

Large-Scale Bose-Einstein Condensation in a Vapor of Cesium Atoms at Normal Temperature (T=353K)

You PL*

Institute of Quantum Electronics, Guangdong Ocean University, Zhanjiang, China

Abstract

Large-scale BEC of cesium at T=353 K was first observed. Until now, scientists have applied magnetic fields and lasers, but never applied electric fields, and atoms are oriented at random, so observation of BEC is very difficult. Our innovation lies in the application of electric fields. We theoretically proved that alkali atom (include Cs) may be polar atom doesn't conflict with quantum mechanics. Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom. Cs vapor was filled in cylindrical capacitor. Our experiment shows that Cs is polar atom because its capacitance is related to temperature. In the past, to realize the phase transition, ultralow temperature is necessary. But now we don't require ultralow temperature, because we use the critical voltage V_c to achieve the phase transition. From the entropy $S=Nk \ln 2\pi e / a=0$, $a=dV/kTH = 2\pi e$, $V_c \approx 63$ volts. When $V < V_c$, $S > 0$; when $V > V_c$, $S < 0$, phase transition occurred. When $V=350$ volts, the capacitance decreased from $C=1.97C_0$ to $C \approx C_0$ (C_0 is the vacuum capacitance), this result implies that almost all Cs atoms (more than 98.9%), like as dipoles, are aligned with the field. We create BEC with 1.928×10^{17} atoms, these atoms have the same momentum. Cs material with purity 99.95% was supplied by Strem Chemicals Co., USA. Both BEC and superconductivity are condensed in the momentum space, therefore these two kinds of condensation can't be observed with the naked eye. When superconductivity occurs, the resistance $R \approx 0$, a simple and direct method to observe superconductivity is to measure the resistance by voltammetry. Similarly, when BEC occurs, the electric susceptibility $\chi_e = C/C_0 - 1 \approx 0$, a simple and direct method to observe BEC is to measure the capacitance by cylindrical capacitor. BEC is also a quasi-superconducting state.

Keywords: PACS numbers; Bose-Einstein condensation; Entropy; Polar atom; Cylindrical capacitor; Permanent dipole moment; Quasi-superconducting state

Introduction

BEC experiment is quite intriguing and a challenging task for the experimentalists. In 1925 Einstein pointed out that I maintain that, in this case, a number of molecules steadily growing with increasing density goes over in the first quantum state (which has zero kinetic energy) while the remaining molecules distribute themselves according to the parameter value $\lambda=1$. "A separation is effected; one part condenses, the rest remains a 'saturated ideal gas' ($A=0, \lambda=1$)" [1]. The famous physicist Abraham Pais emphasized that this is now called Bose-Einstein condensation. Now we estimate how many atoms in this part of Bose gas. In order to achieve BEC, the suitable density of Bose gas is $10^{13} - 10^{15} \text{ cm}^{-3}$, and its volume is usually greater than 10 cm^3 , so according to Einstein's prediction, the number of condensed atoms at least greater than $10^{13} - 10^{15}$ (take 1/10 as "one part"). But what is about the actual situation? A typical example of BEC is provided by the dilute gases of alkali-metal atoms that can be prepared inside magnetic ion traps [2-6]. All stable alkali species---Li (2), Na (3), K (4), Rb (5), and Cs (6) ---have been condensed. The atoms, usually only $10^4 - 10^6$ of them, can be trapped and cooled [2-6]. And therefore the previous condensate fraction is usually less than 10^{-9} . This result shows that in the one billion alkali atoms which participating in the experiment, equivalent to only one atom was condensed. Strictly speaking, the results of previous experiments did not conform to Einstein's predictions.

In the past, there are two ways to achieve phase transition: given atomic density, lower the temperature of Bose gas, making $T < T_c = \frac{2\pi\hbar^2}{km} \left(\frac{n}{2.612} \right)^{2/3}$; or given temperature, increase the density of Bose gas, making $n > n_c$ (the critical density $n_c = 2.612 \left(\frac{mkT}{2\pi\hbar^2} \right)^{3/2}$). But the latter method is also not successful, because the density of Bose gas can't

be increased indefinitely, and therefore the laser cooling is now widely used. What is the definition of BEC? "Bose-Einstein condensation is a macroscopic occupation of the ground state" [7]. Until now, scientists have applied magnetic field (used to trap atoms) and laser (used to cool atoms), but never applied electric field, despite the use of many advanced technology, condensed atomic number is still very small. This fact shows that according to the current method, the "macroscopic occupation" is very difficult to achieve.

Can we find a new way to get out of the current predicament? This article provides a new idea for the implementation of BEC. In BEC experiments, scientists never applied an electric field because they think that alkali atoms are non-polar atoms. If alkali atom is non-polar atom, has been verified by experiment? Answer: no! No such experimental results have been reported in the history of physics! In fact, this traditional concept is an untested hypothesis, and it has misled physicists all over the world. Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom. If alkali atom is a non-polar atom, its capacitance should be independent of the temperature due to the nucleus located at the center of the electron cloud. We measured the capacitance at different temperatures. Our experiment proved that Cs atom is polar atom, and it becomes a dipole. When an electric field is applied, the dipoles tend

*Corresponding author: You PL, Institute of Quantum Electronics, Guangdong Ocean University, Zhanjiang, 524025 China, Tel: 41446334570; E-mail: youpeli@163.com

Received August 01, 2016; Accepted August 12, 2016; Published August 22, 2016

Citation: You PL (2016) Large-Scale Bose-Einstein Condensation in a Vapor of Cesium Atoms at Normal Temperature (T=353K). J Material Sci Eng 5: 276. doi:10.4172/2169-0022.1000276

Copyright: © 2016 You PL. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

to align with the field because of the torque it experiences. So a large-scale BEC at normal temperature can be observed.

Our previous research found that rubidium atom has a non-zero permanent dipole moment (PDM), $d_{\text{Rb}} \geq 8.6 \times 10^{-9} \text{ e.cm}$, the saturation polarization of rubidium vapor has been observed [8,9]. In particular, recent report showed that sodium is polar atom, BEC of Na atoms at normal temperature has been observed [10]. This article has been rigorously peer reviewed for up to two months. This reviewer has published three theoretical articles about BEC in Physical Review, and these interesting theoretical results may be useful for the design of BEC experiment [11-13]. This fact indicates that this peer review is authoritative and persuasive. Reviewer comments pointed out that "The author presented a good idea (using the critical voltage) to observe the BEC. Moreover, the author shows that the ultra-low temperature is not a necessary condition to verify the existence of BEC. This paper is interesting and certainly deserves publication in the journal". This objective evaluation is a great inspiration to us, and encourages us to report the results of new BEC experiment of cesium atoms.

Let us take the iron filings as an example for further explanation. In the absence of magnetic field, iron filings are oriented at random, although use laser cooling technology, but the condensation of these iron filings is still very difficult. When a magnetic field is applied, however, the iron filings immediately arranged along tangent to the magnetic field lines like as little compass needles, and the condensation of these iron filings is easily observed. This example shows the previous research is caught in a misunderstanding because they never applied an electric field, and therefore scientists have missed out on this significant discovery.

