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Hom-Algebras: Generalizations, Cohomology, and Deformations

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Introduction

The study of algebraic structures has seen remarkable advancements, particularly with the introduction of Hom-type algebras. These structures, characterized by the incorporation of a linear self-map, or deformation map, provide a generalized framework for classical algebraic identities. They offer new perspectives on deformation theory and have found applications in various fields of mathematics and physics, especially in understanding quantum deformations and related geometric structures. This collection of papers explores a diverse range of Hom-type algebras, delving into their fundamental properties, representations, cohomology theories, and various generalizations.

One fundamental area of focus involves Hom-Lie algebroids. This concept generalizes traditional Lie algebroids by introducing a Hom-Jacobi identity, where the Lie bracket interacts with a deformation map. The development of a robust cohomology theory for these structures is critical, not only for establishing their inherent properties but also for understanding their relationship to standard Lie algebroid cohomology in specific, limiting cases. This foundational work is essential for anyone studying geometric structures under the influence of deformation maps [1].

Beyond Hom-Lie algebroids, Hom-pre-Lie algebras represent another crucial class of generalized structures. Researchers delve into the representations and develop a comprehensive cohomology theory for these algebras. Critically, these investigations establish explicit relationships between Hom-pre-Lie algebras, Hom-Lie algebras, and their respective cohomologies. These insights are invaluable for comprehending the algebraic properties and deformation theory of these generalized structures [2]. This connection is further highlighted in studies exploring central extensions of Hom-Lie algebras. Here, Hom-pre-Lie algebras serve as an intermediate structure, providing a systematic approach to constructing and classifying these extensions. This research underscores the intricate interplay between different types of Hom-algebras, crucial for understanding their underlying architectures [5].

The generalizations extend to Hom-Lie-Yamaguti algebras, which are themselves an extension of classical Lie-Yamaguti algebras, enriched with a linear self-map. For these structures, representations are defined and a comprehensive cohomology theory is constructed. This theory is then applied to examine central extensions, offering vital tools for their classification and detailed deformation analysis [4]. The utility of this framework is further expanded by extending the concept to superalgebras, leading to the definition of Hom-Lie-Yamaguti superalgebras. A detailed cohomology theory is then developed for these superalgebras, rigorously applied to analyze their deformations. This work significantly contributes to the understanding of graded algebraic structures within the broader Hom-setting [9].

The influence of deformation maps on classical equations and operators is also a prominent area of inquiry. For instance, the Hom-associative Yang-Baxter equation is investigated within the context of Hom-Lie algebras. Solutions are presented, and their connections to r-matrices and other algebraic structures are explored. This advances the understanding of how deformation maps alter the Yang-Baxter equation in non-associative environments [3]. Similarly, the concept of relative Rota-Baxter operators is introduced and studied on Hom-Lie algebras. These operators are vital for understanding integrable systems and quantum groups. Their properties and relationships with other algebraic structures are established, extending classical Rota-Baxter theory into the Hom-framework [6]. Another significant contribution involves the deformation theory of Hom-Nijenhuis operators and their natural relationship to Hom-post-Lie algebras. A dedicated cohomology theory for Hom-Nijenhuis operators is provided, demonstrating how these operators inherently give rise to Hom-post-Lie algebras, thereby offering critical insights into the integrability conditions and deformations of various Hom-structures [8].

Further broadening the scope, the studies generalize Jordan triple systems and Lie triple systems by introducing Hom-Jordan triple systems and Hom-Lie triple systems. Researchers investigate the properties of these novel algebraic structures, exploring their connections and demonstrating how Hom-structures effectively provide a framework for deformed versions of classical triple systems [7]. Additionally, new classes of Hom-pre-Lie algebras, specifically those of type (r, s), are introduced. These generalize the standard Hom-pre-Lie algebras, and their algebraic properties, constructions, and relationships with other types of Hom-algebras are meticulously examined, enriching the overall theory of non-associative algebras equipped with deformation maps [10].

Collectively, these papers highlight the dynamic and expanding landscape of Hom-algebraic research. They not only generalize existing algebraic concepts but also develop sophisticated theoretical tools, such as cohomology theories, to understand the intricate effects of deformation maps. This body of work establishes a robust foundation for continued exploration into deformed algebraic structures and their multifaceted applications in geometry, mathematical physics, and pure algebra.

Description

The papers collected here provide a comprehensive overview of recent advancements in Hom-algebraic structures. This field centers on algebras where identities are modified by a linear self-map, often termed a deformation map. This approach allows for the generalization of classical algebraic systems, revealing new properties and relationships. A key contribution involves Hom-Lie algebroids, a direct generalization of Lie algebroids. The research introduces their concept, defining the Lie bracket with a Hom-Jacobi identity, and subsequently develops a comprehensive cohomology theory. This theory establishes fundamental properties and clarifies its relation to standard Lie algebroid cohomology in specific contexts, offering a strong base for understanding geometric structures with deformation maps [1].

