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Homotopy Perturbation and Adomian Decomposition Methods for a Quadratic Integral Equations with Erdelyi-Kober Fractional Operator

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Abstract

This paper is devoted with two analytical methods; Homotopy perturbation method (HPM) and Adomian decomposition method(ADM). We display an efficient application of the ADM and HPM methods to the nonlinear fractional quadratic integral equations of *Erdelyi-kober* type. The existence and uniqueness of the solution and convergence will be discussed. In particular, the well-known Chandrasekhar integral equation also belong to this class, recent will be discussed. Finally, two numerical examples demonstrate the efficiency of the method.

Keywords: Adomian decomposition; Integral equations; Homotopy; Functional equations

Introduction

It is well-known that the theory of integral equations has many applications in describing numerous events and problems of the real world. Nonlinear quadratic integral equations (NQIE)are also often encountered in the theories of radiative transfer and neutron transport [1,2].

Many research about (QIE) appear in the literaturely, numerous research papers have appeared devoted to nonlinear fractional quadratic integral equation (NFQIE) [3-8]. However, there few work on NFQIE with *Erdelyi-kober* fractional operator.

Hashem [9], studied the existence of maximal and minimal at least one continuous solution for NQIE of *Erdelyi-kober* type

$$x(t)=a(t)+g(t,x(t))\int_0^t (t^m-s^m)^{\alpha-1}\Gamma(\alpha)ms^{m-1}f(s,x(s)),\quad t\in I,\alpha>0,m>0 \quad (1)$$

In this paper, we investigated the existence, uniqueness of the solution and convergence for NFQIE (1), using two methods; HPM and ADM. The homotopy perturbation method (HPM) was suggested by Ji-Huan [10-15] in 1999. In this method, the solution can be expressed by an infinite series, which commonly converges fast to the exact solution. It is a coupling of the traditional perturbation method and homotopy in topology, which is solved differential and integral equations, linear and nonlinear. The HPM does not require a small parameter in equations. Also, it has an important advantage which enlarges the application of nonlinear problem in applied science.

The Adomian decomposition method (ADM) solves many of functional equations, for example, differential, integro-differential, differential-delay, and partial differential equations. The solution usually appears in a series form, this method has many significant advantages, it does not require linearization, perturbation and other restrictive methods. Also, it might change the problem to a solved one [16-19]. It is worth mentioning that our results are motiviated by the generalization of the work.

Theorem 1: Assume that

$f:[0,1]\to R_+=[0,+\infty)$ is acontinuous function on [0,1],

$g,u:[0,1]\times R_+\to R_+$ are continuous and bounded with

$M_1=\sup_{(t,x)\in[0,1]\times R_+}|g(t,x)|$ and $M_2=\sup_{(t,x)\in[0,1]\times R_+}|u(t,x)|$,

there exist constant L_1 and L_2 such that

$|a(t,x)-a(t,y)|\le L_1|x-y|$,

$(L_1M_2+L_2M_1)<1$.

Then the nonLinear fractional quadratic integral (Theorem 1) has a unique positive solution $x\in C$.

proof: For $x,y\in S$ and for each $t\in[0,1]$, we obtain

$$(Tx)(t)-(Ty)(t)=g(t,x)\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)u(s,x(s))ds$$

$$-g(t,y)\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)u(s,y(s))ds$$

$$+g(t,x)\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)u(s,y(s))ds$$

$$-g(t,x)\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)u(s,y(s))ds,$$

$$=[g(t,x)-g(t,y)]\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)u(s,y(s))ds$$

$$+g(t,x)\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)[u(s,x(s))-u(s,y(s))]ds,$$

$$|(Tx)(t)-(Ty)(t)|\le M_2L_1|x-y|\left|\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)ds\right.$$

$$+M_1L_2\int_0^t \alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)|x(s)-y(s)|ds,$$

$$\|(Tx)(t)-(Ty)(t)\|=max_{t\in I}|(Tx)(t)-(Ty)(t)|,$$

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$$\leq (L_1M_2 + L_2M_1)\|x-y\| t^{\alpha\beta}\Gamma(\beta+1),$$

$$\leq (L_1M_2 + L_2M_1)\|x-y\|.$$

By (H4), The operator T is a contraction map from S into S, hence the conclusion of the theorem follows.

