

In-plane Displacement Measurement for Analytical Strain Determination

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Abstract

We present an optical experimental approach and an analytical model to calculate the strain on a brick when an external force-pulse is indirectly applied on its surface. We assumed that the displacement produced on the brick is energy that it transforms to deformation. To measure the displacement, we applied one laser-beam to the brick and took image data of the speckles created by its reflection. The speckles were processed using the digital image correlation technique and allowed us to evaluate the relative displacements caused on the surface of the brick. Then, based on the kinetic energy of forced and damped oscillations, we calculated analytical strain values of the order of $\epsilon \approx 1.52 \times 10^{-6}$.

Keywords: Laser speckle; In-plane displacement measurement; Digital image correlation; Force-pulse; Strain

Introduction

The strain measurement in materials is a relevant study as structures are involved in different stresses. It is well-known, from the stress-strain relation that every material eventually fails when it's close to the rupture part. Due to this possible consequence, the development of non-destructive strain measurement methods is a challenge for the assessment of structural elements in existing constructions.

Optical methods are an example of non-destructive methods, which have been used widely for measuring in-plane strain, stress, displacements, among others [1-3]. In the structural field, there is a method to measure structural displacement using a charge-coupled-device camera plus digital signal processing [4]. Other methods are for instance the one developed by Jong-Woong et al. whom used two web cams to evaluate the horizontal displacement of high-rise buildings [5]; Hyoung-Suk et al. employed digital image processing, introducing a new dynamic vision system to measure displacements [6]; Sung-Wang and Nam-Sik [7] proposed digital image processing plus correlation to measure displacements in a structure model using a single point and multiple points.

As an alternative to measure displacements with a non-destructive technique, we implemented In-plane displacement measurements using Digital Image Correlation technique, which is able to measure an external force-pulse signal displacement and the strain caused by it. It is known that the simplest image matching procedure is Cross Correlation (CC) (which can be performed either in physical space [8,9] or in Fourier space), the use of Fast Fourier Transform (FFT) helps to evaluate the CC function quickly [10]. Anuta [11] took advantage of the high speed of FFT algorithm doing digital multispectral and multi-temporal statistical pattern-recognition. One of the advantage in doing Digital Image Correlation, is that users can create their own codes, as it is image processing, it is possible to process the images for a better result, such as selecting the best zone of interest.

To test our proposed method, we measured the displacement provoked by the propagation of an external force-pulse, using a laser beam focusing on the cross-section of a brick sample. The brick is a construction material made of backed clay, generally in a rectangular form. The material is in rest position and as the incoming force-pulse is propagated and strikes it, the brick will move according to the energy and will return to its initial, rest position.

The novelty in our proposed method is in coupling a displacement measurement to an analytical strain calculus. The main advantage is that this technique uses few optical components and is cheap for further applications: such as an optical displacement sensor.

It is well known that structures are under different kind of loads and also to external incoming-energies, thus it is important to have a non-destructive method to measure displacements and then calculate the strain. So, the aim of our study is to present a new alternative method to calculate the strain when the material is affected by an incoming force.

Methodology

Laser speckle generation

A laser beam propagates as a nearly unidirectional wave with little divergence and with finite cross-section [12]. The simplest beam and the most known is the Gaussian, because its characteristics and evolution are well-known [13]. The intensity distribution of a Gaussian beam is given by:

$$I(r, z) = I_0 \exp\left[-\frac{r^2}{\omega^2(z)}\right] \quad (1)$$

where I_0 is the intensity of the beam at $z=0$, $r=(x^2 + y^2)^{1/2}$ and $\omega(z)$ is the beam spot size at z plane [14]. In the present work, the beam propagates through a lens with a focal length f and is focused to the cross-section of a brick sample. To study this phenomenon, we need the Fresnel diffraction equation and the transformation due to the transmission through a lens [15] given by:

$$U_f(u, v) = \frac{\exp\left[\frac{ik}{2f}(u^2 + v^2)\right]}{i\lambda f} \iint_{-\infty}^{\infty} U(x, y) \cdot t_A \cdot \exp\left[-\frac{ik}{f}(ux + vy)\right] dx dy \quad (2)$$

where $U_f(u, v)$ is the distribution, k is the wave number equal to $2\pi/\lambda$