Theoretical Breakthrough

Normally atoms do not have PDM because of their spherical symmetry; however our article proved that alkali atom forms an exception. *ns* and *np* states of alkali atoms are not degenerate, and therefore the expectation value of PDM is zero: $\langle \psi_E | -er | \psi_E \rangle = 0$ ($d = -er$ is the dipole moment operator). However, many physicists strictly proved that the state function $|\psi_E\rangle$ doesn't describe an individual particle but an ensemble of particles with the same energy [14-16]. "The expectation value is the average of repeated measurements on an ensemble of identically prepared systems, not the average of repeated measurements on one and the same system" [17]. So $\langle \psi_E | -er | \psi_E \rangle = 0$ only means that the average PDM of large number of alkali atoms is zero, but doesn't mean that the PDM of individual alkali atom is zero.

The hydrogen atom is a typical example. "The shift in the energy levels of an atom in an electric field is known as the Stark effect" [14]. Normally the effect is quadratic in the field intensity, which corresponds to an induced electric dipole moment (EDM) [14]. The quadratic Stark effect occurs in general in all states. But LD Landau once stated that "the hydrogen atom forms an exception; here the Stark effect is linear in the field". "The energy levels of the hydrogen atom, unlike those of other atoms, undergo a splitting proportional to the field (the *linear Stark effect*)" [18]. Evidently, the linear effect corresponds to a PDM. This effect showed that hydrogen atom ($n=2$) has a PDM of magnitude $-3ea_0 = 1.59 \times 10^{-8} \text{ e.cm}$ (a_0 is the Bohr radius) [14,18]. LD Landau once stated that "The presence of the linear effect means that, in the unperturbed state ψ_{2lm} , the hydrogen atom has a dipole moment whose mean value is $d = -3ea_0$ " [18]. LI Schiff also stated that "It is also possible, as in the case of the hydrogen atom, that unperturbed degenerate states of opposite parities can give rise to a

permanent electric dipole moment" [19]. That is, $d(\psi_{200}) \neq 0$, $d(\psi_{210}) \neq 0$, $d(\psi_{211}) \neq 0$ and $d(\psi_{21-1}) \neq 0$.

However, quantum mechanical calculations indicated that although ψ_{2lm} is four-fold degenerate, but the expectation value of PDM is zero: $\langle \psi_{2lm} | -er | \psi_{2lm} \rangle = 0$! That is, $d(\psi_{200}) = d(\psi_{210}) = d(\psi_{211}) = d(\psi_{21-1}) = 0$. Evidently, the zero result is inconsistent with above conclusion. Up to now, no quantum mechanical textbook explains this contradictory result. This fact shows that individual hydrogen atom ($n=2$) has a non-zero PDM, but quantum mechanics can't obtain this non-zero result in any way. It convincingly proved that $\langle \psi_{2lm} | -er | \psi_{2lm} \rangle = 0$ only means that the average PDM is zero, but an individual hydrogen ($n=2$) may have a non-zero PDM.

Recall that alkali atoms having only one valence electron in the outermost shell can be described as hydrogen-like atoms [20]. So similar to the first excited state of hydrogen, the ground state of alkali atom may be polar atom doesn't conflict with quantum mechanics. A neutral alkali atom (include Cs) is or is not polar atom must be determined by experiments.

A new formula of atomic PDM is obtained for the first time

Experiments to search for PDM of atoms began half a century ago. In all experiments, they measured the spin resonance frequency ν of individual atom by $h\nu = 2\mu B \pm 2dE$, where h is Planck's constant, μ and d is magnetic and electric dipole moments [21,22]. But "experimental searches for PDM have so far yielded null results" [21]. This fact shows that this formula is not successful, because it measured the microscopic quantity d by using another microscopic quantity ν .

However, measuring the average kinetic energy of a gas molecule using the temperature is easy: $E_k = 3kT/2$. Similarly, we measure the PDM of an atom using the change of the capacitance is easy: $d = (C - C_0)V/L(a) nS$. This formula is easy to verify. The magnitude of the PDM is $d = er$. $L(a)$ equals the percentage of Cs atoms lined up along an electric field, n is its density, S is the plate area. When the electric field is applied, the change of the charge of the capacitor is $\Delta Q = (C - C_0)V$. On the other hand, its volume is SH , the total number of oriented atoms of the capacitor is $SH n L(a)$. The number of layers of oriented atoms is H/r . Because inside the Cs vapor the positive and negative charges cancel out each other, the polarization only gives rise to a net positive charge on one side of the capacitor, and a net negative charge on the opposite side. Therefore $\Delta Q = SH n L(a)e/(H/r) = n S L(a)e r = n S L(a) d = (C - C_0)V$, so $d = (C - C_0)V/n L(a) S$.

Our Innovations

According to W. Ketterle's standard, our experiment is a truly ideal BEC

Wolfgang Ketterle, in the Nobel Prize winning paper, proposed the objective standard of an ideal BEC: "An ideal Bose condensate shows a macroscopic population of the ground state of the trapping potential. This picture is modified for a weakly interacting Bose gas" [3]. This standard emphasizes that the ground state is the ground state of the trapping potential, which is very important correction. Because this description doesn't involve temperature, and therefore this standard is also suitable for the evaluation of our experiments. Cesium has a non-zero PDM, and it becomes an electric dipole. When an electric field is applied, the dipoles tend to orient in the direction of the field because of the torque it experiences. The magnitude of the torque is $\tau = dE \sin\theta$, and θ is the angle between d and E . When $\theta=0$, the dipole is in equilibrium. In our experiment, the trapping potential is the electric

potential energy of Cs atoms: $\mathcal{E} = -dE\cos\theta$. When $\theta=0$, the potential energy is a minimum, which indicates that the dipole is oriented parallel to the field. But atomic collisions tend to disarrange the dipoles. When $V < V_c$, many atoms are in random directions, this state has high entropy $S > 0$; when $V > V_c$, the atoms become aligned with the field, this state has low entropy $S < 0$, phase transition occurred. When $V \gg V_c$, $C \approx C_0$, this result implies that the alignment would be perfect, these atoms have the same momentum, this is condensation in momentum space. In effect, BEC is the perfect alignment of bosons. So BEC at normal temperature can be observed. When $V \gg V_c$, Bose condensate contained up to 1.928×10^{17} atoms really achieved "macroscopic population". The atoms condensed into the ground state of trapping potential, because their electric potential energy is a minimum along the field. Therefore, Ketterle's standard proved that although we didn't use laser cooling atoms, but our experiment is an ideal BEC. There are few groups that use electric fields to cool atoms. This article will not discuss this situation.

Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom experimentally. The classical electrostatics textbook plotted the relationship between χ_c and $1/T$ [23].

$$\text{For the polar atom } \chi_c = A + B/T, \text{ for non-polar atom } \chi_c = A \quad (1)$$

Where, A and B is constant [23]. If Cs is polar atom, the form $\chi_c = A + B/T$ should be expected. As a contrast, the capacitance of Hg has been measured, but its capacitance is independent of temperature, Hg is non-polar atom.