Main Results

In this section, we prove the existence and uniqueness of continuous solutions and the convergence for Equation

$$x(t) = a(t) + g(t,x(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} f(s,x(s)) \quad (2)$$

we denote by $C=C(I)$ the space of all real-valued functions which are continuous on $I=[0,1]$.We can transform (2) into an equivalent fixed point problem $Tx=x$, where the operator T:$C\to C$ is defined by

$$(Tx)(t) = f(t) + g(t,x(t))\int_0^t \alpha s^{\alpha-1}(t^\alpha - s^\alpha)^{\beta-r}\Gamma(\beta)u(s,x(s))ds, t\in[0,1] \quad (3)$$

Observe that the existence of a fixed point for the operator T implies the existence of a solution for the (2).

Now define a subset S of C as

$$S = \{x \in C : |x - f(t)| \leq k\}, k = M_1M_2\Gamma(\beta+1),$$

Then operator T maps S into S, since for $x\in S$

$$|x(t) - f(t)| \leq M_1M_2\int_0^t \alpha s^{\alpha-1}(t^\alpha - s^\alpha)^{\beta-r}\Gamma(\beta)ds = M_1M_2\Gamma(\beta+1)t^{\alpha\beta} \leq M_1M_2\Gamma(\beta+1),$$

It is clear that S is a closed subset of C.

Homotopy Perturbation Method

The He's homotopy perturbation technique [10,11] defines the homotopy $u(t,p):\Omega\times[0,1]\to\Re$ which satisfies

$$H(u,p) = (1-p)F(u) + pL(u) = 0 \quad (4)$$

Where $t\in\Omega$ and $p\in[0,1]$ is an impeding parameter, u_0 is an initial approximation which satisfies the boundary conditions, we can define $H(u,p)$ by

$$H(u,0) = F(u), H(u,1) = L(u),$$

where $F(u)$ is an integral operator such that $F(u)=u(t)-a(t)$, and $L(u)$ has the form,

$$L(u) = u(t) - a(t) - g(t,x(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} f(s,x(s))ds \quad (5)$$

and continuously trace an implicitly defined curve from a starting point $H(u_0,0)$ to a solution function $H(x,t)$. The embedding parameter p monotonically increases from zero to one as the trivial problem $F(u)=0$ is continuously deformed to the original problem $L(u)=0$.

The embedding parameter $p\in(0,1]$ can be considered as an expanding parameter [20].

$$u = \sum_{n=0}^{\infty} p^n u_n, \quad (6)$$

when $p\to 1$, (6) corresponds to (4) and give an approximation to the solution of (2) as follows,

$$x(t) = \lim_{p\to 1} u = \sum_{n=0}^{\infty} u_n, \quad (7)$$

The series(7) converges in most cases, and the rate of convergence depends on $L(u)$ [21].

We substitute (6) into (4) and equate the terms with identical powers of p, obtaining

$$p^0 : u_0(t) = a(t),$$

$$p^1 : u_1(t) = g(t,u_0(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} H_0(s)ds,$$

$$p^2 : u_2(t) = g(t,u_0(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} H_1(s)ds$$

$$+g(t,u_1(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha).m.s^{m-1}.H_0(s)ds,$$

$$p^n : u_n(t) = g(t,u_0(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} H_{n-1}(s)ds$$

$$+g(t,u_1(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} H_{n-2}(s)ds$$

$$+ .... + g(t,u_{n-1}(t))\int_0^t (t^m - s^m)^{\alpha-1}\Gamma(\alpha) m s^{m-1} H_0(s)ds, \quad n = 1,2,3,...$$

Where the H_n are the so-called He's polynomials [22] which can be calculated by using the formula

$$H_n(u_0,....,u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}((\sum_{k=0}^{n} p^k u_k)^r)_{p=0}, \quad n = 0,1,2,...$$

Adomian Decomposition Method (ADM)

