Heat and Mass Transfer in MHD Mixed Convection Flow on a Moving Inclined Porous Plate

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Abstract
The effects of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous, incompressible and electrically conducting fluid on a moving inclined heated porous plate is analyzed. The non-linear coupled partial differential equations are solved by perturbation technique. The influence of different pertinent parameters such as Grashof number (Gr), modified Grashof number (Gm), magnetic field parameter (M), heat source parameter (ϕ), chemical reaction parameter (γ), Schmidt number (Sc) and angle of inclination (α) on velocity, temperature and concentration distribution have been studied and analyzed with the help of graphs. An analysis of the coupled heat and mass transfer phenomena is provided in detail. We observed that velocity decreases for increasing values of the angle of inclination α. The results of the present study are compared with the results obtained by Chaudhary et al. and Dulal et al. in the absence of angle of inclination; our results appear to be in good agreement with the existing results.

Keywords: Heat and mass transfer; Viscous and Ohmic dissipation; Radiation; Chemical reaction; Inclined; Porous plate

Introduction
The convective heat and mass transfer flows in porous medium find a number of applications in many branches of science and technology like chemical industry, cooling of nuclear reactors. MHD power generators, geothermal energy extractions processes, petroleum engineering etc. [1,2]. Convective boundary layer flow problems in the cases of horizontal and vertical flat plates have been investigated quite extensively. The boundary layer flows adjacent to inclined plates or wedges have received less attention. The study of Sparrow et al. [3] is related to the convection flow about an inclined surface in which the combined forced and free boundary layer problem has been discussed using the similarity method. Chaudhury [1] have analyzed the effect of radiation on heat transfer in MHD mixed convection flow with simultaneous thermal and mass diffusion from an infinite vertical plate with viscous dissipation and Ohmic heating. Dulal [2] studied the combined effect of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium. Singh [4] studied Heat and Mass Transfer in MHD Boundary Layer Flow past an Inclined Plate with Viscous Dissipation in Porous Medium. Cheng [5] studied the convective flow problem about an inclined surface through porous medium. Mansour [6] studied the boundary layer analysis has been presented for the free convection flow past an inclined surface in a Newtonian fluid-saturated porous medium. Chen [7] investigated the heat and mass transfer characteristics of MHD natural convection flow over a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of Ohmic heating and viscous dissipation. Hossain [8] have studied the convection flow from an isolated plate inclined at a small angle to the horizontal. Dulal pal [9] investigated the unsteady mixed convection with thermal radiation and first order chemical reaction on magnetohydrodynamic. Noor [10] studied the problem involving the conjugate phenomenon of heat and mass transfer analytically. Bhuvaneswari [11] analyzed on the magnetohydrodynamic (MHD) free convection flow with simultaneous effects of heat and mass transfer.

Ziyauddin [12] developed closed form exact solutions for the unsteady MHD free convection flow of a viscous fluid over an inclined plate with variable heat and mass transfer in a porous medium. Alam [13] studied the combined effect of viscous dissipation and Joule heating on steady MHD free convective heat and mass transfer flow of a viscous incompressible fluid past a semi-infinite inclined radiate isothermal permeable moving surface in the presence of thermophoresis. Ganesan et al. [14] studied the problem of unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an inclined plate with variable heat and mass flux’s. Orthan Aydm et al. [15] studied MHD mixed convective heat transfer flow about an inclined plate. Thermo diffusion and chemical effects with simultaneous thermal and mass diffusio in MHD mixed convection flow with Ohmic heating was studied by Reddy et al. [16]. Unsteady MHD radiative and chemically reactive free convection flow near a moving vertical plate in porous medium was studied by Reddy et al. [17]. MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating was considered by Raju et al. [18]. Combined effects of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate were investigated by Ravikumar et al. [19]. Analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction was carried out by Raju et al. [20]. Thermal diffusion effect on MHD mixed convection flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating.

