Heat and Mass Balance for Baking Process

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Abstract

The main objective of this research was to develop a mathematical model of heat and mass balance of the bread baking process to predict the temperature and water content of bread at different heating temperatures and times. The model was able to predict the bread temperature and water content at different oven temperatures (180, 190, 200, 210 and 220°C). The results showed that the bread temperature and weight loss of bread increase with increasing oven temperature, when, the oven temperature increased from 180 to 220°C, the temperature of bread increased from 112.73 to 168.49°C and the weight loss of bread increased from 22.40 to 52.46%. The results also showed that the bread temperature and weight loss of bread increase with increasing time, this increment starts to decline after 18-20 min, until it reach equilibrium after 30 min. The weight loss of bread increases with increasing bread temperature. It indicates that when the bread temperature increased from 20.00 to 131.69°C, the weight loss of bread increased from 0.00 to 40.79% at 200°C oven temperature. The model was validated with an experimental data and showed a reasonable agreement with those measured, where; it ranged 20.00 to 131.69°C theoretically while it was from 25.00 to 119.00°C experimentally during the baking at 200°C oven temperature. The weight loss data was in a reasonable agreement with those measured, where; it ranged 0.00 to 40.79% theoretically while it was from 0.00 to 48.69% experimentally during the baking at 200°C oven temperature.

Keywords: Heat balance; Mass balance; Mathematical model; Bread temperature; Baking

Introduction

Baking is the final and most important step in bread production, and can be defined as the process which transforms dough, basically made of flour, water and leavening agents, in a food with unique sensory features by application of heat inside an oven. In particular, white or French bread is the most popular type of bread, and is distinguished for having a crunchy and golden-yellow (or brown) crust, a sponge and light crumb with soft texture and intermediate moisture, and a typical flavour. All these quality aspects are the result of a series of physical and chemical changes produced by simultaneous heat and mass transfer occurring within the product during baking [1-6].

The first bread was made around 10,000 years BC or over 12,000 years in the past, which may have been developed by deliberate experimentation with water and grain flour. Egyptians are the pioneers who make the art of bread making popular throughout the world. Control of the production and distribution of bread has been used as a means of exercising political influence over the populace for at least the last 2000 years. Even today, a shortage of bread is synonymous with hard times, while the promise of its service is used as rallying call for better life [2].

Simulation can be defined as the process of developing a model of a real system and carrying out experiments through the model, with the aim of studying, analyzing, designing or re-designing, controlling and predicting a certain real process. Besides the validation of the model, numerical simulation does not imply field experiments; it only includes the development of a mathematical model and computational effort, mostly more inexpensive than real tests. In addition, simulation gives the possibility of working under standardized operative conditions, minimizing the uncertainties of complex processes, especially those which are traditional non-automated processes. In this way, it will be very useful to have an accurate mathematical model for bread baking simulation [7].

There are several researchers have been studied the baking process, [8] studied of sponge cake batter baking process (Modeling and parameter estimation) and [9] developed a model of heat and mass transfer phenomena and quality changes during continuous biscuit baking using both deductive and inductive (neural network) modelling principles. [10,11] developed a simulation of bread making process using a direct 3D numerical method at microscale (Analysis of foaming phase during proofing, and Analysis of baking step). Also, [12] studied behavior of baking of semi-sweet short dough biscuits mathematically.

Bread is closely related to people's daily life and bread making is an important unit operation in food industry. Although people have practiced baking for a very long time, quantitative understanding of physical processes in baking is still very limited, therefore, understanding the physical processes of baking such as volume expansion, heating temperature, moisture change is the main aim of this work which could be achieved by developing a mathematical model of the heat and mass balance of the baking process to study the heating temperature and bread moisture content.