The ADM suggest the solution $x(t)$ be decomposed by infinite series solution

$$x(t) = \sum_{i=0}^{\infty} x_i(t) \quad (8)$$

and the nonlinear functions $g(t,x(t))$ and $f(t,x(t))$ of Equation (2), represented by Adomian polynomials as follows

$$A_n = 1n!d^n d\lambda^n[g(t,\sum_{i=0}^{\infty}\lambda^i x_i)]_{\lambda=0} \quad (9)$$

$$B_n = 1n!d^n d\lambda^n[f(t,\sum_{i=0}^{\infty}\lambda^i x_i)]_{\lambda=0} \quad (10)$$

substituting (9) and (10) into (2) gives the following recursive scheme

$$x_0(t) = f(t),$$

$$x_i(t) = A_{i-1}(t)^{\beta}_{\alpha} \beta_{i-1}(t) \quad (11)$$

Theorem 2: Assume that the solution of the (2)exist . If $|x_1(t)| < m, m$ is a positive constant, then the series solution (8) of the(2) converges.

Proof: Define the Sequence $\{S_p\}$ such that $S_p = \sum_{i=0}^{p} x_i(t)$ is the sequence of partial sums from the series solution $\sum_{i=0}^{\infty} x_i(t)$ ', and we have

$$g(t,x) = \sum_{i=0}^{\infty} A_i, u(t,x) = \sum_{i=0}^{\infty} B_i,$$

Let S_p and s_q be two arbitrary partial sums with $p>q$ Now, We are going to prove that $\{S_p\}$ is a cauchy Sequence in the Banach Space E [23-25].

$$S_p - S_q = \sum_{i=0}^{p} x_i - \sum_{i=0}^{q} x_i,$$

$$= \sum_{i=0}^{p} A_{i-1}(I^{\beta}_{\alpha}\sum_{i=0}^{p}\beta_{i-1}(t)) - \sum_{i=0}^{q} A_{i-1}(I^{\beta}_{\alpha}\sum_{i=0}^{q}\beta_{i-1}(t)),$$

$$- \sum_{i=0}^{q} A_{i-1}(I^{\beta}_{\alpha}\sum_{i=0}^{p}\beta_{i-1}(t)) + \sum_{i=0}^{q} A_{i-1}(I^{\beta}_{\alpha}\sum_{i=0}^{p}\beta_{i-1}(t)),$$

$$= [\sum_{i=0}^{p} A_{i-1}(t) - \sum_{i=0}^{q} A_{i-1}(t)] \quad (I^{\beta}_{\alpha}\sum_{i=0}^{p} B_{i-1}(t)),$$

$$+ \sum_{i=0}^{2} A_{i-1}(t)(I^{\beta}_{\alpha}[\sum_{i=0}^{p}\beta_{i-1}(t) - \sum_{i=0}^{q}\beta_{i-1}(t)])$$

$$\|S_p - S_q\| \leq \max_{t\in I} |\sum_{i=q+1}^{p} A_{i-1}(t)(I^{\beta}_{\alpha}\sum_{i=0}^{p}\beta_{i-1}(t))|,$$

$$+\max_{t\in I}\ |\sum_{i=0}^{q}A_{i-1}(t)(I_\alpha^\beta\sum_{i=q+1}^{p}\beta_{i-1}(t))|,$$

$$\le \max_{t\in I}\ |\sum_{i=q}^{p-1}A_i(t)||I_\alpha^\beta\sum_{i=0}^{p}\beta_{i-1}(t)|,$$

$$+\max_{t\in I}|\sum_{i=0}^{q}A_{i-1}(t)||I_\alpha^\beta\sum_{i=q}^{p-1}\beta_i(t)|,$$

$$\le \max_{t\in I}|g(t,s_{p-1})-g(t,s_{q-1})|\int_0^t|\alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)|u(s,s_p)|ds$$

$$+\max_{t\in I}|g(t,s_q)|\int_0^t|\alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)|u(s,s_{p-1})-u(s,s_{q-1})|ds,$$

$$\le \max_{t\in I}L_1|s_{p-1}-s_{q-1}|\cdot\int_0^t|\alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)|u(s,s_p)|ds$$

