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How to Prove the Riemann Hypothesis

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Abstract

The aim of this paper is to prove the celebrated Riemann Hypothesis. I have already discovered a simple proof of the Riemann Hypothesis. The hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5. I assume that any such zero is s=a+bi. I use integral calculus in the first part of the proof. In the second part I employ variational calculus. Through equations (50) to (59) I consider (a) as a fixed exponent, and verify that a=0.5. From equation (60) onward I view (a) as a parameter (a <0.5) and arrive at a contradiction. At the end of the proof (from equation (73)) and through the assumption that (a) is a parameter, I verify again that a=0.5.

Keywords: Definite integral; Indefinite integral; Variational calculus

Introduction

The Riemann zeta function is the function of the complex variable $s = a + bi$ $\left(i = \sqrt{-1}\right)$, defined in the half plane $a > 1$ by the absolute convergent series.

$$\zeta(s) = \sum_{1}^{\infty} \frac{1}{n^{s}} \tag{1}$$

and in the whole complex plane by analytic continuation.

The function $\zeta(S)$ has zeros at the negative even integers -2, -4, … and one refers to them as the trivial zeros. The Riemann hypothesis states that the nontrivial zeros of $\zeta(S)$ have real part equal to 0.5 [1].

Proof of the Hypothesis

We begin with the equation,

$$\zeta(S)=0 \tag{2}$$

And with,

$$s=a+bi \tag{3}$$

$$\zeta(a+bi)=0 \tag{4}$$

It is known that the nontrivial zeros of $\zeta(S)$ are all complex. Their real parts lie between zero and one.

If $0 < a < 1$ then,

$$\zeta(s) = s\int_{0}^{\infty} \frac{[x]-x}{x^{s+1}} dx \quad (0 < a < 1) \tag{5}$$

$[x]$ is the integer function.

Hence,

$$\int_{0}^{\infty} \frac{[x]-x}{x^{s+1}} dx = 0 \tag{6}$$

Therefore,

$$\int_{0}^{\infty} ([x]-x) x^{-1-a-bi} dx = 0 \tag{7}$$

$$\int_{0}^{\infty} ([x]-x) x^{-1-a} x^{-bi} dx = 0 \tag{8}$$

$$\int^{\infty} x^{- \quad -} \, ([x]-x)(\cos(b\log x) - i\sin(b\log x)) dx = 0 \tag{9}$$

Separating the real and imaginary parts we get,

$$\int_{0}^{\infty} x^{-1-a} ([x]-x) \cos(b\log x) dx = 0 \tag{10}$$

$$\int_{0}^{\infty} x^{-1-a} ([x]-x) \sin(b\log x) dx = 0 \tag{11}$$

According to the functional equation, if $\zeta(S)=0$ then $\zeta(1-S)=0$. Hence we get besides equation (11)

$$\int_{0}^{\infty} y^{-1-a} ([y]-y) \sin(b\log y) dy = 0 \tag{12}$$

In equation (11) replace the dummy variable x by the dummy variable y,

$$\int_{0}^{\infty} y^{-1-a} ([y]-y) \sin(b\log y) dy = 0 \tag{13}$$

We form the product of the integrals (12) and (13).This is justified by the fact that both integrals (12) and (13) are absolutely convergent. As to integral (12) we notice that [2]

$$\int_{0}^{\infty} x^{-2+a} ([x]-x) \sin(b\log x) dx \le \int_{0}^{\infty} \left| x^{-2+\mathrm{a}} \right.$$

$$\left. ([x]-x) \ \sin\ (b\log x) \right| dx \le \int_{0}^{\infty} x^{-2+a} ((x)) dx$$

(where $((z))$ is the fractional part of z, $0 \le ((z)) < 1$)

$$= \lim(t \to 0) \int_{0}^{1-t} x^{-1+a} dx + \lim(t \to 0) \int_{1+t}^{\infty} x^{-2+a} ((x)) dx$$