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λ , λ , is the wavelength of the laser beam, f is the focal length of a lens and u, v are the Fourier domain frequencies. The transmission through a lens t_A is given by:

$$t_A = \exp\left[\frac{ik}{2f}(x^2 + y^2)\right] \quad (3)$$

and as our intensity is produced by a laser beam with an electromagnetic transverse mode TEM_{00} according to the manufacturer [16], we can write the signal distribution $U(x, y)$ as:

$$U(x, y) = \exp\left[-\frac{(x^2 + y^2)}{\omega^2}\right] \quad (4)$$

Introducing equations (3) and (4) in (2), expanding formally and changing to polar coordinates we obtain:

$$U(\rho) = \frac{\exp[C\rho^2]}{i\lambda} \quad (5)$$

where $\rho^2 = u^2 + v^2$,

$$C = \frac{ik}{2f} + \frac{b^2}{4a} \quad (6)$$

and

$$a = \frac{ik}{2f} - \frac{1}{\omega^2} \quad (7)$$

$$b = \frac{ik}{f} \quad (8)$$

As equation (5) shows, we can assume that the intensity arrived at the focal length point is still a Gaussian distribution and as it impinges to a rough-like material, all the incoming light is transformed to random intensity distribution; thus, a speckle pattern is formed [17]. Those patterns are formed when coherent light is reflected from a rough surface or when light is propagated through a medium with random refractive index fluctuations [18].

In general the statistical properties of speckle patterns depend both on the coherence of the incident light and the detailed properties of the random surface or medium. Generally these kind of distributions are asymptotically Gaussian as it follows from the central limit theorem [19].

Correlation technique

Digital Image correlation (DIC), is an optical method that examines image data taken while samples of materials (bricks, metals, among others) are subjected to mechanical tests. This technique consists on capture consecutive images with a digital camera to evaluate the change in surface characteristics and understand the behavior of the material while it is subjected to an incoming force.

For correlation, it is necessary to obtain the spectrum between two images; let's assume that A and B are a pair of 2-D images, where A represents the reference image and B the displaced image [20]; therefore, it is taken the FFT for both images and the Cross-spectrum (C_s) is defined by:

$$C_s(u, v) = F(u, v) \cdot G^*(u, v) \quad (9)$$

where F is the Fourier transform of the first image and G^* is the complex-conjugate Fourier transform of the second image; once C_s is obtained, the cross-correlation (CC) is defined by:

$$C_c = \frac{\text{Re}(F^{-1}[C_s])}{s_1^2 s_2^2 M} \quad (10)$$

where s_1^2 and s_2^2 are the standard deviation of images A and B , respectively and M is the image size (in our case, we worked with 1080*1080 images). Thus, we normalize as we are assuming a totally Gaussian distribution in the speckle pattern.

Theoretical analysis

As we are indirectly applying force-pulses, we assume that the brick is affected first by the force impact, causing a peak amplitude, it can be modeled as a short time force oscillatory wave given by:

$$f(t) = A_p \cdot \sin(w_p t - \alpha) \quad (11)$$

where A_p is the amplitude of $f(t)$, w_p and α are the frequency and the phase respectively [21]. The structure will react to the force and return to its initial point according to damped oscillations:

$$g(t) = A_d \cdot \exp(-\gamma t) \cdot \sin(w_d t - \alpha) \quad (12)$$

where A_d is the amplitude, γ is the damping coefficient and w_d is the frequency of the response [21]. In the present research, the brick sample is completely still at time $t=0$, the incoming force P and the related energy E_p would be set by [21,22]:

$$P = \sigma_p \cdot A_i \quad (13)$$

$$E_p = P \cdot \bar{d} \quad (14)$$

where σ_p is the stress, A_i is the impact area and \bar{d} is the maximum displacement registered by DIC technique [22,23]. As the impact is along one axis of the plane, we can relate the total energy as a sum of both forced and damped Kinetic Energy waves:

$$E_k = \frac{1}{2} \cdot m[f'(t)^2 + g'(t)^2] = E_{kp} + E_{kd} \quad (15)$$

Where $f'(t)$ and $g'(t)$ are the velocity obtained through deriving the signal, E_{kp} and E_{kd} are the Kinetic Energy of the forced and damped oscillations, respectively. The force impacts the surface of the table and the energy E_p produced by this force is propagated through the brick. By using the energy law we can establish that:

$$E_p = E_k \quad (16)$$

Substituting equations (13) and (14) in (16), we can obtain the strain provoked by the force by:

$$\varepsilon = \frac{E_k}{EA\bar{d}} \quad (17)$$

where ε is the dimensionless strain and E is the Young modulus of our material. We assume that every strain calculated remains in the elastic part of the stress-strain curve as the induced force is weak enough to provoke damage on the material.

Experimental Set-up and Data Processing

A table (T) is used to propagate four pulses to a brick with a Young's modulus of 15 GPa [24]. The brick (Sa) was placed over the table and all pulses were induced through the longitudinal direction (Lon) of the brick. In Figure 1 is represented schematically our experimental set-up. The output of a Diode-Pumped Solid-State Laser (L) with a wavelength $\lambda=532$ nm and power of 220 mW [16], is placed at 20 cm from the sample, the wave is propagated through a converging lens (l) with a focal distance of $f=7$ cm which is placed in front of the sample (Sa) in order to irradiate the cross-section face and the scattering reflection impacts a screen (Sc) which is placed in front of the sample at 15 cm; the brick is first completely still, then the incoming force-pulse strikes horizontally to the table causing movement, while this action is taking place, the speckles generated are recorded with a Video Camera (VC).

The video camera used for this experiment can record in a rate of 240 frames per second with a 1980x1080 image resolution. Once the video was recorded, we used Video to JPG free software [25]; this software enables the user to split the video into each possible frame. We wrote a program in Matlab for doing Digital Image Correlation (DIC) in which we load and process all the frames. The image processing is depicted in the following images: In Figure 2a it is shown how the code loads the image and then it is converted into a gray-scale image (Figure 2b). Therefore the image is crop into 1080x1080 to begin the FFT analysis of all depicted images (Figure 2c), taking the first as the reference and the others as displaced images.

Results and Discussions

As a forementioned, we indirectly applied external force-pulses to a brick, and measured the induced displacement by means of in-plane DIC; from this we got an analytical approach of the strain provoked by the induced pulse. All incoming force-pulses are studied in the focal plane of the lens and along to the longitudinal direction of the brick. In Figures 3-7 we show the obtained results.

Figure 3 shows the result of the first test: in (a) We can see the response of the system table-brick to the induced force-pulse. We can see in (b) the signal obtained from DIC technique, shows a 23 mm maximum amplitude, the decay of the recorded pulse last 10 sec approximately. Figure 3b is the analytical approach to the movement

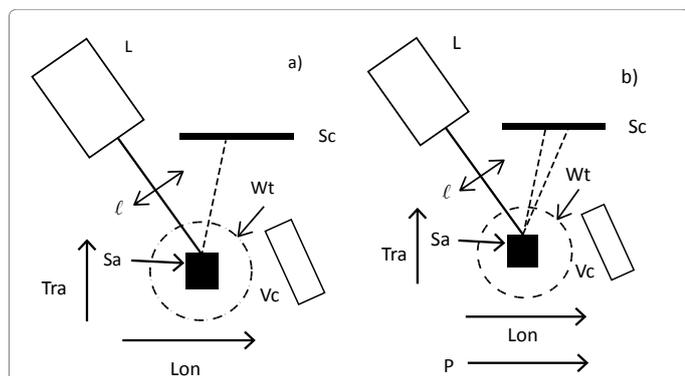


Figure 1: Experimental set-up: (a) Brick totally still at t=0; (b) Movement registration when the force-pulse is applied.