Our experimental type is quantitative

The condensate fraction is a very important physical quantity in BEC, but previous experiments didn't provide a suitable formula, and they are qualitative. We strictly proved that the Langevin function $L(a)$ equals the condensate fraction, and it can be expressed as $L(a) = a(C - C_0)/\eta$, where η is the capacitance constant, and $(C - C_0)$ is the change of the capacitance. For example, when the voltage $V = 350$ volts, $\eta = 192$ pF, $C - C_0 = 2$ pF and the coefficient $a = 95$, we obtain $L(a) = 0.9896$. These facts indicate that our experiments not only don't conflict with their experiments, but also this experimental type is quantitative.

The most striking characteristic of BEC is that BEC is condensed in momentum space

Einstein first noticed that Bose gas would condense to the lowest energy state. However, the result of the condensation doesn't produce crystals, this fact shows that BEC doesn't occur in the position space but occurs in the momentum space. "The term 'condensation' often implies a condensation in space, as when liquid water condenses on a cold window in a steamy bathroom. However, for Bose-Einstein condensation it is a condensation in k -space, with a macroscopic occupation of the lowest energy state" [7]. Note that the wave vector k equals the momentum p divided by Planck constant \hbar , and therefore BEC is a condensation in momentum space. Superconductivity is also the condensation in the momentum space because the Cooper pairs act like bosons, therefore these two kinds of condensation can't be observed with the naked eye. When superconductivity occurs, the resistance $R \approx 0$ (the resistance of vacuum is zero). Although BCS theory is difficult, however, a relatively simple and direct observation method is to measure the resistance by voltammetry. Similarly, when BEC occurs, the electric susceptibility $\chi_c = C/C_0 - 1 \approx 0$ or $C \approx C_0$, a simple and direct observation method is to measure the capacitance. BEC is quasi superconducting state. Note that the resistance and capacitance are two

macroscopic electrical quantities, and their changes are the decisive evidence of these two kinds of condensation. When they (R or C) are reduced to close to the vacuum value, these two kinds of condensation occur. This fact fully reflects the harmony and unity of nature.

The entropy of a system is a measure of the disorder of molecular or atomic motion. No doubt, it is the most important concept in BEC. Consider a system composed of N cesium atoms which are placed in an electric field E , θ is the angle between d and E . Note that the collision between Cs atoms is always through their mass centers, and therefore the nucleus has no contribution to the rotational energy of the atom. When orientation polarization occurs, its rotational energy can be neglected. The potential energy of Cs atom in the field can be expressed as $\varepsilon = -dE \cos\theta$. Unlike the orientation quantization of magnetic moment, the orientation of Cs atoms can be changed continuously in the field. Note that $\beta = 1/kT$ and the chemical potential $\mu \approx 0$ [7], the partition function is given by

$$Z = \int_0^{2\pi} d\phi \int_0^\pi e^{-\beta\varepsilon} \sin\theta d\theta = \int_0^{2\pi} d\phi \int_0^\pi e^{dE \cos\theta / kT} \sin\theta d\theta = 2\pi kT (e^{dE/kT} - e^{-dE/kT}) / dE \quad (2)$$

The entropy is given by $S = Nk(\ln Z + T \frac{\partial}{\partial T} \ln Z)$ [7]. Let the coefficient $a = dE/kT = dV/kTH$, we obtain

$$S = Nk [\ln 2\pi e (e^a - e^{-a}) / a - a \coth a] \quad (3)$$

When $a = dE/kT \gg 1$ or $V \geq 37$ volts ($a \geq 10$), $e^{-a} \approx 0$ and $\coth a \approx 1$, we obtain a simplified formulas

$$S = Nk \ln 2\pi e / a \quad (4)$$

The critical coefficient is $a_c = 2\pi e \approx 17.08$. The formula contains two fundamental constants in nature ($\pi \approx 3.14159$ and $e \approx 2.71828$), it reflects the objective laws of BEC.

Experiment and Interpretation

The preparatory experiment: we measured the density of Cs vapor. The longitudinal section of the apparatus is shown in Figure 1. This closed glass container resembles a Dewar flask in shape. Its internal and external diameters are $D_1 = 5.6$ cm and $D_2 = 8.12$ cm respectively. The external and internal surfaces was paste with aluminum foil, also can be plated with silver, they form the outer and inner electrode (Figure 1) [10]. Their length is $L = 33.4$ cm. This capacitor is equivalently connected in series by two capacitors. One is called C' , and contains the Cs vapor of thickness $H_0 = 9.60$ mm; the other is called C'' , and contains the glass medium of thickness $h = 1.5$ mm. The total capacitance is $C = C'C'' / (C' + C'')$, where C'' and C can be directly measured. The magnitude of capacitance was measured by a digital capacitance meter. The model of the meter is DM6031A and was made in Shenzhen, China. The accuracy of the meter was 0.5%, the measuring voltage was $V_0 = 1.2$ volts, the measuring frequency was 800Hz, and the definition was 0.1

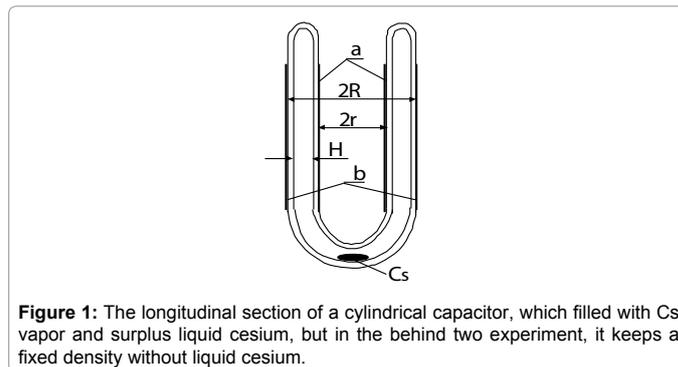


Figure 1: The longitudinal section of a cylindrical capacitor, which filled with Cs vapor and surplus liquid cesium, but in the behind two experiment, it keeps a fixed density without liquid cesium.