(t is a very small positive number) (Since $((x))=x$ whenever $0 \le x < 1$)

$$= \frac{1}{a} + \lim(t \to 0) \int_{1+t}^{\infty} x^{-2+a} ((x)) dx$$

$$< \frac{1}{a} + \lim(t \to 0) \int_{1+t}^{\infty} x^{-2+a} dx = \frac{1}{a} + \frac{1}{a-1}$$

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And as to integral (13) $\int_0^\infty y^{-1-a}([y]-y)\sin(b\log y)dy$

$$\leq \int_0^\infty \left|y^{-1-a}([y]-y)\sin(b\log y)\right|dy$$

$$\leq \int_0^\infty y^{-1-a}((y))dy$$

$$= \lim(t\to 0)\int_0^{1-t} y^{-a}dy + \lim(t\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

(t is a very small positive number) (since $((y))=y$ whenever $0\leq y<1$)

$$=\frac{1}{1-a}+\lim(t\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

$$<\frac{1}{1-a}+\int_{1+t}^\infty y^{-1-a}dy=\frac{1}{1-a}+\frac{1}{a}$$

Since the limits of integration do not involve x or y, the product can be expressed as the double integral,

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\sin(b\log y)\sin(b\log x)dxdy=0 \quad (14)$$

Thus $\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)$ (15)

$$(\cos(b\log y+b\log x)-\cos(b\log y-b\log x))dxdy=0$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)(\cos(b\log xy)-\cos(b\log\frac{y}{x}))dxdy=0 \quad (16)$$

That is,

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy= \quad (17)$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log\frac{y}{x})dxdy$$

Consider the integral on the right-hand side of equation (17)

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log\frac{y}{x})dxdy \quad (18)$$

In this integral make the substitution $x=\frac{1}{z}$ $dx=\frac{-dz}{z^2}$

The integral becomes,

$$\int_\infty^0\int_\infty^0 z^{2-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)\frac{-dz}{z^2}dy \quad (19)$$

That is,

$$-\int_0^\infty\int_0^\infty z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (20)$$

This is equivalent to,

$$\int_0^\infty\int_0^\infty z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (21)$$

If we replace the dummy variable z by the dummy variable x, the integral takes the form [3]

$$\int_0^\infty\int_0^\infty x^{-a}y^{-1-a}([\frac{1}{x}]-\frac{1}{x})([y]-y)\cos(b\log xy)dxdy \quad (22)$$

Rewrite this integral in the equivalent form,

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy \quad (23)$$

Thus equation 17 becomes,

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy=$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy \quad (24)$$

Write the last equation in the form,

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([y]-y)\cos(b\log xy)\left\{\left(x^{2-2a}[\frac{}{x}]-\frac{2\ \ 2}{x}\right)-([x]-x)\right\}dx \quad (25)$$

$dy=0$

Let $p<0$ be an arbitrary small positive number. We consider the following regions in the $x-y$ plane [4].

The region of integration $I=[0,\infty)\times[0,\infty)$ (26)

The large region $I1=[p,\infty)\times[p,\infty)$ (27)

The narrow strip $I2=[p,\infty)\times[0,p]$ (28)

The narrow strip $I3=[0,p]\times[0,\infty)$ (29)

Note that,

$I=I1\cup I2\cup I3$ (30)

Denote the integer and in the left hand side of equation (25) by:

$$F(x,y)=x^{-2+a}y^{-1-a}([y]-y)\cos(b\log xy)$$
$$\left\{\left((x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x}\right)-([x]-x)\right\} \quad (31)$$

Let us find the limit of $F(x,y)$ as $x\to\infty$ and $y\to\infty$. This limit is given by:

$$\lim x^{-a}y^{-1-a}\left[-((y))\right]\cos(\text{blog }xy)\left[-\left(\left(\frac{1}{x}\right)\right)+((x))x^{2a-2}\right] \quad (32)$$