Model Development

The model includes the main distinguishing features of bread baking, i.e. the rapid heating of bread core and the development of a dry outer crust. Bread is modeled as a system containing three different regions: (1) crumb: wet inner zone, where temperature does not exceed 100°C and dehydration does not occur; (2) crust: dry outer zone, where temperature exceeds 100°C and dehydration occurs; (3) evaporation front: between the crumb and crust, where temperature is 100°C and water evaporates (liquid-vapour transition) [7,13].

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Mathematically, the Moving Boundary Problem (MBP) is formulated using a physical approach, where the enthalpy jump corresponding to phase change is incorporated in the model by defining equivalent thermo physical properties [14]. Such definition states that evaporation occurs within a temperature range rather than at a fixed temperature. Other major assumptions of the model are the following:

- Bread is homogeneous and continuous; the concept of porous medium is included through effective or apparent thermo physical properties.

- Heat is transported by conduction from surface to bread core according to Fourier's law, but an effective thermal conductivity is used to incorporate the evaporation-condensation mechanism in heat transfer.

- Only liquid diffusion in the crumb and only vapour diffusion in the crust are assumed to occur [15].

- Volume change is neglected.

- Energy from oven ambient to bread surface is transferred by convection and radiation.

- Liquid water migrates from the core towards to the evaporation front under a water content gradient and liquid water flux is described by Fick's diffusion law.

- Water vapour migrates from the evaporation front to bread surface under a water vapour concentration gradient and mass flux is described by Fick's diffusion law.

- Water vapour is transferred to oven ambient through convective flux.

Governing Equations

Bread is considered as an infinite cylinder of radius r, so the problem is reduced to a single dimension via the axial symmetry assumption and it is assumed that the volume. For initial conditions, uniform temperature and water content are assumed. Figure 1 represents the heat and mass balance for bread. Heat balance include: heat conduction, heat convection and heat radiation.
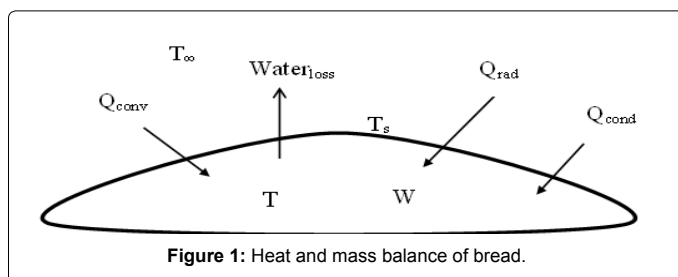
Heat balance equation

$$\rho C_P \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) \tag{1}$$

Boundary conditions

The heat arrives to the bread surface by convection and radiation, and is balanced by conduction inside the bread:

$$-k\frac{\partial T}{\partial r} = h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_\infty^4\right) \tag{2}$$

Figure 1: Heat and mass balance of bread.

Mass balance equation

$$\frac{\partial W}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial W}{\partial r}\right) \tag{3}$$

Boundary conditions

The water migrating towards the bread surface is balanced by convective flux:

$$-D\rho_s\frac{\partial W}{\partial t} = k_g\left(P_s\left(T_s\right) - P_\infty\left(T_\infty\right)\right) \tag{4}$$

$$p_s = a_w p_{sat}\left(T_s\right) \tag{5}$$

$$p_\infty = \left(\frac{RH}{100}\right)p_{sat}\left(T_\infty\right) \tag{6}$$

At the center, i.e. r = 0

$$\frac{\partial T}{\partial t} = 0 \tag{7}$$

$$\frac{\partial W}{\partial t} = 0 \tag{8}$$

Thermo physical properties

According to the MBP formulation, equivalent thermo physical properties are defined including the phase transition occurring during the process, i.e. an equivalent property is valid for dough/ crumb and crust [7,16].

