation and left multiplicative generalized Jordan triple derivation of 2-torsion free semiprime rings are left multiplicative generalized derivation.

Preliminaries

Throughout this paper R will be denote an associative ring with the center $Z(R)$. If $n>1$, a ring R is said to be n-torsion free, if for $x\in R$, $nx=0$ implies $x=0$. Recall that a ring R is called prime if for any $x,y\in R$, $xRy=\{0\}$ implies that either $x=0$ or $y=0$. And R is a semiprime if $xRx=\{0\}$ implies $x=0$. An additive mapping $T:R\to R$ is said to be a left centralizer if $T(xy)=T(x)y$ (resp. $T(x^2)=T(x)x$), for all $x,y\in R$. An additive mapping $T:R\to R$ is said to be a right centralizer if $T(xy)=yT(x)$ (resp. $T(x^2)=xT(x)$), for all $x,y\in R$. An additive mapping $D:R\to R$ is called a derivation (resp. Jordan derivation) if $D(xy)=D(x)y+xD(y)$ (resp. $D(x^2)=D(x)x+xD(x)$), for all $x,y\in R$. An additive mapping $D:R\to R$ is called a left derivation (resp. Jordan left derivation) if $D(xy)=xD(y)+yD(x)$ (resp. $D(x^2)=xD(x)+xD(x)$), for all $x,y\in R$. A mapping $F:R\to R$ is called centralizing on S if $[f(x),x]\in Z$ for all $x\in S$ and is called commuting on S if $[F(x),x]=0$ for all $x\in S$. An additive mapping $F:R\to R$ is called a generalized derivation if there exists a derivation $D:R\to R$ such that (resp. generalized Jordan derivation) $F(xy)=F(x)y+xD(y)$ (resp. $F(x^2)=F(x)x+xD(x)$), for all $x,y\in R$. An additive mapping $F:R\to R$ is called a left generalized derivation if there exists a derivation $D:R\to R$ such that (resp. left generalized Jordan derivation) $F(xy)=xF(y)+D(x)y$ (resp. $F(x^2)=xF(x)+D(x)x$), for all $x,y\in R$. An additive mapping $D:R\to R$ is called Jordan triple derivation if $D(xyx)=D(x)yx+xD(y)x+xyD(x)$, for all $x,y\in R$. An additive mapping $F:R\to R$ generalized Jordan triple derivation if $F(xyx)=F(x)yx+xD(y)x+xyD(x)$, for all $x,y\in R$ where D is a Jordan triple derivation. An additive mapping $F:R\to R$ left multiplicative generalized Jordan triple derivation if $F(xyx)=xyF(x)+D(x)yx+xD(y)x$, for all $x,y\in R$ where D is a Jordan triple derivation.

Introduction

Bresar [1] has proved that any Jordan triple derivation on 2-torsion free semiprime ring is a derivation. A classical result of Herstein [2] asserts that any Jordan derivation on a 2-torsion free prime ring is a derivation. A brief proof of Herstein's result can be found Bresar M et al., [3]. Cusak [4] studied Jordan derivations on prime rings. Zalar [5] has proved that any left Jordan centralizer on a 2-torsion free semiprime ring is a left centralizer. Recently, Jing and Lu [6] introduced a concept of generalized Jordan derivation and generalized Jordan triple derivation. Vukman and kosi-Ulbl [7] studied an equation related to centralizers in semiprime rings. Molnar [8] studied on centralizers of an H*-algebra. Subba Reddy et al. [9-12] studied left multiplicative generalized derivations in prime and semiprime rings. Vukman [13] studied a note on generalized derivations of semiprime rings. In this paper, we can extended some results on left multiplicative generalized Jordan derivations of semiprime rings.

Theorem 1

Let R be a 2-torsion free semiprime ring and let $F:R\to R$ be a left multiplicative generalized Jordan derivation. Then prove that F is a left multiplicative generalized derivation.

Proof: We have therefore the relation,

$$F(x^2)=xF(x)+D(x)x, \text{ for all } x\in R. \quad (1)$$

Here D is a Jordan derivation on R.

Since R is a semiprime ring one can conclude that D is a derivation.

Let us denote $F-D$ by T.

Then we have, $T(x^2)=F(x^2)-D(x^2)$

$=xF(x)+D(x)x-D(x)x-xD(x)$

$=xF(x)-xD(x)$

$=x(F(x)-D(x))$

$T(x^2)=xT(x)$.

We have, therefore $T(x^2)=xT(x)$, for all $x\in R$. In other words, T is a right Jordan centralizer of R. Since R is a 2-torsion free semiprime ring. One can conclude that T is a right centralizer in ref. [5]. Hence F is of the form $F=D+T$. Where D is a derivation and T is a right centralizer of R. This means that F is a left multiplicative generalized derivation.

Theorem 2

Let R be a 2-torsion free semiprime ring and let F: $R\to R$ be a left multiplicative generalized Jordan triple derivation. Then prove that F is a left multiplicative generalized derivation.

Proof: We have therefore the relation,

$F(xyx)=xyF(x)+D(x)yx+xD(y)x$, for all $x,y\in R$.

where D is a Jordan triple derivation of R.

Since R is a semiprime ring one can conclude that, D is a derivation by theorem A in ref. [1].

Let us denote $F-D$ by T.

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We have $T(xyx)=F(xyx)-D(xyx)$

$=xyF(x)+D(x)yx+xD(y)x-D(x)yx-xD(y)x-xyD(x)$.

$=xyF(x)-xyD(x)$

$=xy(F(x)-D(x))$

$T(xyx)=xyT(x)$.

We have therefore $T(xyx)=xyT(x)$, for all $x,y \in R$.

Conclusion

By theorem in ref. [14] one can conclude that T is a right centralizer. We proved that F can be written as $F=D+T$, where D is a derivation and T is a right centralizer, which means that F is a left multiplicative generalized derivation.

Example: The following example express as a centralizer of a ring R is both left and right centralizer of a additive mapping T, i.e., $T(xy)=T(x)y=xT(y)$, for all $x,y \in R$.

Consider the ring:

$$R=\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} / a,b \in S\right\}$$

Where S is any ring. Define $T: R \to R$, by

$$T\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}, \text{ for all } a,b \in S.$$

We can show that T is a centralizer.

Let $x=\begin{pmatrix} a_1 & 0 \\ 0 & b_1 \end{pmatrix}$, $y=\begin{pmatrix} a_2 & 0 \\ 0 & b_2 \end{pmatrix}$, where $x,y \in R$ and $a_1, b_1, a_2, b_2 \in S$.

Applying T, we find that:

$$T(xy)=\begin{pmatrix} a_1a_2 & 0 \\ 0 & 0 \end{pmatrix}=T(x)y=xT(y)\cdot$$

Thus, we obtain T is a centralizer.

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