$((z))$ is the fractional part of the number z, $0\leq((z))<1$

The above limit vanishes, since all the functions $\left[-((y))\right]$, $\cos(\text{blog }xy)$, $-((\frac{1}{x}))$, and $((x))$ remain bounded as $x\to\infty$ and $y\to\infty$

Note that the function $F(x,y)$ is defined and bounded in the region I 1. We can prove that the integral,

$\iint_{I1}F(x,y)\,dx\,dy$ is bounded as follows (33)

$$\leq\left|\iint_{I1}x^{-a}y^{-1-a}\left[-((y))\right]\cos(\text{blog }xy)[-((\frac{1}{x}))+((x))x^{2a-2}]\,dx\,dy\right|$$

$$\leq\left|\iint_{I1}x^{-a}y^{-1-a}\left[-((y))\right]\cos(\text{blog }xy)[-((\frac{1}{x}))+((x))x^{2a-2}]\,dx\,dy\right|$$

$$=\left|\int_p^\infty\left(\int_p^\infty x^{-a}\cos(\text{blog }xy)\,[-((\frac{1}{x}))+((x))x^{2a-2}]\,dx\right)y^{-1-a}\left[-((y))\right]dy\right|$$

$$\leq\int_p^\infty\left|\left(\int_p^\infty x^{-a}\cos(\text{blog }xy)\,[-((\frac{1}{x}))+((x))x^{2a-2}]\,dx\right)\right|\left|y^{-1-a}\left[-((y))\right]\right|dy$$

$$\leq\int_p^\infty\left(\int_p^\infty x^{-a}\left|\cos(\text{blog }xy)\right|\left|[-((\frac{1}{x}))+((x))x^{2a-2}]\right|dx\right)\left|y^{-1-a}\left[-((y))\right]\right|dy$$

$$<\int_p^\infty x^{-a}[\,((\tfrac{1}{x}))\;+\;((x))x^{2a-2}]\,dx\int_p^\infty y^{-1-a}$$

$$=\frac{1}{ap^a}\int_p^\infty x^{-a}[((\tfrac{1}{x}\,))+((x))x^{2a-2}]\,dx \tag{34}$$

$$=\frac{1}{ap^a}\{\,\lim(t\to 0)\int_p^{1-t} x^{-a}[\,((\tfrac{1}{x}))+((x))x^{2a-2}]dx+\lim(t\to 0)\int_{1+t}^\infty x^{-a}[\,((\tfrac{1}{x}))+((x))x^{2a-2}]dx\}$$

Where t is a very small arbitrary positive number. Since the integral:

$\lim(t\to 0)\int_P^{1-t} x^{-a}[\,((\tfrac{1}{x}))+((x))x^{2a-2}]dx$ is bounded, it remains to show that lim(t→0)

$\int_{1+t}^\infty x^{-a}[((\tfrac{1}{x}))+((x))x^{2a-2}]dx$ is bounded.

Since $x>1$, then $\left(\left(\frac{1}{x}\right)\right)=\frac{1}{x}$ and we have $=\lim(t\to 0)\int_{1+t}^\infty x^{-a}[\frac{1}{x}+((x))x^{2a-2}]dx$

$$=\lim(t\to 0)\int_{1+t}^\infty x^{-a}[\frac{1}{x}+((x))x^{2a-2}]dx$$

$$=\lim(t\to 0)\int_{1+t}^\infty [x^{-a-1}+((x))x^{a-2}]dx$$

$$<\;\lim(t\to 0)\int_{1+t}^\infty [x^{-a-1}+x^{a-2}]dx$$

$$=\frac{1}{a(1-a)}$$

Hence the boundedness of the integral $\iint_{I1} F(x,y)\;dx\;dy$ is proved.