$$+\max_{t\in I}L_2|g(t,s_q)|\int_0^t|\alpha s^{\alpha-1}(t^\alpha-s^\alpha)^{\beta-1}\Gamma(\beta)|s_{p-1}-s_{q-1}|ds,$$

$$\le M_2L_1 1\Gamma(\beta+1)\|s_{p-1}-s_{q-1}\|$$

$$+M_1L_2 1\Gamma(\beta+1)\|s_{p-1}-s_{q-1}\|,$$

$$=1\Gamma(\beta+1)[M_2L_1+M_1L_2]\|s_{p-1}-s_{q-1}\|,$$

$$\le h\|s_{p-1}-s_{q-1}\|,$$

Let $p=q+1$ then

$$\|s_{q+1}-s_q\|\le h\|s_q-s_{q-1}\|\le h^2\|s_{q-1}-s_{q-2}\|,$$

$$\le\ldots\le h^q\|s_1-s_0\|,$$

$$s_p-s_q=\sum_{i=q+1}^{p}x_i=x_{q+1}+x_{q+2}+\ldots+x_{p-1}+x_p,$$

$$\|s_p-s_q\|=\|s_p-s_q+s_{q+1}-s_{q+1}+s_{q+2}-s_{q+2}+\ldots+s_{p-1}-s_p\|,$$

$$\le\|s_{q+1}-s_q\|+\|s_{q+2}-s_{q+1}\|+\ldots+\|s_p-s_{p-1}\|,$$

$$\le h^q\|s_1-s_0\|+h^{q+1}\|s_1-s_0\|+\ldots+h^{p-1}\|s_1-s_0\|,$$

$$=h^q\|s_1-s_0\|[1+h+h^2+\ldots+h^{p-q-1}],$$

$$=h^q\|s_1-s_0\|1-h^{p-q}1-h,$$

$$\le h^q\left[1-h^{p-q}1-h\right]\|x_1\|,$$

$$\|s_p-s_q\|\le h^q1-h\max_{t\in I}|x_1(t)|,\quad 1-h^{p-q}<1.$$

Numerical Example

In this section, We shall study some numerical examples and applying HPM and ADM methods, then comparing the result [26-28].

Example 1: Consider the following nonlinear (FQIE),

$$x(t)=t^2-t^{10}35+t5x(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2x^2(s)ds\ , \qquad (12)$$

and has the exact solution $x(t)=t^2$. First applying homotopy perturbation method .

Case 1: We can be constructed a homotopy as follows

$$H(u,p)=(1-p)\big(u(t)-g(t)\big)+p\Big(u(t)-g(t)$$
$$\left.-\frac{t}{5}u(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2u^2(s)ds\right)=0\ , \qquad (13)$$

substituting (6) into (13), and equating the same powers of p

$$p^0:u_0(t)=t^2-t^{10}35,$$

$$p^1:u_1(t)=t5u_0(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_0(s)ds,$$

$$p^2:u_2(t)=t5u_0(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_1(s)ds$$
$$+t5u_1(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_0(s)ds,$$

and so on. Then the approximate solution is

$$x(t)=\lim_{p\to 1}u=u_0+u_1+u_2+\ldots+u_n,$$

Second applying (ADM) to equation (12), We get

$$x_0(t)=(t^2-t^{10}35),$$

$$x_i(t)=t5x_{i-1}(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2A_{i-1}(s)ds,\quad i\ge 1,$$

Where A_i are Adomian polynomials of the nonlinear term x^2, and the solution will be

$$x(t)=\sum_{i=0}^{q}x_i(t),$$

Table 1 shows a comparison between the absolute error of (HPM) (when n=1) and (ADM) solutions (when q=1), (Figure 1).

