

## Hesitant Fuzzy Sets and Hesitant Fuzzy Preference Relations

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Zadeh' fuzzy sets [1] are a powerful tool to address fuzziness, which have wide applications. As an extension of Zadeh's fuzzy sets [2], hesitant fuzzy sets (HFSs), originally introduced by Torra [3], are new tools for handling inaccuracy whereby two or more sources of vagueness appear simultaneously. Being distinct from other existing fuzzy sets, Torra [3] argued that when defining the membership of an element, the difficulty of establishing the membership degree is not a margin of error (as in intuitionistic fuzzy sets [4], or some possibility distributions (as in type 2 fuzzy sets [5]), but a set of possible values.

Due to the advantage of HFSs considering the hesitancy, HFSs have become a hot topic and received more and more attentions. For example, Xia and Xu [6] proposed a concept of hesitant fuzzy elements (HFEs), and defined some basic operations of HFEs. Then the studies on HFSs came to the aggregation operators [6-11], the distance and similarity measures [12], entropy and cross-entropy [13], etc. Under the linguistic environment, Rodríguez, Martínez and Herrera [14] developed the hesitant fuzzy linguistic term sets (HFLTSS) to improve the modeling and computational abilities of traditional linguistic approaches, whose envelopes are uncertain linguistic values [1,15]. HFLTSS increase the flexibility of the elicitation of linguistic information that allows the DMs hesitate about several possible values to assess a linguistic variable. The advantage of using HFLTSS is that they use richer expressions than a single linguistic term for the linguistic variables. Then Rodríguez et al. [16] further developed a group decision making model based on the hesitant fuzzy linguistic information.

To apply fuzzy sets to decision making, preference relations are a common tool used for collecting preferences provided by the decision makers (DMs). The fuzzy preference relations [5,17], the multiplicative preference relations [18,19], and the linguistic preference relations [20,21] have been widely used in decision making problems.

Considering the hesitant fuzzy information, Zhu [10] introduced the concept of hesitant fuzzy preference relation (HFPR), and proposed a regression-based method to transform a HFPR to a FPR with the highest consistency degree. The HFPR is a matrix consisting of HFEs which indicate all the possible preference degrees of an alternative over another. As an extension of the FPRs, HFPRs allow the DMs hesitate about some possible values of the membership when providing preferences in decision making problems, which is practical in applications.

For example, if the DMs have strongly divergent opinions, and cannot reach a consensus with respect to some preferences, this can be considered as a hesitant case, and all the hesitant preferences can be collected by the HFPR. This is different from an uncertain case modeled by an interval measure which uses an interval of numerical values to present a margin of error, such as the interval-MPRs [22], the interval-FPRs [23], the discrete preferences can be explicitly presented and collected in the HFPRs.

On the basis of HFLTSS, and taking linguistic information into account, Zhu and Xu [24] proposed the concept of hesitant fuzzy linguistic preference relation (HFLPR). To guarantee the DMs being neither random nor illogical in their preferences, they further developed some consistency measures to obtain HFLPRs with the

perfect consistency or the acceptable consistency [25]. These results are significant for further research of applying HFLTSS to decision making in practice.

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### References

- Xu Z (2004) Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environmen. *Inf Sci* 168: 171-184.
- Zadeh LA (1965) Fuzzy sets. *Inf Control* 8: 338-353.
- Torra V (2010) Hesitant fuzzy sets. *Int J Intelligent Sys* 25: 529-539.
- Atanassov K (1986) Intuitionistic fuzzy sets. *Fuzzy sets Sys* 20: 87-96.
- Herrera-Viedma E, Herrera F, Chiclana F, Luque M (2004) Some issues on consistency of fuzzy preference relations. *Eur J Oper Res* 154: 98-109.
- Xia MM, Xu ZS (2011) Hesitant fuzzy information aggregation in decision making. *Int J Approx Reason* 52: 395-407.
- Wei GW (2012) Hesitant fuzzy prioritized operators and their application to multiple attribute decision making. *Knowledge-Based Systems* 36: 176-182.
- Xia MM, Xu Z, Chen N (2013) Some hesitant fuzzy aggregation operators with their application in group decision making. *Group Decision and Negotiation* 22: 259-279.
- Zhu B, Xu Z, Xia MM (2012) Hesitant fuzzy geometric Bonferroni means. *Inf Sci* 205: 72-85.
- Zhu B (2013) Studies on Consistency Measure of Hesitant Fuzzy Preference Relations. *Proceed Com Sci* 17: 457-464.
- Zhu B, Xu Z (2013) Hesitant fuzzy Bonferroni means for multi-criteria decision making. *J Oper Res Soc*.
- Xu Z, Xia MM (2011) Distance and similarity measures for hesitant fuzzy sets. *Inf Sci* 181: 2128-2138.
- Xu Z, Xia MM (2012) Hesitant fuzzy entropy and cross-entropy and their use in multiattribute decision making. *Int J Intelligent Sys* 27: 799-822.
- Rodríguez RM, Martínez L, Herrera F (2012) Hesitant fuzzy linguistic term sets for decision making. *Fuzzy Systems, IEEE Trans* 20: 109-119.
- Xu Z (2006) An approach based on the uncertain LOWG and the induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations. *Deci Supp Sys* 41: 488-499.
- Rodríguez RM, Martínez L, Herrera F (2013) A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Inf Sci* 241: 28-42.

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17. Chiclana F, Herrera F, Herrera-Viedma E (1998) Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. *Fuzzy sets Sys* 97: 33-48.
18. Chiclana F, Herrera F, Herrera-Viedma E (2001) Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations. *Fuzzy sets and sys* 122: 277-291.
19. Herrera F, Herrera-Viedma E, Chiclana F (2001) Multiperson decision-making based on multiplicative preference relations. *Eur J Oper Res* 129: 372-385.
20. Fan ZP, Chen X (2005) Consensus measures and adjusting inconsistency of linguistic preference relations in group decision making. *Fuzzy Sys Know Dis* 3613: 475-475.
21. Herrera-Viedma E, Martinez L, Mata F, Chiclana F (2005) A consensus support system model for group decision-making problems with multigranular linguistic preference relations. *Fuzzy Systems, IEEE Trans* 13: 644-658.
22. Saaty TL, Vargas LG (1984) Comparison of eigenvalue, logarithmic least squares and least squares methods in estimating ratios. *Math Model* 5: 309-324.
23. Xu Z (2004) On compatibility of interval fuzzy preference relations. *Fuzzy Opt Decision Making* 3: 217-225.
24. Zhu B, Xu ZS (2013) Consistency measures for hesitant fuzzy linguistic preference relations. *Fuzzy Systems, IEEE Trans*, 10.1109/TFUZZ.2013.2245136.
25. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning--I. *Inf Sci* 8: 199-249.