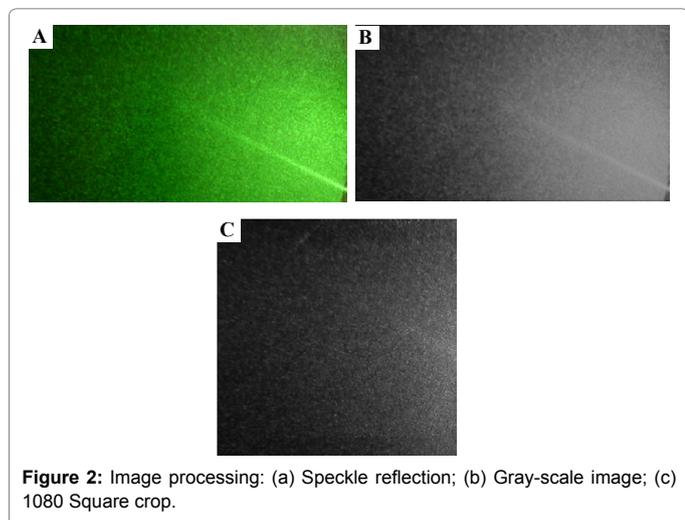


Figure 2: Image processing: (a) Speckle reflection; (b) Gray-scale image; (c) 1080 Square crop.

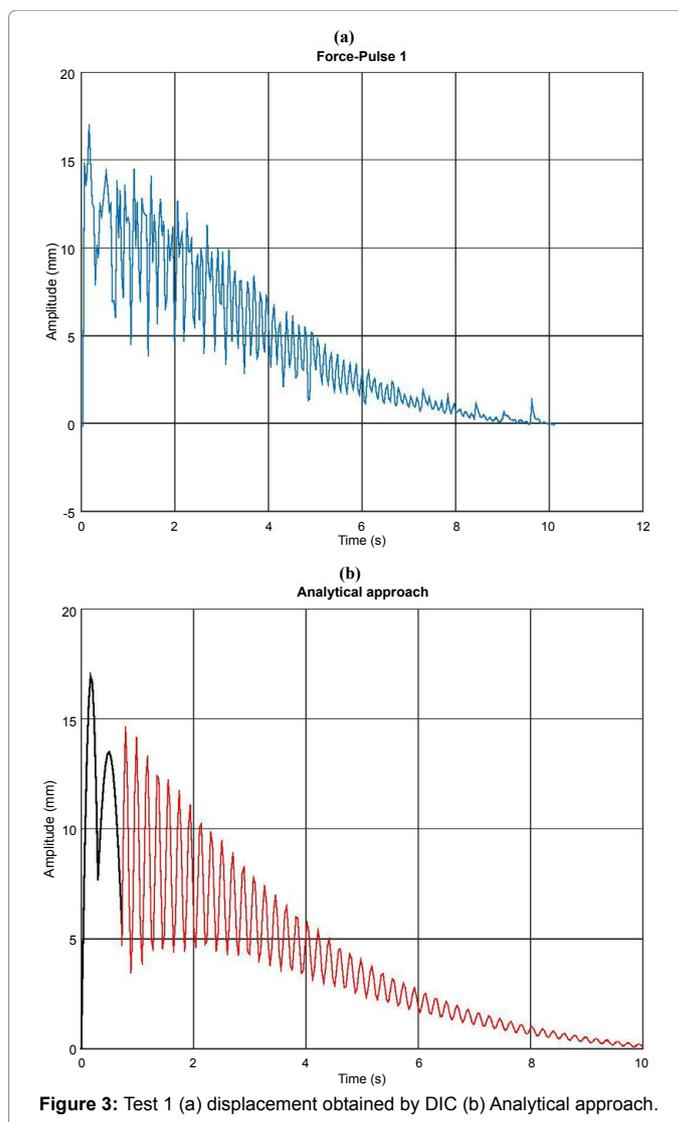


Figure 3: Test 1 (a) displacement obtained by DIC (b) Analytical approach.