Consider the region:

$$I4=I2\cup I3 \tag{35}$$

We know that,

$$0=\iint_I F(x,y)\;dx\;dy=\iint_{I1} F(x,y)\;dx\;dy+\iint_{I4} F(x,y)\;dx\;dy \tag{36}$$

and that,

$$\iint_{I1} F(x,y)\;dx\;dy\;\text{ is bounded} \tag{37}$$

From which we deduce that the integral [5]

$$\iint_{I4} F(x,y)\;dx\;dy\;\text{ is bounded} \tag{38}$$

Remember that,

$$\iint_{I4} F(x,y)\;dx\;dy=\iint_{I2} F(x,y)\;dx\;dy+\iint_{I3} F(x,y)\;dx\;dy \tag{39}$$

Consider the integral,

$$\iint_{I2} F(x,y)\;dx\;dy\le\left|\;\iint_{I2} F(x,y)\;dx\;dy\right| \tag{40}$$

$$\left|\;\int_0^P(\int_P^\infty x^{-a}\{\,((\tfrac{1}{x}))-\;((x))x^{2a-2}\}\cos(b\log xy)dx)\frac{1}{y^a}dy\right|$$

$$\le\int_0^P\;\left|\;\int_P^\infty(x^{-a}\{\,((\tfrac{1}{x}))-\;((x))x^{2a-2}\}\right|\;\left|\;\cos(b\log xy)dx)\right|\;\frac{1}{y^a}dy$$

$$\le\int_0^P\;\left|\;\int_P^\infty(x^{-a}\{\,((\tfrac{1}{x}))-\;((x))x^{2a-2}\}\right|\;\left|\;\cos(b\log xy)\right|\;dx)\frac{1}{y^a}dy$$

$$\le\int_P^\infty\;\left|\;x^{-a}\{\,((\tfrac{1}{x}))-\;((x))x^{2a-2}\}\right|\;dx\times\int_0^P\;\frac{1}{y^a}dy$$

(This is because in this region $((y))=y$). It is evident that the integral $\int_P^\infty\left|\;x^{-a}\{\,((\tfrac{1}{x}))-\;((x))x^{2a-2}\}\right|\;dx$ is bounded, this was proved in the course of proving that the integral $\iint_{I1} F(x,y)\;dx\;dy$ is bounded. Also it is evident that the integral,

$$\int_0^P\;\frac{1}{y^a}dy\,,$$

is bounded. Thus we deduce that the integral (40) $\iint_{I2} F(x,y)\;dx\;dy$ is bounded.

Hence, according to equation (39), the integral $\iint_{I3} F(x,y)\;dx\;dy$ is bounded [6].

Now consider the integral,

$$\iint_{I3} F(x,y)\;dx\;dy \tag{41}$$

We write it in the form:

$$\iint_{I3} F(x,y)\;dx\;dy=\int_0^p(\int_0^\infty\;y^{-1-a}((y))\cos(b\;\log xy)dy)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}dx \tag{42}$$

(This is because in this region $((x))=x$)

$$\le\left|\;\int_0^p(\int_0^\infty\;y^{-1-a}((y))\cos(b\;\log xy)dy)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}dx\right|$$

$$\le\int_0^p\;\left|\;(\int_0^\infty\;y^{-1-a}((y))\;\cos\;(b\;\log xy)\;dy\;)\right|\;\left|\;\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\right|dx$$

$$\le\int_0^p(\int_0^\infty\;y^{-1-a}((y))\;dy\;)\left|\;\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\right|\;dx$$

Now we consider the integral with respect to *y*,

$$\int_0^\infty\;yy^{-1-a}((y))\;dy \tag{43}$$

$$=\;(\lim\;t\to 0)\int_0^{1-t}\;y^{-1-a}\times y\;dy+\;(\lim\;t\to 0)\int_{1+t}^\infty\;y^{-1-a}((y))\;dy$$

(where *t* is a very small arbitrary positive number). (Note that $((y))=y$ whenever $0\le y<1$).