Case 2: We can be constructed a distinct convex homotopy as follows

$$H(u,p)=(1-p)u(t)+p\big(u(t)-g(t)$$
$$\left.-\frac{t}{5}.u(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2u^2(s)ds\right)=0\ , \qquad (14)$$

It can continuously trace an implicity defined curve from a starting point $H(u,0)$ to a solution function $H(u,1)$, and equating the coefficients of the same powers of p, we obtain

$$p^0:u_0(t)=0,$$

$$p^1:u_1(t)=t^2-t^{10}35t5u_0(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_0(s)ds,$$
$$=t^2-t^{10}35,$$

$$p^2:u_2(t)=t5u_0(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_1(s)ds$$
$$+t5u_1(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_0(s)ds,$$
$$=0,$$

$$p^3:u_3(t)=t5u_0(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_2(s)ds$$
$$+t5u_1(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_1(s)ds$$
$$+t5u_2(t)\int_0^t 12s^{-12}\Gamma(12)(t^{12}-s^{12})^{12}s^2H_0(s)ds,$$
$$=0,$$

and so on.

t	u_{HPM}	u_{ADM}	u_{Exact}	$\lvert u_{HPM}-u_{Exact}\rvert$	$\lvert u_{ADM}-u_{Exact}\rvert$
0.1	0.01000000	0.01000000	0.01000000	0.00000000	0.00000000
0.2	0.04000002	0.04000002	0.04000000	0.00000002	0.00000002
0.3	0.09000065	0.09000065	0.09000000	0.00000065	0.00000065
0.4	0.16000868	0.16000868	0.16000000	0.00000868	0.00000868
0.5	0.25006406	0.25006406	0.25000000	0.00006406	0.00006406
0.6	0.36032343	0.36032343	0.36000000	0.00032343	0.00032343
0.7	0.49125223	0.49125223	0.49000000	0.00125223	0.00125223
0.8	0.64396212	0.64396212	0.64000000	0.00396212	0.00396212
0.9	0.82057130	0.82057130	0.81000000	0.01057130	0.01057130
1	1.02378744	1.02378744	1.00000000	0.02378744	0.02378744

Table 1: Comparison betwee n the absolute error of (HPM) (when n=1) and (ADM) solutions (when q=1).

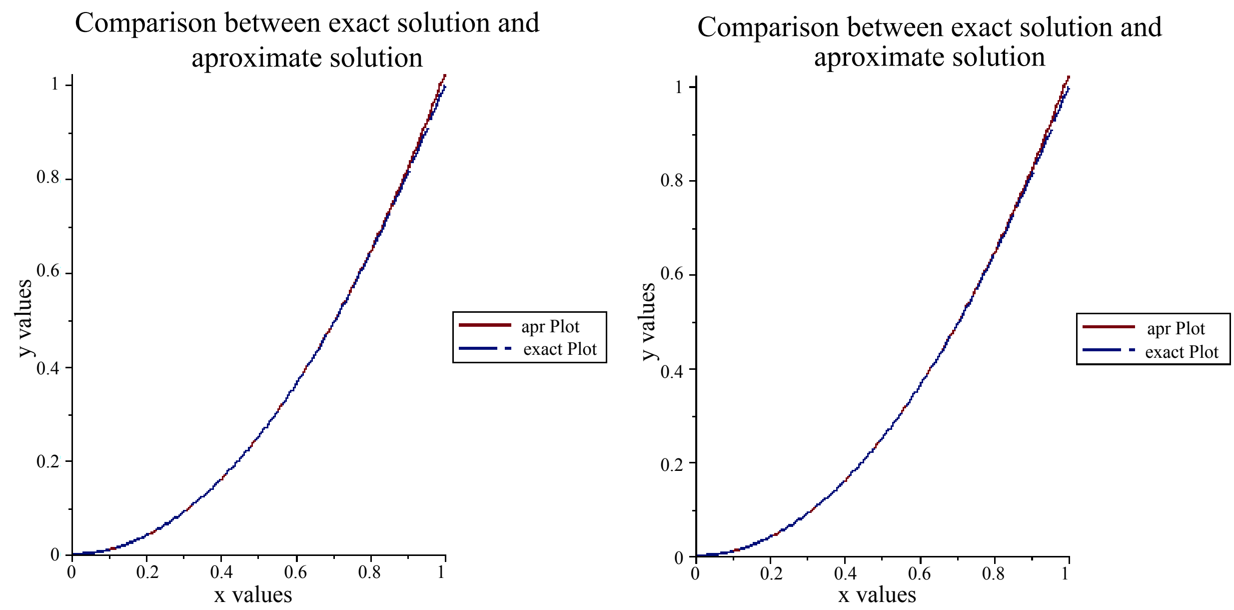

Figure 1: The difference between exact and approximate solutions by (HPM)and (ADM).