pF ($C \leq 200$ pF), 1 pF ($200 \text{ pF} \leq C \leq 2 \text{ nF}$) or 10 pF ($2 \text{ nF} \leq C \leq 20 \text{ nF}$). In order to remove impurities such as oxygen, when the capacitor was empty, it was pumped to vacuum pressure $P \leq 10^{-9}$ torr for 20 hours. The vacuum capacitance is $C'_0 = (54.0 \pm 0.1)$ pF. Next, a small amount of Cs material (about 10 grams) was put into the container, and it is again pumped to $P \leq 10^{-9}$ torr, and then it is sealed. We obtain a cylindrical glass capacitor filled with Cs vapor and surplus liquid cesium (Figure 1) [10]. We put the capacitor into a temperature-control stove, raise its temperature slowly, and keep it at $T_0 = 473$ K for 6 hours. It means that these results are obtained under the saturated vapor pressure. We measured the capacitance is $C'_t = (5140 \pm 10)$ pF. Note that $P = 10^{6.949 - 3833.7/T}$ psi ($473 \text{ K} \leq T \leq 623 \text{ K}$, $1 \text{ psi} = 6894.8 \text{ Pa}$) is the saturated vapor pressure of Cs atoms [24]. We obtained $P = 481.3 \text{ Pa}$ at $T_0 = 473 \text{ K}$. From the ideal gas law, the density of Cs vapor was $n_0 = P/kT_0 = 7.370 \times 10^{16} \text{ cm}^{-3}$, where $k = 1.3807 \times 10^{-23} \text{ J/K}$. The statistical error is $(\Delta n_i/n)^2 = (\Delta P/P)^2 + (\Delta T/T)^2$, due to $\Delta T = 0.5 \text{ K}$ and $\Delta T/T \leq 0.001$, $\Delta P = P(T_0 + \Delta T) - P(T_0) = 9.6 \text{ Pa}$ and $\Delta P/P \leq 0.02$, so $\Delta n_i/n \leq 0.03$. Considering all systematic error, we have that $\Delta n_i/n \leq 0.03$, and the density of Cs vapor is $n_0 = [7.37 \pm 0.22(\text{stat}) \pm 0.22(\text{syst})] \times 10^{16} \text{ cm}^{-3}$.

The first experiment: we measured capacitances of Cs and Hg vapor at different temperatures. A glass cylindrical capacitors fill with cesium at a fixed density n_1 without liquid cesium, and $n_1 \ll n_0$. Another cylindrical capacitors fill with Hg vapor at a fixed density n_3 .

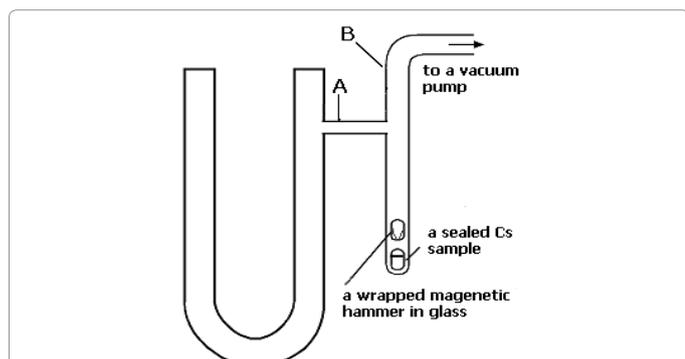


Figure 2: This diagram shows the perfusion method. We let the left capacitor to connect a vacuum pump through a glass tube. In this glass tube, a small magnetic hammer and a sealed cesium sample are put together. When the pump works, the magnetic hammer is raised by a magnet outside the tube. We suddenly release the magnetic hammer, which breaks the bottle sealed cesium sample, and then the tube is sealed at B point. Next step, we put the two containers into the heating furnace, and keep at $T_1 = 453 \text{ K}$ for 6 hours, the glass tube was sealed once again at A point. Thus we obtain a capacitor filled with Cs vapor at a fixed density (i.e. ensure the number of Cs atoms remained constant).

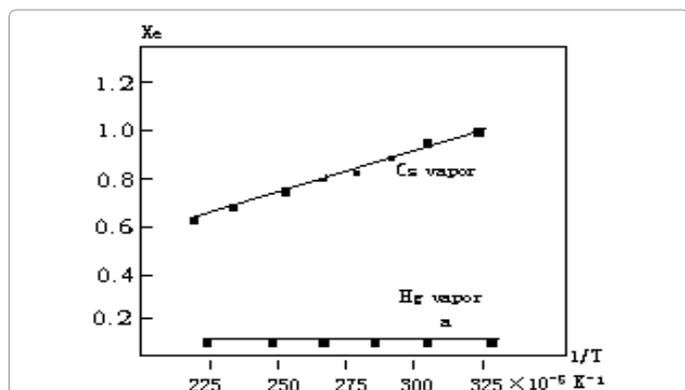


Figure 3: The diagram shows χ_e versus T^{-1} . For Cs vapor $\chi_e = 0.007 + 282.3/T$, but $\chi_e = 0.003$ for Hg vapor. So Cs is polar atom but Hg is non-polar.

T(K)	308	326	345	357	377	392	425	448
1/T($\times 10^{-3}$)	3.2468	3.0675	2.8985	2.8011	2.6525	2.5510	2.3529	2.2321
C(pF)	112.7	109.6	106.9	105.4	102.9	101.3	98.0	95.8
χ_e	0.9232	0.8703	0.8242	0.7986	0.7560	0.7287	0.6724	0.6348

Table 1: The electric susceptibility χ_e of cesium vapor at different temperature T.

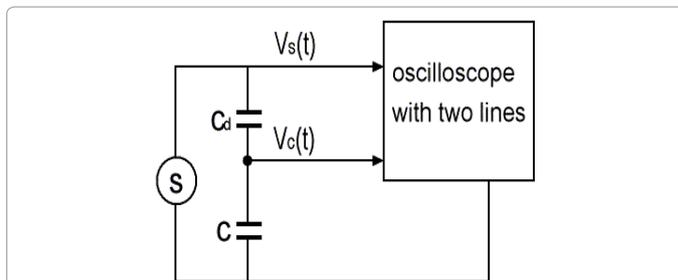


Figure 4: The diagram shows the measuring method: $C = (V_{s0}/V_{c0} - 1)C_d$, where $V_s(t) = V_{s0} \cos \omega t$ and $V_c(t) = V_{c0} \cos \omega t$.

Their capacitances were still measured by the digital meter, the vacuum capacitance is $C_{10} = 58.6$ pF (for Cs) and $C'_{10} = 63.9$ pF (for Hg). Next, a small amount of Cs or Hg material was put into the two capacitors. In order to ensure the number of atoms remained constant, we adopted an ingenious technique (Figure 2) [10]. After fill with Cs or Hg vapor, the capacitances are $C_1 = 112.9$ pF (for Cs) and $C'_1 = 64.1$ pF (for Hg) respectively. We measured their capacitances at different temperatures. As was expected, the capacitance of Hg vapor, $C'_1 = 64.1$ pF, remains constant at different temperatures (i.e., $B=0$) and $\chi_e = A \approx 0.003$. But the capacitance of Cs vapor decreases gradually from 112.7 pF to less than 96 pF, and $\chi_e = A + B/T!$

The Figure 3 shows the two experimental results. Table 1 gives the electric susceptibility of cesium vapor. By least-square method we obtain $B = (282.3 \pm 1.2) \text{ K}$ and $A = 0.007$, because $(\Delta \chi_e / \chi_e)^2 = (\Delta B/B)^2 + (\Delta T/T)^2 = (\Delta C/C)^2 + (\Delta C'_0/C'_0)^2$, so $(\Delta B/B)^2 < (\Delta C/C)^2 + (\Delta C'_0/C'_0)^2$, and $\Delta B/B < 0.004$. They can be expressed as follows

$$\text{For cesium vapor } \chi_e = 0.007 + 282.3/T, \text{ for mercury vapor } \chi_e = 0.003 \quad (5)$$

The two results formed a sharp contrast because Cs atom is polar atom but Hg atom is non-polar.