Specific heat

$$C_p\left(T,W\right) = C_p^*\left(T,W\right) + \lambda_v W\delta \tag{9}$$

$$C_p^*\left(T,W\right) = C_{p,s}\left(T\right) + WC_{p,w}\left(T\right) \tag{10}$$

$$C_{p,s} = 5T + 25 \tag{11}$$

$$C_{p,w} = \left(5.207 - 73.17\times10^{-4}T + 1.35\times10^{-5}T^2\right)\times1000 \tag{12}$$

Latent heat evaporation is calculated with the following set of equations [10]:

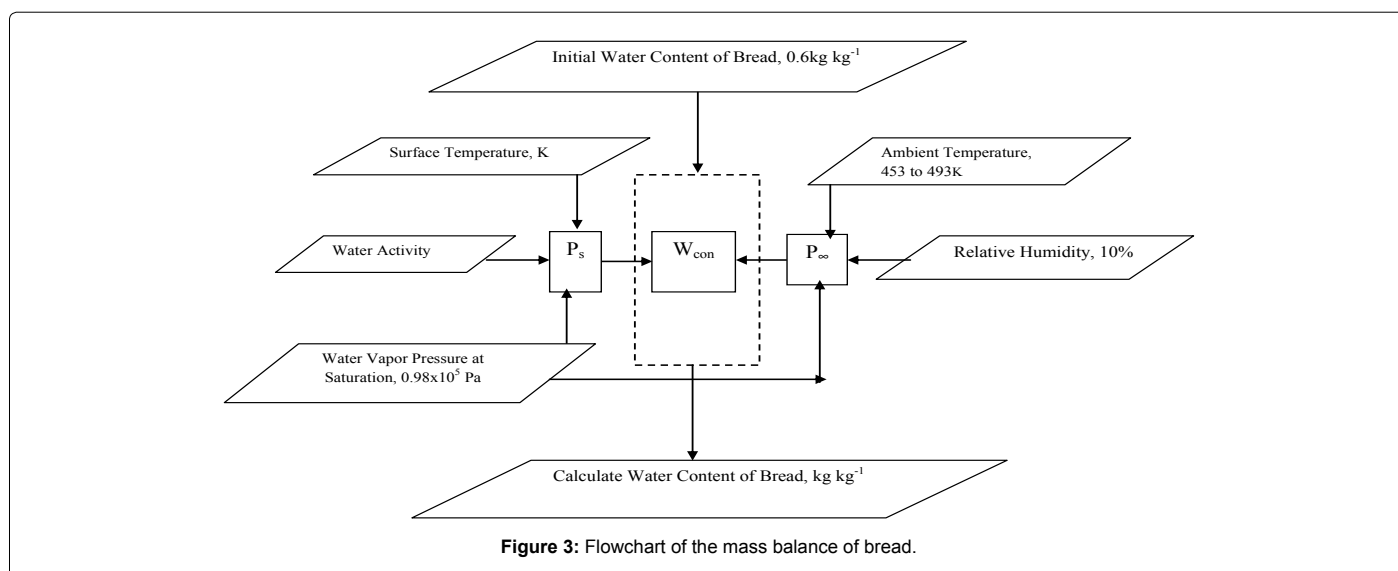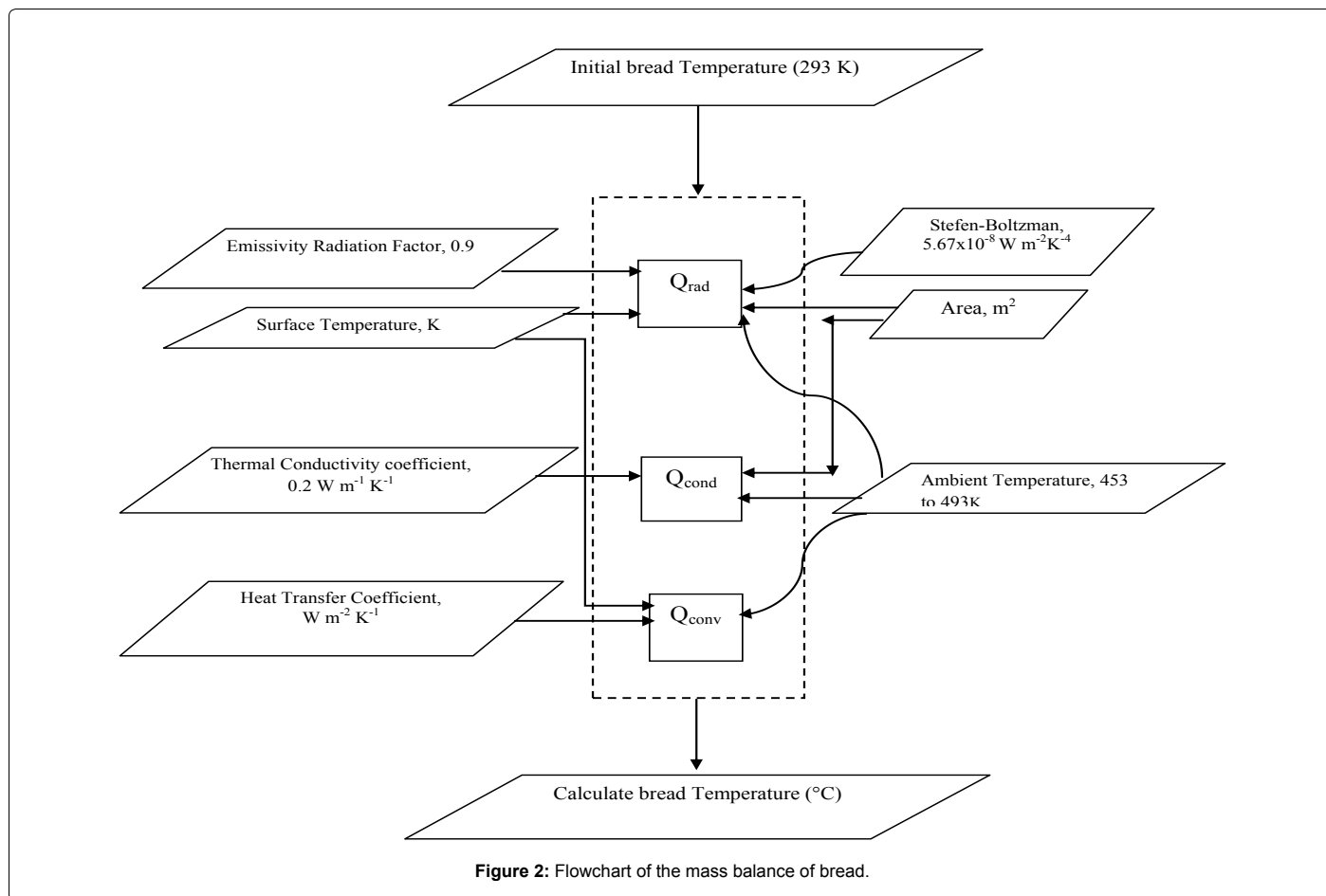$$\lambda_v = \left(2502535.259 - 212.56384\left(T - 273\right)\right) \tag{13}$$