Thus we have $(\lim\;t\to 0)\int_{1+t}^\infty\;y^{-1-a}((y))\;dy<(\lim\;t\to 0)\int_{1+t}^\infty\;y^{-1-a}dy=\frac{1}{a}$

and $(\lim\;t\to 0)\int_0^{1-t}\;y^{-1-a}\times y\;dy=\frac{1}{1-a}$

Hence the integral (43) $\int_0^\infty\;y^{-1-a}((y))\;dy$ is bounded.

Since $\left|\;\int_0^\infty y^{-1-a}((y))\;\cos\;(b\;\log xy)dy\right|\;\le\int_0^\infty\;y^{-1-a}((y))\;dy$, we conclude that the integral $\left|\;\int_0^\infty\;y^{-1-a}((y))\;\cos\;(b\;\log xy)dy\right|$ is a bounded function of *x*. Let this function be $H(x)$. Thus we have,

$$\left|\;\int_0^\infty\;y^{-1-a}((y))\;\cos\;(b\;\log xy)dy\right|\;=H(x)\le K\;(K\text{ is a positive number}) \tag{44}$$

Now equation (44) gives us,

$$-K \le \int_0^{\infty} y^{-1-a}((y)) \cos(b \log xy) dy \le K \qquad (45)$$

According to equation (42) we have,

$$\iint_{I3} F(x,y)\,dx\,dy = \int_0^p (\int_0^{\infty} y^{-1-a}((y)) \cos(b \log xy)\,dy) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx \qquad (46)$$

$$\ge \int_0^p (-K) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx = K \int_p^0 \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx$$

Since $\iint_{I3} F(x,y)\,dx\,dy$ is bounded, then $\int_p^0 \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a}$ dx is also bounded. Therefore the integral,

$$G = \int_0^p \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx \text{ is bounded} \qquad (47)$$

We denote the integer and of (47) by:

$$F = \frac{1}{x^a}\{((\frac{1}{x})) - x^{2a-1}\} \qquad (48)$$

Let $\delta\, G\, [F]$ be the variation of the integral G due to the variation of the integrand $\delta\, F$.

Since,

$G\,[F]=\int F\,dx$ (the integral (49) is indefinite) (49)

(here we do not consider a as a parameter, rather we consider it as a given exponent)

We deduce that $\frac{\delta G[F]}{\delta F(x)} = 1$

that is,

$\delta G\,[F]=\delta F\,(x)$ (50)

But we have,

$$\delta G[F] = \int dx \frac{\delta G[F]}{\delta F(x)} \delta F(x) \text{ (the integral (51) is indefinite)} \qquad (51)$$

Using equation (50) we deduce that,

$\delta G[F]=\int dx F(x)$(the integral (52) is indefinite) (52)

Since $G[F]$ is bounded across the elementary interval $[0,p]$, we must have that,

$\delta G[F]$ is bounded across this interval (53)

From (52) we conclude that,

$$\delta G = \int_0^p dx \delta F(x) = \int_0^p dx \frac{dF}{dx} \delta x = [F\delta x](\text{at } x=p) - [F\delta x](\text{at } x = 0) \qquad (54)$$

Since the value of $[F\delta x]$ (at $x=p$)is bounded, we deduce from equation (54) that,

$\lim (x \to 0)\, F\, \delta\, x$ must remain bounded. (55)

Thus we must have that

$$(\lim x \to 0)\ [\delta x \frac{1}{x^a}\{ ((\frac{1}{x})) - x^{2a-1}\}] \text{ is bounded.} \qquad (56)$$

First we compute,

$$(\lim x \to 0) \frac{\delta x}{x^a} \qquad (57)$$

Applying L 'Hospital ' rule we get,

$$(\lim x \to 0) \frac{\delta x}{x^a} = (\lim x \to 0) \frac{1}{a} \times x^{1-a} \times \frac{d(\delta x)}{dx} = 0 \qquad (58)$$

We conclude from (56) that the product,

$0 \times (\lim x \to 0)\{((\frac{1}{x})) - x^{2a-1}\}$ must remain bounded. (59)

Assume that a=0.5. (Remember that we considered a as a given exponent)This value a=0.5 will guarantee that the quantity $\{((\frac{1}{x})) - x^{2a-1}\}$

will remain bounded in the limit as ($x \to 0$). Therefore, in this case (a=0.5) (56) will approach zero as ($x \to 0$) and hence remain bounded.