The second experiment: we measured the capacitance of Cs vapor at different voltages under a fixed density n_2 and $T_2 = 353 \text{ K}$. The vacuum capacitance of the apparatus is $C_{20} = (66.0 \pm 0.1)$ pF, where $H_2 = 6.8 \text{ mm}$. The Figure 4 shows the experimental method [10]. C is the measured capacitor, which filled with cesium vapor. C_d is a reference capacitor. Two signals $V_c(t) = V_{c0} \cos \omega t$ and $V_s(t) = V_{s0} \cos \omega t$ were measured by a two channel digital oscilloscope (Tektronix TDS 210 USA). From Figure 4, we have $(V_s - V_c)/V_c = C/C_d$ and $C = (V_{s0}/V_{c0} - 1)C_d$. The voltages V_{s0} could be adjusted from zero to 800 V. The frequency could be adjusted from one to 10^6 Hz. The measurement was started in 0.01 volt. When $V_1 = V_{c0} \leq 0.3$ volts, $C_1 = 130.0$ pF is approximately constant. When $V < V_c$, the peak difference of the two signals waveforms is large, this image means that $C \gg C_0$ (Figure 5). When $V_2 = V_{c0} = 350$ volts $\gg V_c$, $C_2 = 68.0 \text{ pF} \approx C_0$. This oscilloscope shows that the peak of the two signals is very close and they almost overlap, this precious image means that large-scale BEC at normal temperature has been observed (Figure 6). If almost all the dipoles in a gas were to line up with an external electric field, this effect is called the saturation polarization. The experimental C-V curve shows that the saturation polarization of Cs vapor is easily observed when $V_2 \gg V_c$ (Figure 7) [10]. Table 2 gives the measured values of Cs

vapor at different voltages.

RP Feynman, he was awarded the 1965 Nobel Prize, once stated that “The electric field tends to line up the individual dipoles to produce a net moment per unit volume. If all the dipoles in a gas were to line up, there would be a very large polarization, but that does not happen” [25]. Our experiment confirmed his prediction, and it tells us that the saturation polarization of Cs vapor is an unexpected discovery. When the peak of the two signal waveforms is very close, and the two waveforms are almost overlapped, this image means the BEC occurs. The capacitance $C \approx C_0$, it implies that Cs vapor entered a quasi-vacuum state! Figure 8 shows that the condensate fraction versus the external voltages, a large-

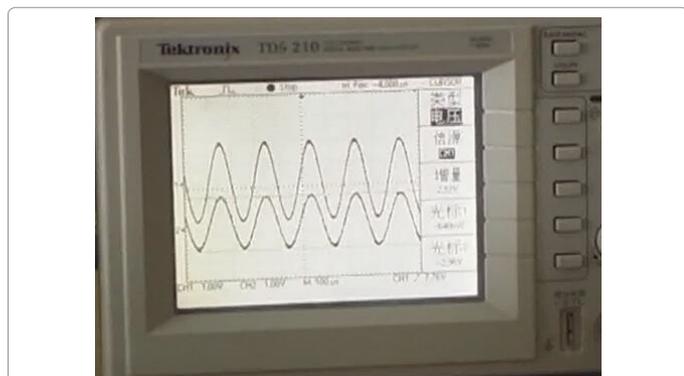


Figure 5: This diagram shows that when $V_{\infty} < V_c$, the peak difference of the two signals is large, this image means that $C \gg C_0$ (C_0 is the vacuum capacitance). The calculated result shows that this state is far from BEC.

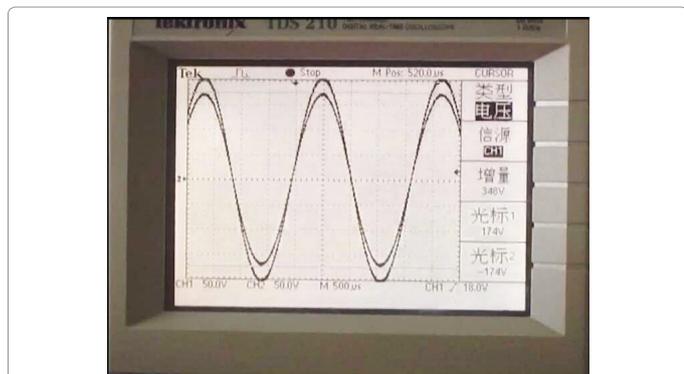


Figure 6: This diagram shows that when $V_{\infty} = 350V \gg V_c$, the two signals almost overlap, and $C \approx C_0$, this precious image means that large scale BEC at normal temperature has been observed. Note that this oscilloscope uses an attenuator, its attenuation rate is 2.

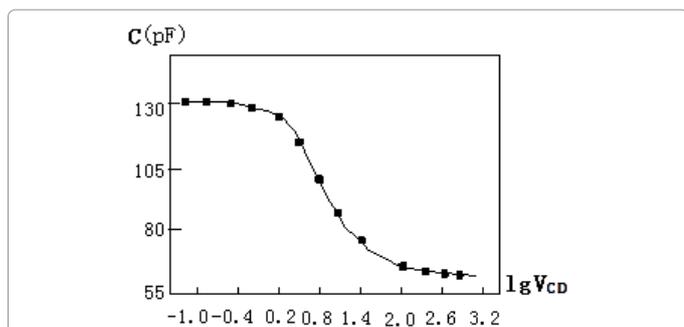


Figure 7: The $C - V$ curve shows that when $V \gg V_c$, the capacitance decreased to $C \approx C_0$, it implies that almost all the Cs atoms were to line up along the field, and BEC occurs.

V(volt)	0.01	0.3	7.1	63	173	232	350	∞
C(p F)	130	130	118	76.6	70	69	68	66
χ_e	0.9697	0.9697	0.7879	0.1606	0.0606	0.0454	0.0303	0
P(e. cm ⁻²)	7.9×10^3	2.4×10^6	4.6×10^6	8.2×10^6	8.5×10^6	8.58×10^6	8.63×10^6	8.72×10^6

Table 2: The measured values of Cs vapor at different voltages T=353K.

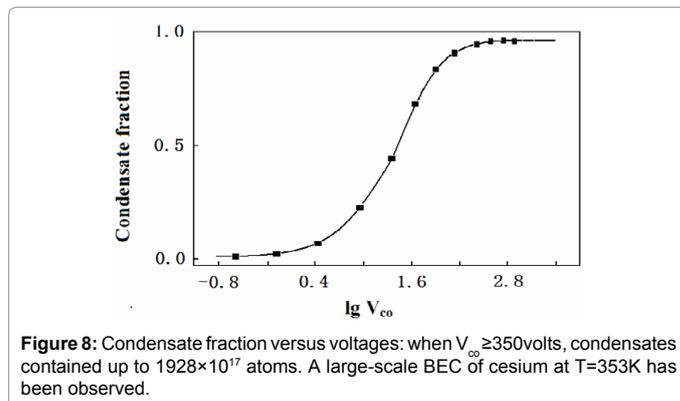


Figure 8: Condensate fraction versus voltages: when $V_{\infty} \geq 350$ volts, condensates contained up to 1928×10^{17} atoms. A large-scale BEC of cesium at T=353K has been observed.

scale BEC has been observed when $V_2 \geq 350$ volts [10]. When $V \gg V_c$, the capacitance $C \approx C_0$, $\chi_e \approx 0$, and the polarization increase to $P = P_{\max}$, the changes of these electrical quantities show that a large-scale BEC at normal temperature has been observed.