Received September 04, 2015; Accepted September 26, 2015; Published September 29, 2015


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unsteady flow of a micro polar fluid past a semi-infinite vertical porous plate with radiation and mass transfer was studied by Mamatha et al. [21]. Unsteady MHD free convection flow of a Kuvshinski fluid past a vertical porous plate in the presence of chemical reaction and heat source/sink was studied by Reddy et al. [22]. Motivated by the above studies, in this manuscript an attempt is made to investigate the effects of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous, incompressible and electrically conducting fluid on a moving inclined heated porous plate.

Mathematical Formulation

Consider a laminar boundary layer flow of a viscous incompressible electrically conducting and heat absorbing fluid past a semi-infinite moving permeable plate inclined at an angle \( \alpha \) in vertical direction embedded in a uniform porous medium, which is subject to thermal and concentration buoyancy effects. The wall is maintained at a constant temperature \( T_w \) and concentration \( C_w \) higher than the ambient and concentration buoyancy effects. The wall is maintained at a constant temperature \( T_{\infty} \) and concentration \( C_{\infty} \) respectively. Also it is assumed that there exists a homogeneous chemical reaction of first order with rate constant \( R \) between the diffusing species and the fluid. With these physical considerations, the equations governing the fluid. Based on the above assumptions the governing equations that describe the physical situation can be given in Cartesian frame of reference as

Continuity Equation:

\[
\frac{\partial \nu}{\partial y} = 0 \Rightarrow \nu = -\nu_0 \tag{1}
\]

Momentum Equation:

\[
\rho \nu \frac{\partial u}{\partial y} - \mu \frac{\partial^2 u}{\partial y^2} = -\sigma K_i u + \rho g \beta T \left[ T^2 - T_\infty \right] \cos \alpha + \rho g \beta \nu \left( C - C_\infty \right) \cos \alpha \tag{2}
\]

Energy Equation:

\[
\frac{\rho C_v \nu}{\partial y} = -\sigma K_i u + \nu \frac{\partial^2 u}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y}\right)^2 - \rho_0 \left[ T^2 - T_\infty \right] \cos \alpha \tag{3}
\]

Concentration Equation:

\[
y \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \lambda \left( C - C_\infty \right) \tag{4}
\]

The radiative heat flux is given by

\[
\frac{\partial Q^*}{\partial y} = 4 \left( T^2 - T_\infty \right) \frac{dI}{dy} \tag{5}
\]

Where \( I = \int_0^\infty K_{I_{\infty}} \frac{dI}{dy} \, d\lambda \), \( K_{I_{\infty}} \) is the absorption coefficient at the wall and \( \epsilon_{\infty} \) is Planck’s function. Under these assumptions the appropriate boundary conditions for velocity, temperature and concentration fields are defined as

\[
u = 0, T = T_w, C = C_{\infty} \quad \text{at} \quad y = 0 \tag{6}
\]

\[
u \rightarrow 0, T \rightarrow T_{\infty}, C \rightarrow C_{\infty} \quad \text{as} \quad y \rightarrow \infty \tag{7}
\]

Introducing the following non-dimensional quantities

\[
y^* = \frac{y}{y_0}, \quad u^* = \frac{u}{u_0}, \quad T^* = \frac{T}{T_0}, \quad C^* = \frac{C}{C_0}, \quad \nu^* = \frac{\nu}{\nu_0}, \quad \epsilon^* = \frac{\epsilon}{\epsilon_0} \tag{8}
\]

The basic field eqs (2) – (4), can be expressed in non-dimensional form as

\[
\frac{d^2 u}{dy^2} + \frac{d(u)}{dy} \left( M^2 + \frac{1}{K} \right) u = -Gr \cos \alpha - Gm \cos \alpha \tag{9}
\]

\[
\frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + Pr Ec \left( \frac{d\theta}{dy} \right)^2 - Pr \left( F + \phi \right) + Pr Ec M^2 u^2 = 0 \tag{10}
\]

\[
\frac{d^2 C}{dy^2} + Sc \frac{dC}{dy} - ScyC = 0 \tag{11}
\]