Water activity

$$a_w\left(T,W\right) = \left(\left(\frac{100W}{\exp\left(-0.0056T + 5.5\right)}\right)^{-\frac{1}{0.38}} + 1\right)^{-1} \tag{14}$$

The heat transfer coefficient (h) is a model input for process simulation, and the mass transfer coefficient (kg) is determined by using the Chilton–Colburn (or heat–mass) analogy and a correction factor [7]:

$$\frac{h}{k_g^*} = \frac{M_{air}}{M_w}P_{atm}C_{p,air}\left(\frac{Sc}{Pr}\right)^{\frac{2}{3}} \tag{15}$$

$$k_g = 7.83\times10^{-3}k_g^* \tag{16}$$

$$Sc = \frac{v}{D} \tag{17}$$

Figure 2: Flowchart of the mass balance of bread.

Figure 3: Flowchart of the mass balance of bread.

$$Pr = \frac{v}{\alpha} \qquad (18)$$

Regarding heat transfer by radiation, the emissivity of bread surface is considered equal to 0.9 [17,18].

All computational procedures of the model were carried out using Excel spreadsheet. The computer program consisted of two parts in addition to the input parts. The first part was devoted to heat balance for predicting the temperature at the middle cross-section of bread. The second part was devoted to mass balance for predicting water content of bread. Figures 2 and 3 show the flowcharts of the model steps and sequences. They are representing the heat and mass balance and the

Parameter	Units	Value	Source
Thermal conductivity, k	W m^{-1} K^{-1}	0.20	[6]
Bread density, ρ	kg m^{-3}	321.31	[18]
Density of solid, ρ$_s$	kg m^{-3}	241.76	[18]
Water diffusion coefficient, D	m^2 s^{-1}	1×10^{-4}	[19]
Water vapor pressure at saturation, P$_{sat}$	Pa	0.98×10^5	[20]
Mass transfer coefficient, k$_g$*	kg Pa^{-1} m^{-2} s^1	1.1×10^{-7}	[5]
Emissivity, ε		0.90	[18]
Kinematic viscosity, v	m^2 s^{-1}	28.86×10^{-6}	[21]
Thermal diffusivity, α	m^2 s^{-1}	1.17×10^{-7}	[21]
Bread radius, r	M	0.03	
Relative humidity, RH	%	10	
Delta-type function, δ		1	
Stefan-Boltzmann constant, σ	W m^{-2} K^{-4}	5.67 × 10^{-8}	
Initial temperature	K	293	
Initial water content	kg kg^{-1}	0.6	

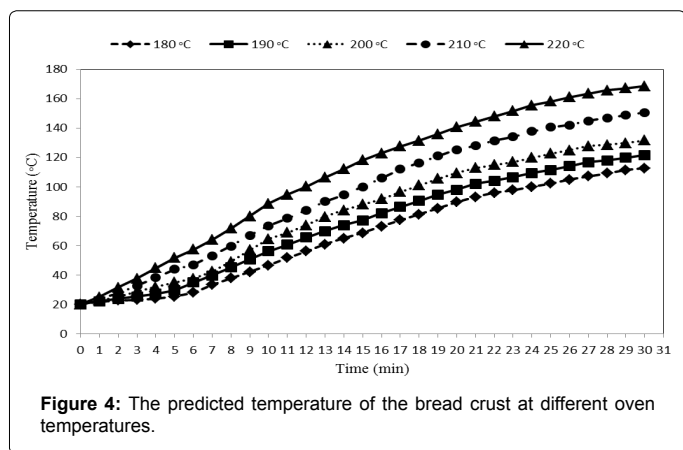**Table 1:** The parameters used in the heat and mass balance.

Figure 4: The predicted temperature of the bread crust at different oven temperatures.
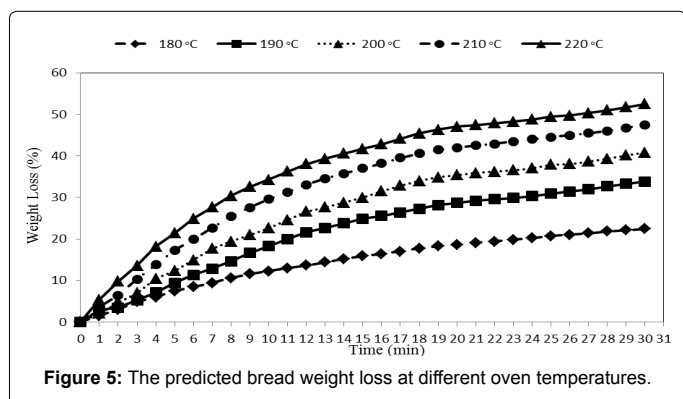

Figure 5: The predicted bread weight loss at different oven temperatures.

calculation steps of their model. Table 1 show the parameters used in the model.