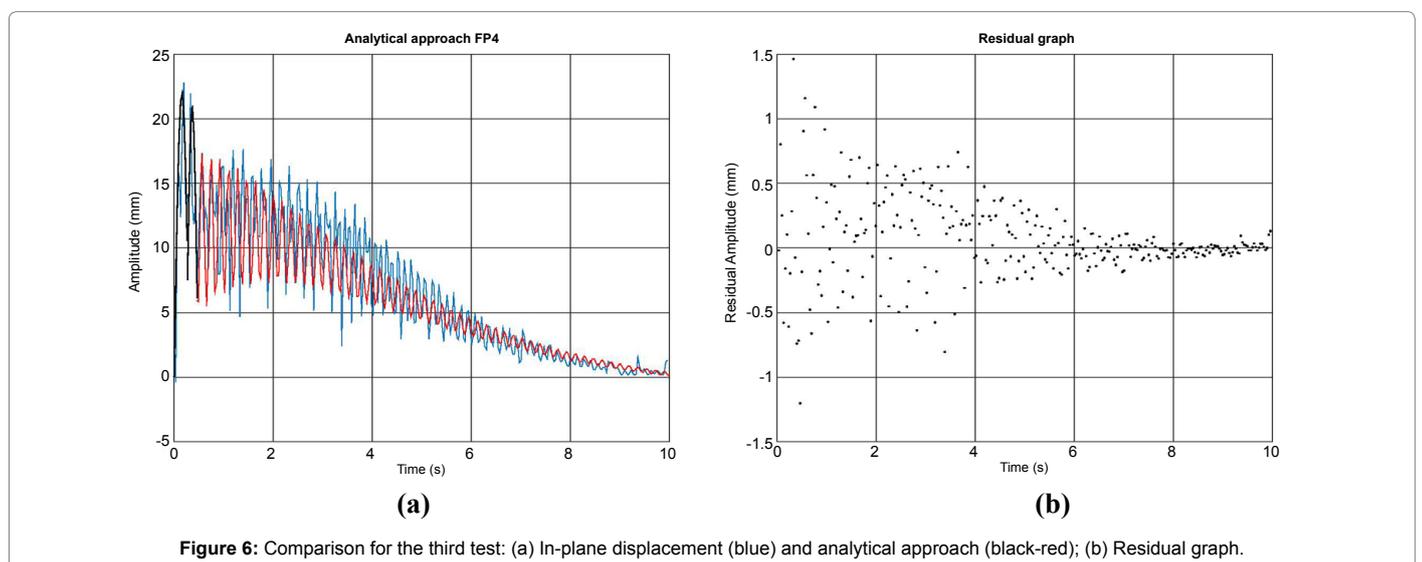
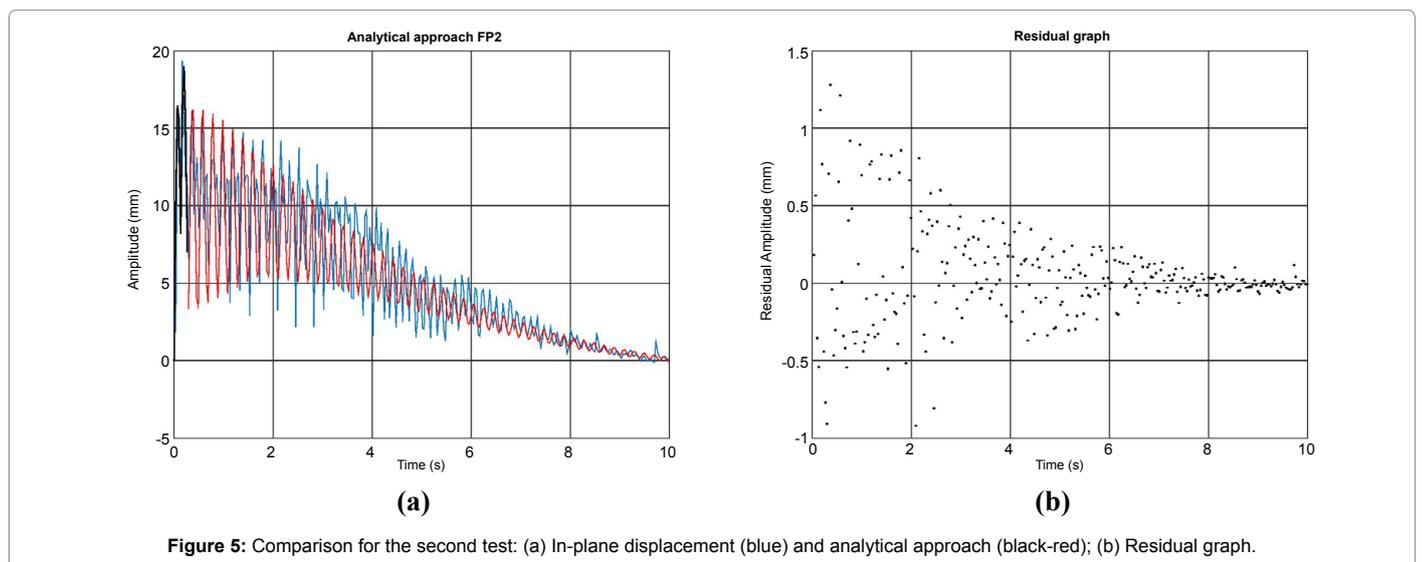
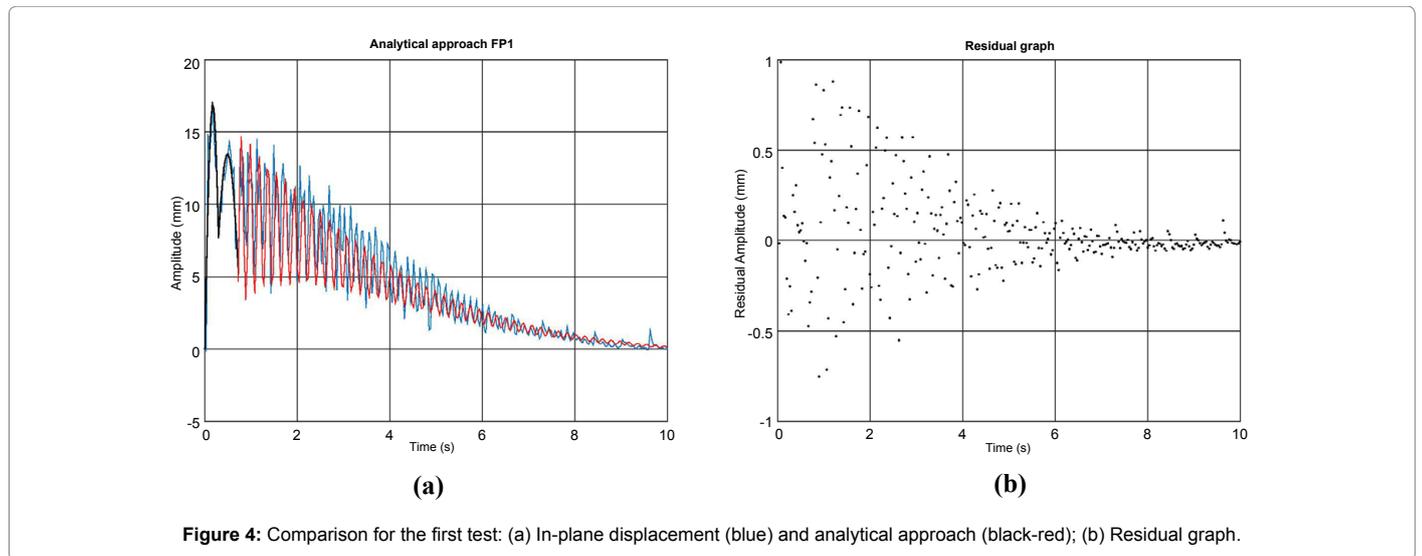
according to Equations (11) (black plot) and (12) (red plot), the values used to do the analytical approach are in Table 1. In Figure 4 we show the comparison between the displacement measured and the analytical approach.