The condensate fraction

The local field acting on a molecule or atom in a gas is almost the same as the external field E [23]. For polar molecules or atoms, $\chi_e = na + n d_0 L(a) / \epsilon_0 E$, where $a = d_0 V / kTH$. Langevin function $L(a) = [(e^a + e^{-a}) / (e^a - e^{-a})] - 1/a = \coth a - 1/a$. $L(a)$ equals the average value of $\cos \theta$ [26]:

$$L(a) = \langle \cos \theta \rangle = f \int_0^\pi \cos \theta \exp(d_0 E \cos \theta / kT) \sin \theta d\theta,$$

$$f = \left[\int_0^\pi \exp(d_0 E \cos \theta / kT) \sin \theta d\theta \right]^{-1} \quad (6)$$

Where, f is normalized constant, θ is the angle between d_0 and E [25]. The electric polarizability of cesium is $\alpha = 59.6 \times 10^{-30} \text{ m}^3$ [27], the density $n < 8.0 \times 10^{22} \text{ m}^{-3}$, and induced susceptibility $\chi_e = n\alpha < 4.8 \times 10^{-6}$ can be neglected. We obtain $\chi_e = n d L(a) / \epsilon_0 E$, where d is PDM of Cs atom. When $a \ll 1$, $L(a) \approx a/3$, and $\chi_e = n d_0^2 / 3kT\epsilon_0$, this is the familiar Langevin formula. The polarization P, the average dipole moment per unit volume, is defined as

$$P = \chi_e \epsilon_0 E = n d L(a) \quad (7)$$

Formula (6) and (7) are clearly indicated that $L(a)$ equals the condensing fraction of BEC. When $a \gg 1$, $L(a) \approx 1$ and $P_{\max} \approx nd$, it implies the saturation polarization. Note that $E = V/H$ and the formula of the parallel-plate capacitor $\epsilon_0 = C_0 H/S$, from $\chi_e = n d L(a) / \epsilon_0 E$, we obtain

$$\text{Condensate fraction: } L(a) = a(C - C_0) / \eta \text{ or } C = \eta L(a) / a + C_0 \quad (8)$$

Where, $\eta = Snd^2 / kTH$ is the capacitance constant. When $V_1 = 0.3 \text{ V}$, $a_1 \ll 1$, $L(a) = a/3$ and $\eta = 3(C_1 - C_{20}) = 192 \text{ pF}$. When $V_2 = 350 \text{ V}$, $C_2 - C_{20} = 2.0 \text{ pF}$, $a_2 = \eta L(a_2) / C_2 - C_{20} = 95$, $L(95) = 0.9896$. The critical voltage is $V_c = V_2 2\pi e / a_2 = 62.9 \text{ V}$.

Notice that we deduced equation (8) from the formula of the

V(volt)	0.01	0.30	7.1	63	173	232	350	∞
C(p F)	130	130	118	76.6	70.0	69.0	68.0	66.0
a	2.7×10 ⁻³	0.0814	1.93	17.1	47.0	63.0	95.0	∞
L(a)	9×10 ⁻⁴	0.02714	0.5249	0.9415	0.9787	0.9841	0.9896	1.0
N _c (10 ¹⁷)	0.0018	0.0529	1.023	1.834	1.906	1.917	1.928	1.948
S (Nk)	2.531	2.530	2.076	-1.2×10 ⁻⁴	-1.012	-1.305	-1.716	-ln∞

Table 3: a, L(a), N_c and S of Cs vapor at different voltages (V) T=353K.

Electric susceptibility of alkali atoms	$\chi_e = A$	$\chi_e = A + B/T$
PDM of alkali atom	d=0	d= (C - C ₀)/V/L(a)Sn
Entropy of the system		S=Nk ln 2π e /a
The energy density of the system		U=-n d L(a) E
Condensate fraction formula		L(a)=a (C - C ₀)/η(η is constant)
The density of alkali gas	10 ¹³ -10 ¹⁵ cm ⁻³	5.65×10 ¹⁴ cm ⁻³
Critical condition of BEC	Tc ≤ 10 μK	Vc= 63 volts
Number of condensed atoms	10 ⁴ -10 ⁶	1.928×10 ¹⁷
Most atoms are in BEC	T << Tc is difficult	V >> Vc is easily
Macroscopic occupation of ground state	No	Yes
A large-scale BEC	No	Yes

Table 4: A sharp contrast between previous and our BEC experiment.

parallel-plate capacitor $\epsilon_0 = C_0 H/S$, so the cylindrical capacitor must be regarded as an equivalent parallel-plate capacitor with the plate area $S = C_0 H/\epsilon_0$. In the three experiment, the equivalent area is $S'_0 = C'_0 H_0/\epsilon_0 = 5.855 \times 10^{-2} \text{ m}^2$ and $S'_2 = C_{20} H_2/\epsilon_0 = 5.069 \times 10^{-2} \text{ m}^2$. Since the density n_2 is unknown, from equation (8) we obtain $n_2 = (C_2 - C_{20}) V_2 L(a) S'_0 n'_0 / (C'_2 - C'_0) V_0 L(a_2) S'_2 = 5.652 \times 10^{14} \text{ cm}^{-3}$, where $a = a_2 V_0 T_2 H_2 / T_0 H_0 V_2 = 0.1722$ and $L(a) = 0.05729$. According to the root mean square rule, the statistical error of the measured value is $\Delta n/n \leq 0.05$, and the systematic error is $\Delta n/n \leq 0.04$, so $n_2 = [5.65 \pm 0.28(\text{stat}) \pm 0.23(\text{syst})] \times 10^{14} \text{ cm}^{-3}$. The volume of C_{20} is $S'_2 H_2 = 3.447 \times 10^2 \text{ cm}^3$, the total number of Cs atoms is $N = 1.948 \times 10^{17}$ and the number of condensing atoms is $N_c = N L(a)$. Table 3 provides a complete analysis of BEC of Cs atoms.

From equation (8) we can deduce the formula of the atomic PDM

$$d = (C - C_0) V / L(a) n S \quad (9)$$

For example, by second experimental data, we obtain $d_{Cs} = 2.469 \times 10^{-29} \text{ C.m} = 1.543 \times 10^{-8} \text{ e.cm}$, where $(C - C_0) = 2 \text{ pF}$, $V = 350 \text{ volts}$, $L(a) = 0.9896$, $S = S'_2 = 5.069 \times 10^{-2} \text{ m}^2$ and $n = n_2 = 5.652 \times 10^{20} \text{ m}^{-3}$. The statistical error of the measured value is $\Delta d_1/d \leq 0.08$ due to $\Delta C'_t/C'_t \leq 0.005$, $\Delta C'_0/C'_0 \leq 0.003$, $\Delta V_0/V_0 \leq 0.03$, $\Delta n_0/n_0 \leq 0.06$, $\Delta S'_0/S'_0 \leq 0.04$. Considering all systematic error, $\Delta d_2/d \leq 0.06$, so $d_{Cs} = [1.54 \pm 0.12(\text{stat}) \pm 0.09(\text{syst})] \times 10^{-8} \text{ e.cm}$.