t	u_{HPM}	u_{ADM}	u_{Exact}	$\|u_{HPM}-u_{Exact}\|$	$\|u_{ADM}-u_{Exact}\|$
0.1	0.01000000	0.01000000	0.01000000	0.00000000	0.00000000
0.2	0.04000000	0.04000002	0.04000000	0.00000000	0.00000002
0.3	0.08999983	0.09000065	0.09000000	0.00000017	0.00000065
0.4	0.15999700	0.16000868	0.16000000	0.00000300	0.00000868
0.5	0.24997210	0.25006406	0.25000000	0.00002790	0.00006406
0.6	0.35982724	0.36032343	0.36000000	0.00017276	0.00032343
0.7	0.48919293	0.49125223	0.49000000	0.00080707	0.00125223
0.8	0.63693217	0.64396212	0.64000000	0.00306783	0.00396212
0.9	0.80003776	0.82057130	0.81000000	0.00996224	0.01057130
1	0.97142857	1.02378744	1.00000000	0.02857143	0.02378744

Table 2: Comparison between the absolute error of (HPM) (when n=1) and (ADM) solutions (when q=1).

Table 2 shows a comparison between the absolute error of (HPM) (when n=1) and (ADM) solutions (when q=1), (Figure 2).

Example 2

$$x(t)=t^3+14(t^2+1)+14(t^2+1)x(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}\cos\left(x^2(s)1+x^2(s)\right)ds \quad (15)$$

Case 1: First applying homotopy perturbation method, we can be constructed a homotopy as follows

$$H(u,p)=(1-p)\left(u(t)-g(t)\right)+p\left(u(t)-g(t)\right.$$
$$\left.-14(t^2+1)u(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}\cos(u^2(s)1+u^2(s))ds\right)=0 \quad (16)$$

substituting (6) into (16), and equating the same powers of p

$$p^0: u_0(t)=t^3+14(t^2+1),$$
$$p^1: u_1(t)=14(t^2+1)u_0(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}H_0(s)ds,$$
$$p^2: u_2(t)=14(t^2+1)u_0(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}H_1(s)ds$$
$$+14(t^2+1)u_1(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}H_0(s)ds,$$

and so on. Then the approximate solution is

$$x(t)=\lim_{p\to 1}u=u_0+u_1+u_2+....+u_n,$$

Second applying (ADM) to equation (15), we get

$$x_0(t)=t^3+14(t^2+1),$$
$$x_i(t)=14(t^2+1)x_{i-1}(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}A_{i-1}(s)ds.$$

Where A_i are Adomian polynomials of the nonlinear term $\cos(x^2 1+x^2)$ and the solution will be

$$x(t)=\sum_{i=0}^{q}x_i(t),$$

Table 3 shows a comparisons between (HPM) and (ADM) solutions (when n=2, q=2), (Figure 3).

Case 2: We can be constructed a distinct convex homotopy as follows

$$H(u,p)=(1-p)u(t)+p\left(u(t)-g(t)\right.$$
$$-14(t^2+1)u(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}\cos$$

It can continuously trace an implicity defined curve from a starting point $H(u,0)$ to a solution function $H(u,1)$, and equating the coefficients of the same powers of p, we obtain

$$p^0: u_0(t)=0,$$
$$p^1: u_1(t)=14(t^2+1)u_0(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7}H_0(s)ds,$$

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$$= t^3 + 14t^2 + 14,$$

$$p^2 : u_2(t) = 14(t^2+1)u_0(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7} H_1(s)ds$$

$$+14(t^2+1)u_1(t)\int_0^t 1\Gamma(0.3)(t-s)^{0.7} H_0(s)ds,$$

$$= 1.114242508\left(14t^2+14\right).\left(t^3+14t^2+14\right)t^{310}.$$

and so on.

Table 4 shows a co.parison between the absolute error between (HPM) and (ADM) (when n=2, q=2), (Figure 4).