In this first test, the analytical approach has a correlation coefficient of 0.83 and has a mean accuracy of 94% from the digital signal acquired. The accuracy has obtained by calculating first the relative error:

$$e_{rr} = \frac{V_r - V_e}{V_r} \quad (18)$$

In which V_r and V_e are the real and experimental data. Once calculated a mean error we can obtain the mean accuracy by doing $1 - e_{rr}$. Next, we present the results derived from the application of another three pulses to the system table-brick.

In the comparison of the fourth tests, we can observe that the analytical model fits to the signal acquired by in-plane DIC, the mean correlation coefficient for them were 0.8, which is a good approximation to the real signal. Residual graphs were obtained by subtracting the analytical approach from the experimental data obtained by DIC, in them, we can observe that for the first 6 sec we have the most differences,



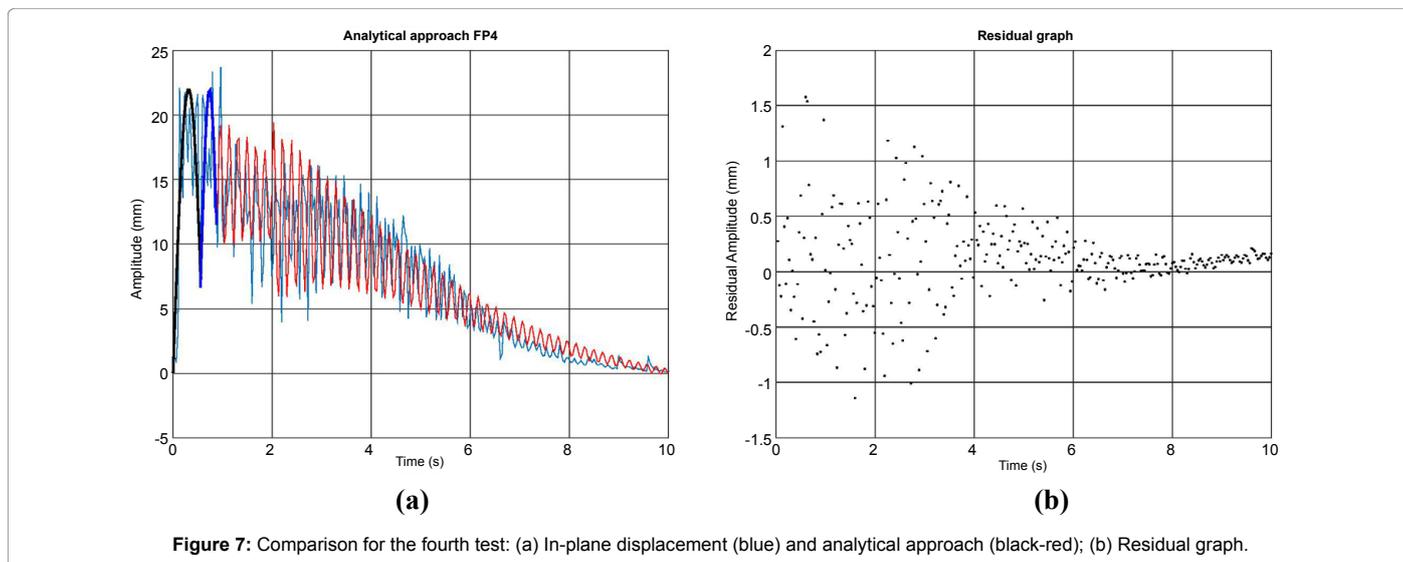


Figure 7: Comparison for the fourth test: (a) In-plane displacement (blue) and analytical approach (black-red); (b) Residual graph.

it is mainly because the analytical model follows a prescribed equation and the signal we acquire has fluctuations in time as a natural reaction to the induced force-pulse; in the interval time of 6 to 10 sec, we can observe that the analytical model has a better fit, giving us differences lower than 0.3.

The analytical approach were done by using equations (11) and (12), where the data used for them are described in Table 1.

The data of this table were obtained by means of fitting iteratively the values in the equations (11) and (12). We see that the analytical data used for the tests are almost the same values. We observe that every test has an analytical response with high frequency and the initial amplitude just varied 5 mm.

The strain values are calculated by the analytical model proposed in equation (17) and the data used are shown in Table 2.