In our experiment, the maximum field intensity is $E_{\text{max}} = V_2/H_2 = 5.15 \times 10^4 \text{ V/m}$, and therefore the maximum induced dipole moment is $d_{\text{ind}} \leq 2.72 \times 10^{-35} \text{ C.m} = 1.70 \times 10^{-14} \text{ e.cm}$, and it can be neglected.

Discussion

If cesium atom has a non-zero PDM, why it does not violate the time reversal and parity symmetry?

According to quantum theory, an atom in its ground state at most has an extremely small PDM, $d \leq e \times 10^{-20} \text{ cm}$, it points along the nuclear spin axis and arise mainly from the nuclear spin [19,21,22]. Under time reversal the direction of the spin changes, while the direction of the PDM does not change. Therefore, the PDM of an atom would violate time reversal symmetry. By the CPT theorem it also implies a violation of CP symmetry [21]. A representative result as follows: $d(\text{Hg}) = [0.49 \pm 1.29(\text{stat}) \pm 0.76(\text{syst})] \times 10^{-29} \text{ e.cm}$ [22]. In short, if the PDM of atom violates time reversal symmetry, it must have two characteristics: the

first is very small, $d \ll 10^{-20} \text{ e.cm}$, and the second is arises from the nuclear spin.

However, the linear Stark effect of hydrogen atom provides a new example. This effect showed that the hydrogen atom ($n=2$) has a PDM of magnitude $-3ea_0 = 1.59 \times 10^{-8} \text{ e.cm}$. Since the nuclear spin was completely irrelevant to the calculation of the PDM, and Bohr radius ($a_0 = 0.53 \times 10^{-8} \text{ cm}$) is far greater than the nuclear radius ($r_0 \approx 10^{-11} \text{ cm}$), so this PDM has nothing to do with the nuclear spin, and only arises from the asymmetrical charge distribution of hydrogen atom. LD Landau once stated that the hydrogen atom has a dipole moment whose mean value is $d = -3ea_0$, "This is in accordance with the fact, in a state determined by parabolic quantum numbers, the distribution of the charges in the atom is not symmetrical about the plane $z=0$ " [18]. So similar to the first excited state of hydrogen atom, the PDM of Cs atom doesn't arise from the nuclear spin but from asymmetrical charge distribution, and it doesn't violate time reversal and parity symmetry [21,22].

If cesium atom has a large PDM, why its linear Stark effect has not been observed?

This is a challenging question. As two concrete examples, first let us deal with the fine structure and the linear Stark shifts of the hydrogen ($n=2$). The wavenumber of the fine structure of the hydrogen ($n=2$) is only 0.33 cm^{-1} for the Ha lines of the Balmer series, where $\lambda = 656.3 \text{ nm}$. The splitting is only $\Delta \lambda = 0.33 \times (656.3 \times 10^{-7})^2 = 0.014 \text{ nm}$, therefore the fine structure is difficult to observe [28]. The linear Stark shift of the hydrogen ($n=2$) is proportional to the field intensity: $\Delta W = d_H E = 1.59 \times 10^{-8} E \text{ e.cm}$. When $E = 10^3 \text{ V/cm}$, $\Delta W = 1.59 \times 10^{-3} \text{ eV}$, this corresponds to a wavenumber of 12.8 cm^{-1} . So the linear Stark shifts is $\Delta \lambda = \Delta W \lambda^2 / hc = 12.8 \times (656.3 \times 10^{-7})^2 = 0.55 \text{ nm}$, and the linear Stark shift of the hydrogen ($n=2$) is easily observed [28]. However, when $V = 350 \text{ V}$, the most field intensity is $E_{\text{max}} = V/H = 515 \text{ V/cm}$. When the external electric field increases to $E = E_{\text{max}}$, almost all the Cs atoms (more than 98.9%) were to line up along the field, Cs vapor no longer absorb energy, this Stark effect will not occur. if the PDM of cesium is $d_{Cs} = 1.54 \times 10^{-8} \text{ e.cm}$, and the most splitting of the energy levels is $\Delta W_{\text{max}} = d_{Cs} E_{\text{max}} = 7.93 \times 10^{-6} \text{ eV}$. This corresponds to a wavenumber is $\Delta W_{\text{max}} / hc \leq 0.064 \text{ cm}^{-1}$. On the other hand, observed values for a line pair of the first primary series of Cs atom ($Z=55, n=6$) are $\lambda_1 = 894.3 \text{ nm}$ and $\lambda_2 = 852.1 \text{ nm}$ [24,28]. So the most linear Stark shift of Cs atoms is only $\Delta \lambda = \Delta W (\lambda_1 + \lambda_2)^2 / 4hc = 0.0048$

nm. It is so small, in fact, that a direct observation of the linear Stark shifts of Cs atom is not possible by conventional spectroscopy!

A sharp contrast between previous and our BEC experiment

In order to illustrate the original innovation of our experiments, Table 4 gives a detailed comparison between our and previous BEC experiments [2-6]. From the formula of PDM, if $(C - C_0) \neq 0$, and is bound to get: $d \neq 0$. But in the past few decades, scientists have never measured the capacitance of the alkali atoms, so they missed this significant discovery.

Rigorous mathematical proof that BEC of cesium vapor is bound to occur

From equation (8) $C = \eta L(a)/a + C_0$, we construct a new function

$$f(a) = L(a)/a = [(e^a + e^{-a}) / a (e^a - e^{-a})] - 1/a^2 \quad (10)$$

Now we find the inflection point of the function $f(a)$. From the second order derivative of this function is zero, we obtain $f''(a) = [(2e^{-3a} - 2e^{-a} - 8ae^{-a} + 8a^2e^{-a} + 2e^{3a} - 2e^a + 8ae^a + 8a^2e^a) / a^3(e^a - e^{-a})^3] - 6/a^4 = 0$. From $f''(1.9296812) = -10^{-9} < 0$ and $f''(1.9296814) = 2 \times 10^{-9} > 0$, we obtain its inflection point is $a_p = 1.9296813 \approx 1.93$. The voltages of the inflection point is $V_p = 7.1V$ and $\lg V_p = 0.85$. By contrast with Fig.4, it is clear that our polarization equation Eq.(8) is correct. This result shows that when $\lg V_{CD} < 0.85$, the C-V curve is upper convex; when $\lg V_{CD} > 0.85$, the curve is down convex. This result shows that when the voltage increases to thousands of volts, C will inevitably approach C_0 .

A discussion of many interesting questions, such as the definition of Boltzmann constant, can be found in the reference 10.