Where \( M \) is the magnetic parameter, \( K \) is the permeability parameter, \( Gr \) is the Grashof number, \( Gm \) is the modified Grashof number, \( \alpha \) is the angle of inclination, \( \gamma \) is the chemical reaction parameter, \( \phi \) is the heat source parameter and \( Sc \) is the Schmidt number defined as follows:

\[
Gr = \frac{pg \beta T (T - T_{\infty})}{\nu \mu}, \quad Gm = \frac{pg \beta \nu (C - C_{\infty})}{\nu \mu}, \quad Pr = \frac{\mu}{\rho C_v}, \quad \gamma = \frac{-\gamma v}{C_w}, \quad \phi = \frac{-\phi v}{C_w}, \quad Ec = \frac{v}{D}\tag{12}
\]

The corresponding boundary conditions in non-dimensional form are:

\[
u = 0, \theta = 1, C = 1 \quad \text{at} \quad y = 0 \tag{13}
\]

\[
u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \tag{14}
\]

Method of Solution

The set of partial differential equations (9) – (11) cannot be solved in closed form. However they can be solved analytically after reducing them into a set of ordinary differential equations by taking the expressions for velocity \( u(y) \), temperature \( \theta(y) \) and concentration \( C(y) \) [9] in dimensionless form as follows:

\[
u(y) = u_0(y) + Ec u_1(y) + O\left( Ec^2 \right) \tag{14}
\]

\[
\theta(y) = \theta_0(y) + Ec \theta_1(y) + O\left( Ec^2 \right) \tag{15}
\]

\[
C(y) = C_0(y) + Ec C_1(y) + O\left( Ec^2 \right) \tag{16}
\]

Substituting (14)-(16) in (9) – (11) and equating the coefficients of zeroth order of Eckert number (constants), and neglecting the higher order of \( O\left( Ec^2 \right) \) and simplifying to get the following set of equations

\[
u_0 + u_0' - p u_0 = -G_0 \theta_0 - G_1 C_0 \tag{17}
\]

\[
\theta_0' + Pr \theta_0' - Pr (F + \phi) \theta_0 = 0 \tag{18}
\]

\[
C_0' + Sc C_0' - ScyC_0 = 0 \tag{19}
\]

Equating the coefficients of first order of Eckert number, we get

\[
u_0' + u_0' - pu_0 = -G_0 \theta_1 - G_1 C_1 \tag{20}
\]

\[
\theta_0'' + Pr \theta_0' - Pr (F + \phi) \theta_0 + Pr u_0' + Pr M^2 u_0'' = 0 \tag{21}
\]

\[
C_0'' + Sc C_0' - ScyC_1 = 0 \tag{22}
\]

Where prime denotes ordinary differentiation with respect to ‘\( y \)’ and

\[
p = M^2 + \frac{1}{K}, \quad G_1 = Gr \cos \alpha, \quad G_2 = Gm \cos \alpha \tag{23}
\]

The corresponding boundary conditions are:

\[
u_0 = 0, u_0' = 0, \theta_0 = 1, \theta_0' = 0, C_0 = 1, C_1 = 0 \quad \text{at} \quad y = 0 \tag{24}
\]
\[ u_0 \to 0, u_y \to 0, \theta_0 \to 0, \dot{\theta}_0 \to 0, c_0 \to 0, c_1 \to 0 \quad \text{as} \quad y \to \infty \quad (24) \]

Using boundary conditions (23) and (24), the solutions of (17) – (22), we obtain the following expressions for velocity, temperature and concentration,

\[ u_0 = A_2 \left( e^{-A_1 y} - e^{-A_2 y} \right) + A_6 \left( e^{-A_4 y} - e^{-m_1 y} \right) \quad (25) \]

\[ \theta_0 = e^{-A_3 y} \quad (26) \]

\[ c_0 = e^{-m_1 y} \quad (27) \]

\[ u = B_1 e^{-A_4 y} + B_2 e^{-A_5 y} - B_3 e^{-A_6 y} + B_4 e^{-A_7 y} + B_5 e^{-A_8 y} + B_6 e^{-A_9 y} \quad (28) \]

\[ \theta = B_4 e^{-A_7 y} - B_5 e^{-A_8 y} + B_6 e^{-A_9 y} - B_7 e^{-A_{10} y} + B_8 e^{-A_{11} y} - B_9 e^{-A_{12} y} \quad (29) \]

\[ c_1 = 0 \quad (30) \]