In Table 2, we have a total Kinetic Energy E_k which is the sum of E_{kp} and E_{kd} as we established in the previous section, also it is seen that the impact area A_i has the same value, it is because we used the same object to induce the force-pulses, thus we measure its impact area. Third test has a particular behavior, comparing with first test, the amplitude A_d registered was higher but the strain was lower, according to Equation (17) in which is established that strain is inversely proportional to the maximum displacement registered \bar{d} .

As a summary, in Table 3 we show the number of the applied pulses, the related Kinetic Energy, the strain caused on the brick and the accuracy of the tests.

From Table 3, we can see that this technique allows to measure strain indirectly with a mean value of 1.528×10^{-6} . Also it is seen that there is a relation between energy and strain provoked; in pulses 1 and 3 we can see that there is almost the same total Kinetic Energy compared with pulse 2 and the strain varies in the second test about 0.1×10^{-6} ; for the third test it has a higher strain variation compared with pulses 1 and 2 and we referred it due to the maximum displacement registered in the signal. If we compare fourth test with the first one, we can see about 0.15 J more but the strain caused by it is just 0.1×10^{-6} greater.

Some other methodologies are used to obtain low values of strain such as strain gages, which can get values of strain in the micrometer

Test	$A_p(m)$	W_p (rad/s)	$E_{kp}(J)$	$A_d(m)$	W_d (rad/s)	γ (1/s)	$E_{kd}(J)$
1	0.017	9	0.0754	0.003	33	0.47	0.33471
2	0.017	10	0.0139	0.003	31	0.4	0.3703
3	0.022	10	0.0527	0.0035	35	0.4	0.3535
4	0.022	10	0.023	0.0015	35	0.4	0.5411

Table 1: Data used in Eq. (11) and (12) to compute the analytical model.

Test	E_k (J)	$A_i(m^2)$	\bar{d} (m)	ϵ (m/m)
1	0.4101	1×10^{-5}	0.017	1.608×10^{-6}
2	0.3843	1×10^{-5}	0.017	1.507×10^{-6}
3	0.4062	1×10^{-5}	0.018	1.5045×10^{-6}
4	0.5641	1×10^{-5}	0.022	1.709×10^{-6}

Table 2: Data used in Eq. (17) to compute the analytical strain.

Force pulse	E_k (J)	ϵ (m/m)	Accuracy
1	0.4101	1.608×10^{-6}	94%
2	0.3843	1.507×10^{-6}	90%
3	0.4062	1.5045×10^{-6}	93%
4	0.5641	1.709×10^{-6}	92%

Table 3: Strain caused on the brick by the energy generated from the force-pulse tests.

region [26,27]. Optical methods such as Interferometry used with a wavelength of $0.51 \mu\text{m}$ can generate a resolution of $0.047 \mu\text{m}$ [28]. A micro Moiré Interferometry can obtain values of strain of 0.0036 [29]. Another optical method establish that with beam propagation the user can obtain values in the order of 1×10^{-4} [30,31].

The present technique helps us to obtain values of strain when an incoming force in the range of 1.9-2.5 kN is affecting a sample, it can be mentioned that is a novel procedure. Our proposed technique can be improved by doing standardized force-pulse test, as we see in the results, each signal obtained from the force-pulses was different one from the other and also it can be improved by doing standardized mechanical impact test.

Some of the advantages of this technique are that we can measure indirectly the strain by means of Digital Image Correlation; while the sample is subjected to an induced force movement. With this strain measured, we are able to know a better total stress concentration in a

selected zone [22]. A disadvantage is that in our experimental setup, we use a lens which must be in the focal length distance, if the induced force movement displace the lens, our experiment will experience some trouble in image processing.

Conclusions

We proposed a methodology to analytically calculate the strain on a sample of brick by force-pulses applied indirectly to it. The proposed optical method based on the use of laser speckles and digital image correlation can be used to determine the strain on a material in the micrometric order: $\varepsilon \approx 1.52 \mu\text{m}$ with a correlation coefficient of 0.8 and an accuracy of 92%. This methodology is inexpensive and non-destructive; however, it must be improved by standardizing the force-pulse test.

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