Conclusions

In theory, $\langle \psi_E | -er | \psi_E \rangle = 0$ only means that the average PDM of large number of alkali atoms is zero, but doesn't mean that the PDM of individual alkali atom is zero. Despite 6s and 6p states of cesium are not degenerate, but Cs may be polar atom doesn't conflict with quantum mechanics because it is hydrogen-like atom.

All kinds of alkali atoms are non-polar atom, which is an untested hypothesis, and we must test it by experiments. We measured the capacitance at different temperatures, our experiment proved that Cs atom is polar atom, and it becomes a dipole. But in the past, scientists have never measured the capacitance of alkali atoms, so they missed this significant discovery.

BEC has three main features: BEC is a macroscopic occupation of the ground state; BEC is a condensation in momentum space; Bose gas would undergo a phase transition. Our experiments are fully in line with these three main features, so although we have not used laser cooling techniques, our experiments are an ideal BEC.

Ultra-low temperature is in order to make Bose gas phase transition, and we use the critical voltage V_c to describe phase transition, and therefore ultra-low temperature is not necessary. Because there are only a handful of laboratories in the world that can achieve ultra-low temperatures, it limits the majority of scientists involved in the study. This new technology will completely change the current situation, and it will be used and praised by the vast majority of scientists.

The entropy of a system is a measure of the disorder of molecular or atomic motion. No doubt, it is the most important concept in BEC. This formula, $S = Nk \ln 2\pi e/a$, contains two fundamental constants in nature

($\pi \approx 3.14159$ and $e \approx 2.71828$), it reflects the objective laws of BEC.

The presence of the inflection point proved that the saturation polarization of Cs vapor is inevitable, and therefore BEC of Cs vapor is also a certainty. Including the introduction of entropy, this is the two successful examples of applying mathematics to explain abstruse physical phenomena.

Both BEC and superconductivity are condensed in the momentum space. When the resistance R or the capacitance C was reduced to close to the vacuum value, these two kinds of condensation occur. So BEC is also a quasi-superconducting state. This fact fully reflects the harmony and unity of nature.

Our experiments are easily repeated in other laboratories, because the details have been described in this paper. Once scientists completed these measurements, they will obtain the same results as our experiments. They will discover that a large-scale BEC of cesium atoms is easily observed when an electric field is applied!

Acknowledgements

This research was supported by the NSF of Guangdong Province (Grant No. 021377). The author thanks Prof. Xiang-You Huang (Peking University), Dr. Yu-Sheng Zhang, Director Xun Chen, Engineer Yi-Quan Zhan (Peking University), Engineer Jia You and our colleagues Rui-Hua Zhou, Ming-Jun Zheng, Xue-Ming Yi, Zhao Tang and Xin Huang for their help with this work.

References

1. Pais A (1982) *Subtle is the Lord: The Science and the Life of Albert Einstein*. Oxford University Press, USA. p: 430.
2. Bradley C, Sackett C, Tollett J, Hulet R (1995) Evidence of Bose-Einstein Condensation in an Atomic Gas with Attractive Interactions. *Phys Rev Lett* 75: 1687.
3. Davis K, Mewes MO, Andrews MR, van Druten NJ, Durfee DS, et al. (1995) Bose-Einstein Condensation in a Gas of Sodium Atoms. *Phys Rev Lett* 75: 3969.
4. Modugno G, Ferrari G, Roati G, Brecha RJ, Simoni A, et al. (2001) Bose-Einstein Condensation of Potassium Atoms by Sympathetic Cooling. *Science* 294: 1320-1322.
5. Anderson MH, Ensher J, Matthews M, Wieman C, Cornell E (1995) Observation of Bose-Einstein Condensation in a dilute Atomic Vapour. *Science* 269: 198.
6. Weber T, Herbig J, Mark M, Nägerl H, Grimm R (2003) Bose-Einstein Condensation of Cesium. *Science* 299: 232-235.
7. Blundell SJ, Blundell KM (2009) *Concepts in Thermal Physics*. (2nd edn) Oxford University Press, USA. p: 369.
8. Huang XY, You PL (2002) Permanent Electric Dipole Moment of an Rb Atom. *Chin Phys Lett* 19: 1038.
9. Huang XY, You PL, Du WM (2004) The Experiment on the Saturation Polarization of Rb Vapour. *Chin Phys* 13: 11.
10. You PL (2016) Bose-Einstein Condensation in a Vapor of Sodium Atoms in an Electric Field. *Physica B* 401: 84-92.
11. Yan Z, Konotop VV (2009) Exact Solutions to Three-Dimensional Generalized Nonlinear Schrödinger Equations with Varying Potential and Nonlinearities. *Phys Rev E* 80: 036607.
12. Yan Z, Konotop VV, Yulin AV, Liu WM (2012) Two-dimensional Superfluid Flows in Inhomogeneous Bose-Einstein Condensates. *Phys Rev E* 85: 016601.
13. Yan Z, Konotop V, Akhmediev N (2010) Three-dimensional Rogue Waves in Nonstationary Parabolic Potentials. *Phys Rev E* 82: 036610.
14. Ballentine LE (1998) *Quantum Mechanics a Modern Development*. World Scientific Publishing Co. Pte. p: 286.
15. Ballentine LE (1970) The Statistical Interpretation of Quantum Mechanics. *Rev Mod Phys* 42: 358.

-
16. Ballentine LE, Yang Y, Zibin JP (1994) Inadequacy of Ehrenfest's Theorem to Characterize the Classical Regime. *Phys Rev A* 50: 2854.
 17. Griffiths DJ (2005) *Introduction to Quantum Mechanics*. (2nd edn) Pearson Education. p: 15.
 18. Landau LD, Lifshitz EM (1999) *Quantum Mechanics*. Beijing World Publishing Corporation. p: 285-290.
 19. Schiff LI (1968) *Quantum Mechanics*. McGraw-Hill, New York. p: 252.
 20. Greiner W (1994) *Quantum Mechanics an Introduction*. Springer-Verlag Berlin/Heidelberg. p: 213.
 21. Fortson N, Sandars P, Barr S (2003) The Search for a Permanent Electric Dipole Moment *Physics Today*. pp: 33-39.
 22. Griffith WC, Swallows MD, Loftus TH, Romalis MV, Heckel BR, et al. (2009) Improved Limit on the Permanent Electric Dipole Moment of Hg199. *Phys Rev Lett* 102: 101601.
 23. Jackson JD (1999) *Classical Electrodynamics*. (3rd edn) John Wiley and Sons, Inc, USA. p: 162-173.
 24. Dean JA (1998) *Lange's Handbook of Chemistry*. McGraw-Hill, Inc, New York.
 25. Feynman RP, Leighton RB (1964) *The Feynman Lectures on Physics: Volume 2*. Addison-Wesley Publishing Co, Boston. p: 11-3.
 26. Bottcher CJF (1973) *Theory of Electric Polarization*. Elsevier, Amsterdam. p: 161.
 27. Lide DR (1998) *Handbook of Chemistry and Physics*. CRC Press, Boca Raton New York.
 28. Haken H, Wolf HC (2000) *The Physics of Atoms and Quanta*. Springer-Verlag Berlin Heidelberg. p: 172.