Substituting the above solutions (25)-(30) in (14)-(16), we get the final form of velocity,

Temperature, concentration distributions in the boundary layer as follows

\[ u(y) = A_2 \left( e^{-A_1 y} - e^{-A_2 y} \right) + A_6 \left( e^{-A_4 y} - e^{-m_1 y} \right) \]

\[ + B_1 e^{-A_4 y} + B_2 e^{-A_5 y} - B_3 e^{-A_6 y} + B_4 e^{-A_7 y} + B_5 e^{-A_8 y} + B_6 e^{-A_9 y} \quad (31) \]

\[ \theta(y) = B_4 e^{-A_7 y} - B_5 e^{-A_8 y} + B_6 e^{-A_9 y} - B_7 e^{-A_{10} y} + B_8 e^{-A_{11} y} - B_9 e^{-A_{12} y} \quad (32) \]

\[ C(y) = e^{-m_1 y} \quad (33) \]

The physical quantities of interest are the wall shear stress \( \tau_w \) is given by

\[ \tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} = \rho_0 u'(0) \]

The local skin friction factor \( C_{f_2} \) is given by

\[ C_{f_2} = \frac{\tau_w}{\rho_0 u'^2} = \frac{1}{\rho_0 u'^2} \bigg|_{y=0} = \frac{1}{\rho_0 u'^2} \bigg|_{y=0} \quad (34) \]

The local surface heat flux is given by:

\[ q_w = -\kappa \frac{\partial T}{\partial y} \bigg|_{y=0} \]

Where \( \kappa \) is the effective thermal conductivity. The local Nusselt number \( Nu_x = \frac{q_w}{T_w - T_r} \) can be written as

\[ Nu_x = -\frac{\partial T}{\partial y} \bigg|_{y=0} = \frac{1}{\rho_0 u'^2} \bigg|_{y=0} \quad (35) \]

The local surface mass flux is given by

\[ Sk_x = \frac{\partial c}{\partial y} \bigg|_{y=0} = -m_1 \quad (36) \]

Where \( Re_x = \frac{v_y X}{V} \) is the local Reynolds number.

**Results and Discussion**

The results are showing the nature of the effects of the parameters like Grashof number \( Gr \), modified Grashof number \( Gm \), angle of inclination \( \alpha \), Prandtl number \( Pr \), magnetic field \( M \), radiation parameter \( \Phi \), Schmidt number \( Sc \), heat source parameter \( \phi \), chemical reaction parameter \( \gamma \) and permeability parameter \( K \) on velocity, temperature and concentration profiles are displayed with the help of graphical illustration.

Figure 1 shows the effects of angle of inclination \( \alpha \) on velocity profile. We observed that the velocity decreases for increasing the angle of inclination \( \alpha \). The effects of Grashof number \( Gr \) on velocity distribution are presented in Figure 2, from this figure we noticed that the velocity increases as Grashof number \( Gr \) increases. In Figure 3 the effects of magnetic parameter \( M \) on velocity is shown. From this figure we observed that the velocity decreases as \( M \) increasing in case of cooling of the plate. The effects of heat source parameter \( \phi \), on velocity is shown in Figure 4. It is noticed that the velocity decreases as \( \phi \) increases. In Figures 5 and 6 the velocity profile increases as permeability parameter \( K \) and modified Grashof number \( Gm \) increase respectively. Figure 7 shows that velocity decreases with increase in the value of Radiation parameter \( F \). Figure 8 depicts that velocity decreases as chemical reaction parameter \( \gamma \) increases. In Figure 9 we observe that velocity decreases as Schmidt number \( Sc \) increases. In Figure 10 we observe that velocity decreases as Prandtl number \( Pr \) increases. Figure 11 represents graph of temperature distribution decreases with the increase of radiation parameter \( F \). Similar effect is seen by value of heat Source parameter \( \phi \) as shown in Figure 12. Figures 13 and 14 represent the effects of chemical reaction parameter \( \gamma \) and Schmidt number \( Sc \) on velocity profile.
Figure 3: Velocity profiles for different values of \( M \). \( Pr=0.7, \, Sc=0.6, \, F=3.0, \, K=0.5, \, \gamma=0.1, \, Gm=2.0, \, Gr=4.0, \, \phi=2.0, \, \alpha=30^\circ, \, E=0.01 \).