Now suppose that a< 0.5. In this case we consider a as a parameter. Hence we have,

$$G_a[x] = \int dx \frac{F(x,a)}{x} x \text{ (the integral (60) is indefinite)} \qquad (60)$$

Thus,

$$\frac{\delta G_a[x]}{\delta x} = \frac{F(x,a)}{x} \qquad (61)$$

But we have that,

$$\delta G_a[x] = \int dx \frac{\delta G_a[x]}{\delta x} \delta x \text{ (the integral (62) is indefinite)} \qquad (62)$$

Substituting from (61) we get,

$$\delta G_a[x] = \int dx \frac{F(x,a)}{x} \delta x \text{ (the integral (63) is indefinite)} \qquad (63)$$

We return to equation (49) and write,

$$G = \lim (t \to 0) \int_t^p F dx \quad (t \text{ is a very small positive number } 0 < t < p) \qquad (64)$$

$$= \{F\, x(\text{at } p) - \lim (t \to 0) F x(\text{at } t)\ \} - \lim (t \to 0) \int_t^p x\, dF$$

Let us compute,

$$\lim(t \to 0) F x(\text{at } t) = \lim (t \to 0) t^{1-a}((\frac{1}{t})) - t^a = 0 \qquad (65)$$

Thus equation (64) reduces to,

$$G - Fx(\text{at } p\ ) = -\lim (t \to 0) \int_t^p x dF \qquad (66)$$

Note that the left – hand side of equation (66) is bounded. Equation (63) gives us,

$$\delta G_a = \lim (t \to 0) \int_t^p dx \frac{F}{x} \delta x \qquad (67)$$

(t is the same small positive number $0<t<p$)

We can easily prove that the two integrals $\int_t^p x dF$ and $\int_t^p dx \frac{F}{x} \delta x$ are absolutely convergent. Since the limits of integration do not involve any variable, we form the product of **(66)** and **(67)**

$$K = \lim(t \to 0) \int_t^p \int_t^p x dF \times dx \frac{F}{x} \delta x = \lim(t \to 0) \int_t^p F dF \times \int_t^p \delta x dx \qquad (68)$$

(K is a bounded quantity)

That is,

$$K = \lim(t \to 0)[\frac{F^2}{2}(\text{at } p) - \frac{F^2}{2}(\text{at } t)] \times [\delta x(\text{at } p) - \delta x(\text{at } t)] \qquad (69)$$

We conclude from this equation that,

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$$\{[\frac{F^2}{2}(\text{ at } p) - \lim(t\to 0)\frac{F^2}{2}(\text{at } t)\ ]\times[\delta x(\text{at } p)]\} \text{ is bounded.} \quad (70)$$

(Since $\lim(x\to 0)\delta x=0$, which is the same thing as $\lim (t\to 0)\ \delta x=0$)

Since $\frac{F^2}{2}$ (at p) is bounded, we deduce at once that $\frac{F^2}{2}$ must remain bounded in the limit as ($t\to 0$), which is the same thing as saying that F must remain bounded in the limit as ($x\to 0$). Therefore,

$$\lim\ (x\to 0)\frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \text{ must remain bounded} \quad (71)$$

But,

$$\lim\ (x\to 0)\frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \lim\ (x\to 0)\frac{x^{1-2a}}{x^{1-2a}}\times\frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \quad (72)$$

$$\lim\ (x\to 0)\frac{x^{1-2a}((\frac{1}{x}))-1}{x^{1-a}} = \lim\ (x\to 0)\frac{-1}{x^{1-a}}$$

It is evident that this last limit is unbounded. This contradicts our conclusion (71) that:

$$\lim\ (x\to 0)\frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \text{ must remain bounded (for } a< 0.5\text{)}$$

Therefore the case $a<0.5$ is rejected. We verify here that, for $a=0.5$ (71) remains bounded as ($x\to 0$).