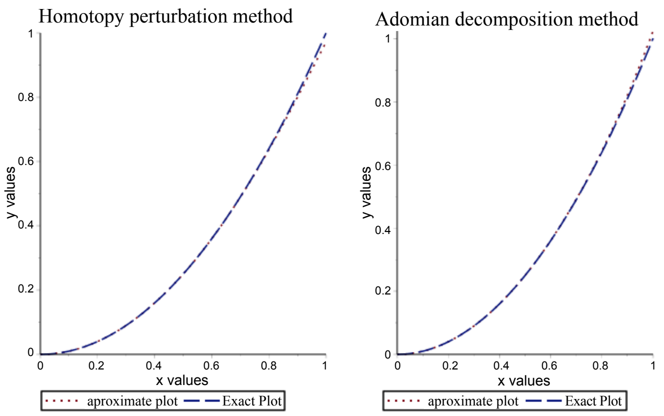

Figure 2: The difference between exact and approximate solutions by (HPM) and (ADM).

t	u_{HPM}	u_{ADM}
0.1	0.29428565	0.28924531
0.2	0.32447042	0.31590677
0.3	0.37627732	0.36286938
0.4	0.46222603	0.44089584
0.5	0.59623287	0.56123678
0.6	0.79544168	0.73670859
0.7	1.08184953	0.98233639
0.8	1.48424305	1.31593221
0.9	2.04062381	1.75867138
1	2.80125802	2.33568188

Table 3: Comparisons between (HPM) and (ADM) solutions (when n=2, q=2).

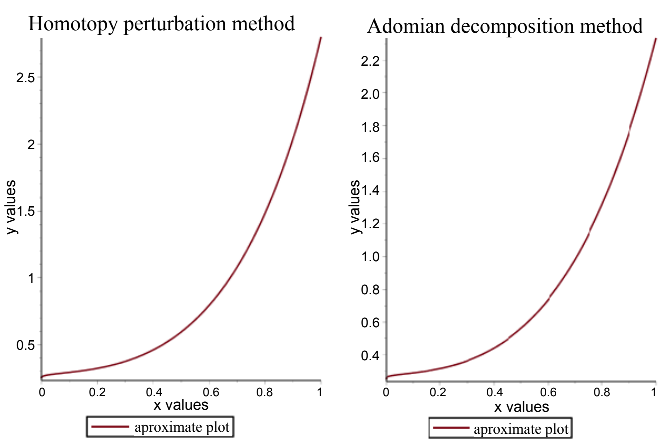

Figure 3: Approximate solution for (HPM)and (ADM).

t	u_{HPM}	u_{ADM}
0.1	0.28925	0.28924531
0.2	0.31591	0.31590677
0.3	0.36287	0.36286938
0.4	0.4409	0.44089584
0.5	0.56124	0.56123678
0.6	0.73671	0.73670859
0.7	0.98234	0.98233639
0.8	1.31593	1.31593221
0.9	1.75867	1.75867138
1	2.33568	2.33568188

Table 4: Comparison between the absolute error between (HPM) and (ADM) (when n=2, q=2).

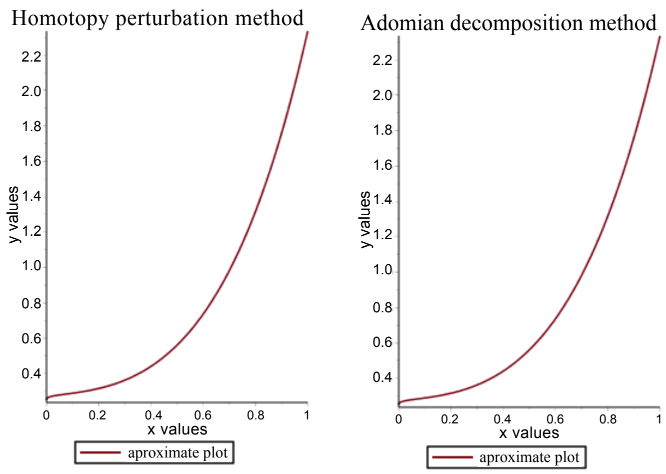

Figure 4: Approximate solution for (HPM) and (ADM).

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