Figure 4: Velocity profiles for different values of \( \phi \). \( Pr=0.7, \, M=2.0, \, Sc=0.6, \, F=3.0, \, K=0.5, \, \gamma=0.1, \, Gm=2.0, \, Gr=4.0, \, \alpha=30^\circ, \, \phi=2.0, \, E=0.01 \).

Figure 5: Velocity profiles for different values of \( K \). \( Pr=0.7, \, M=2.0, \, Sc=0.6, \, F=3.0, \, \gamma=0.1, \, Gm=2.0, \, Gr=4.0, \, \alpha=30^\circ, \, \phi=2.0, \, E=0.01 \).

Figure 6: Velocity profiles for different values of \( Gm \). \( Pr=0.7, \, M=2.0, \, Sc=0.6, \, F=3.0, \, K=0.5, \, \gamma=0.1, \, Gr=4.0, \, \alpha=30^\circ, \, \phi=2.0, \, E=0.01 \).

Figure 7: Velocity profiles for different values of \( F \). \( Pr=0.7, \, M=2.0, \, Sc=0.6, \, K=0.5, \, \gamma=0.1, \, Gm=2.0, \, Gr=4.0, \, \alpha=30^\circ, \, \phi=2.0, \, E=0.01 \).

Figure 8: Velocity profiles for different values of \( \gamma \). \( Pr=0.7, \, M=2.0, \, Sc=0.6, \, K=0.5, \, F=3.0, \, Gm=2.0, \, Gr=4.0, \, \alpha=30^\circ, \, \phi=2.0, \, E=0.01 \).
Figure 9: Velocity profiles for different values of $Sc$. $Pr=0.7$, $M=2.0$, $K=0.5$, $F=3.0$, $Gm=2.0$, $Gr=4.0$, $\gamma=0.1$, $\alpha=30^\circ$, $\varphi=2.0$, $E=0.01$.

Figure 10: Velocity profiles for different values of $Pr$. $M=2.0$, $K=0.5$, $F=3.0$, $Gm=2.0$, $Gr=4.0$, $\gamma=0.1$, $\alpha=30^\circ$, $Sc=0.6$, $E=0.01$.

Figure 11: Temperature profiles for different values of $F$. $Pr=0.7$, $Sc=0.6$, $M=2.0$, $K=0.5$, $Gm=2.0$, $Gr=4.0$, $\gamma=0.1$, $\alpha=30^\circ$, $\varphi=2.0$, $E=0.01$.

Figure 12: Temperature profiles for different values of $\varphi$. $Pr=0.7$, $Sc=0.6$, $M=2.0$, $K=0.5$, $Gm=2.0$, $Gr=4.0$, $\gamma=0.1$, $\alpha=30^\circ$, $E=0.01$.

Figure 13: Concentration profiles for different values of $\gamma$. $Sc=0.6$, $E=0.01$.

Figure 14: Concentration profiles for different values of $Sc$. $\gamma=0.1$, $E=0.01$. 
Table 1: Comparison of present results with those of Chaudhary et al. [1] and Dulal Pal et al. [2] with different values of $F$ for $C_{fx}$, $N_{ux}/Re_x$, $Sh_{x}/Re_x$, $C_{fy}$, $N_{uy}/Re_y$, $Sh_{y}/Re_y$, where as it shows reverse tendency in the case of Grashof number $Gr$, the following conclusions are made.

1. Velocity decreases for increasing values of the angle of inclination $\alpha$.
2. Temperature distribution decreases with an increase in radiation parameter $\phi$.
3. Concentration boundary layer decreases with an increase in chemical reaction parameter $\gamma$ and Schmidt number $Sc$.

**Appendix**

$$m_i = \frac{Sc \sqrt{Sc^2 + 4Sc\gamma}}{2}$$

$$A_i = \frac{G_i}{A_i - A_i - p}$$

$$A_i = \frac{G_i}{m_i - m_i - p}$$

**References**