We have that:

$$((\frac{1}{x})) - x^{2a-1} < 1 - x^{2a-1} \quad (73)$$

Therefore:

$$\lim(a\to 0.5)\ (x\to 0)\frac{((\frac{1}{x})) - x^{2a-1}}{x^a} < \lim(a\to 0.5)\ (x\to 0)\frac{1-x^{2a-1}}{x^a} \quad (74)$$

We consider the limit:

$$\lim(a\to 0.5)\ (x\to 0)\frac{1-x^{2a-1}}{x^a} \quad (75)$$

We write:

$a=(\lim x\to 0)\ (\ 0.5+x\ )$ (76)

Hence we get:

$$\lim(a\to 0.5)\ (x\to 0)\mathrm{x}^{2a-1} = \lim\ (x\to 0)x^{2(0.5+x)-1} = \lim\ (x\to 0)x^{2x} = 1 \quad (77)$$

(Since $\lim(x\to 0)\ x^x=1$)

Therefore we must apply L 'Hospital ' rule with respect to x in the limiting process (75)

$$\lim(a\to 0.5)\ (x\to 0)\frac{1-x^{2a-1}}{x^a} = \lim(a\to 0.5)\ (x\to 0)\frac{-(2a-1)x^{2a-2}}{ax^{a-1}} \quad (78)$$

$$\lim(a\to 0.5)\ (x\to 0)\frac{(\frac{1}{a}-2)}{x^{1-a}}$$

Now we write again,

$a=(\lim x\to 0)\ (\ 0.5+x\ )$ (79)

Thus the limit (78) becomes:

$$\lim(a\to 0.5)\ (x\to 0)\frac{(\frac{1}{a}-2)}{x^{1-a}} = \lim(x\to 0)\frac{(0.5+x)^{-1}-2}{x^{0.5-x}} = \lim\ (x\to 0)\frac{(0.5+x)^{-1}-2}{x^{0.5}\times x^{-x}}$$

$$= \lim(x\to 0)\frac{(0.5+x)^{-1}-2}{x^{0.5}} \text{ (Since } \lim\ (x\to 0)\ x^{-x}=1\text{)} \quad (80)$$

We must apply L 'Hospital ' rule,

$$\lim\ (x\to 0)\frac{(0.5+x)^{-1}-2}{x^{0.5}} = \lim\ (x\to 0)\frac{-(0.5+x)^{-2}}{0.5x^{-0.5}} = \lim\ (x\to 0)\frac{-2\times x^{0.5}}{(0.5+x)^2} = 0 \quad (81)$$

Thus we have verified here that, for a=0.5 (71) approaches zero as ($x\to 0$) and hence remains bounded.

We consider the case a >0.5. This case is also rejected, since according to the functional equation, if (ζ (S)=0) ($s=a+bi$) has a root with a>0.5, then it must have another root with another value of a<0.5. But we have already rejected this last case with a<0.5

Thus we are left with the only possible value of a which is a=0.5

Therefore a=0.5

This proves the Riemann Hypothesis.

Conclusion

The Riemann Hypothesis is now proved. The hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5. I assume that any such zero is $s=a+bi$. I use integral calculus in the first part of the proof. In the second part I employ variational calculus. Through equations (50) to (59) I consider (a) as a fixed exponent, and verify that a=0.5. From equation (60) onward I view (a) as a parameter (a <0.5) and arrive at a contradiction. At the end of the proof (from equation (73)) and through the assumption that (a) is a parameter, I verify again that a=0.5.

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