Another significant area explored is Hom-pre-Lie algebras. Detailed studies address their representations and construct an extensive cohomology theory. These investigations are crucial for illuminating the algebraic properties and deformation theory of these generalized structures, and they explicitly connect Hom-pre-Lie algebras to Hom-Lie algebras and their respective cohomologies [2]. Building on this, the papers delve into central extensions of Hom-Lie algebras. Here, Hom-pre-Lie algebras play an integral role as an intermediate structure, providing a systematic method for constructing and classifying these extensions. This work effectively highlights the intricate interplay between various Hom-algebra types, deepening our understanding of their underlying structural relationships [5]. Further expanding on Hom-pre-Lie algebras, a new class known as Hom-pre-Lie algebras of type (r, s) is introduced. These structures generalize the standard Hom-pre-Lie algebras, and their algebraic properties, unique constructions, and connections to other Hom-algebras are thoroughly examined, considerably enriching the landscape of non-associative algebras that include deformation maps [10].

Generalizations extend to Hom-Lie-Yamaguti algebras, which incorporate a linear self-map into the classical Lie-Yamaguti algebras. This research defines representations for these structures and develops a robust cohomology theory. This theoretical framework is then used to investigate their central extensions, providing essential tools for classification and deformation analysis [4]. Taking this further, the concept is expanded to superalgebras, resulting in the definition of Hom-Lie-Yamaguti superalgebras. A detailed cohomology theory for these superalgebras is established, which is then applied to analyze their deformations. This particular line of inquiry significantly advances the understanding of graded algebraic structures within the specialized Hom-setting [9].

The collection also scrutinizes how deformation maps influence classical operators and equations. The Hom-associative Yang-Baxter equation is investigated within the framework of Hom-Lie algebras, presenting solutions and exploring their connections to r-matrices and other algebraic structures. This significantly advances the understanding of how deformation maps affect the Yang-Baxter equation in non-associative environments [3]. Similarly, the concept of relative Rota-Baxter operators on Hom-Lie algebras is introduced and thoroughly studied. These operators are recognized as essential tools in the theory of integrable systems and quantum groups. The research explores their properties and establishes relationships with other algebraic structures, effectively extending classical Rota-Baxter theory into the Hom-framework [6]. Another notable study concentrates on the deformation theory of Hom-Nijenhuis operators and their intrinsic relationship to Hom-post-Lie algebras. It provides a specific cohomology theory for Hom-Nijenhuis operators and shows how these operators naturally lead to Hom-post-Lie algebras, offering crucial insights into integrability conditions and deformations of Hom-structures [8].

Finally, the papers include generalizations of classical Jordan triple systems and Lie triple systems by introducing Hom-Jordan triple systems and Hom-Lie triple systems. The properties of these new algebraic structures are thoroughly investigated, exploring their connections and demonstrating that Hom-structures offer a robust framework for deformed versions of classical triple systems [7]. Overall, this body of work illustrates a concerted effort to generalize and understand various algebraic structures under the influence of deformation maps, providing a solid theoretical foundation for further research in abstract algebra, differential geometry, and mathematical physics.

Conclusion

The collection of research papers primarily focuses on various generalizations and extensions of algebraic structures in the Hom-setting. This includes deep dives into Hom-Lie algebras, Hom-pre-Lie algebras, and Hom-Lie algebroids, which incorporate a linear self-map to deform classical algebraic identities. A significant recurring theme is the development of cohomology theories for these generalized structures. Several works explore the cohomology of Hom-Lie algebroids [1], Hom-pre-Lie algebras [2], and Hom-Lie-Yamaguti algebras [4]. These cohomology theories are crucial for understanding deformations and classifications of these algebraic systems. For example, the cohomology of Hom-pre-Lie algebras helps in analyzing the deformation theory of related structures. Researchers also investigate the interplay between different Hom-algebraic types. Central extensions of Hom-Lie algebras are examined through the lens of Hom-pre-Lie algebras [5], demonstrating how one structure can illuminate another. The deformation theory of Hom-Nijenhuis operators is linked to Hom-post-Lie algebras, revealing connections to integrability conditions [8]. Further generalizations introduce Hom-Lie-Yamaguti superalgebras, for which a comprehensive cohomology theory and deformation analysis are developed [9]. Other studies extend classical concepts like Rota-Baxter operators to the Hom-Lie algebra setting [6], crucial for integrable systems. Hom-associative Yang-Baxter equations are also studied within Hom-Lie algebras, showcasing how deformation maps affect non-associative settings [3]. The exploration extends to Hom-Jordan triple systems and Hom-Lie triple systems, providing deformed versions of classical triple systems [7]. New classes, such as Hom-pre-Lie algebras of type (r, s), are introduced to further enrich the theory of non-associative algebras with deformation maps [10]. This body of work collectively builds a robust framework for understanding and classifying generalized algebraic structures, their deformations, and their relationships, offering foundational insights for future research in geometry and theoretical physics.

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Conflict of Interest

None.

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