Results and Discussions

Model experimentation

Temperature: Figure 4 shows the predicted temperature of the crust of bread process during the baking at different oven temperatures (180, 190, 200, 210 and 220°C). The results indicate that the bread temperature increases with increasing oven temperature. It also indicates that when the oven temperature increased from 180 to 220°C, the temperature at the crust of bread increased from 112.73 to 168.49°C. The results

show that the temperature increases with increasing time, it increased from 20 to 112.73°C when the time increased from 0 to 30 min at oven temperature 180°C. On the other hand, the temperature increased from 20 to 116.18°C, 20 to 131.69°C, 20 to 144.05°C and 20 to 168.49°C at oven temperature 190, 200, 210 and 220°C, respectively. These results were in agreement with those obtained by [7,16,22-25].

It is worth to note that the bread temperature reached 62.6% of the heat balance at temperature of 180°C, while it reached 76.7% of the heat balance at 220°C oven temperature, this could lead to that using the higher temperature (200-220°C) more effective in utilization of heat energy. Multiple regression analysis was carried out to find a relationship between baking time, oven temperature and the bread temperature. The best form obtained was as follows:

$$T_P = -210.217 + 4.45t + 1.137T_o \qquad R^2 = 0.97 \ (19)$$

Weight loss: Figure 5 shows the predicted bread weight loss during the baking process at different oven temperatures (180, 190, 200, 210 and 220°C). The results indicate that the weight loss increases with increasing oven temperature. It indicates that when the oven temperature increased from 180 to 220°C, the bread weight loss increased from 22.40 to 52.46%. The results show that the weight loss increases with increasing time; this increment starts to decline after 18-20 min, until it reached equilibrium after 30 min. It increased from 0 to 22.40% when the time increased from 0 to 30 min at oven temperature 180°C. On the other hand, the weight loss increased from 0 to 52.46% at oven temperature 220°C. These results were in agreement with those obtained by [16,25]. Weight loss of bread decrease by 22.40, 33.79, 40.49, 47.46 and 52.46% for the oven drying temperatures of 180, 190, 200, 210 and 220°C, respectively.

At the lower oven temperature (180°C), the weight loss reached 22.40% of the total weight which is lower by about 17.60% that should be lost to reach the recommended weight loss of the bread (40.00%, [26]). At the higher oven temperature (220°C), the weight loss reached 52.46% of the total weight which is higher by about 12.46% that should be lost to reach the recommended weight loss of the bread (40%). Multiple regression analysis was carried out to find a relationship between baking time, oven temperature and the bread weight loss. The best form obtained was as follows:

$$WL_P = -103.106 + 1.213t + 0.555T_o \qquad R^2 = 0.90 \ (20)$$

Relationship between the average bread temperature and average weight loss of bread

Figure 6 shows the relationship between the predicted average
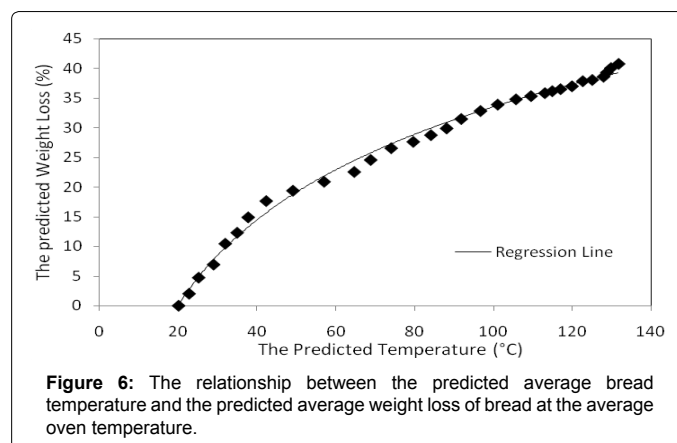

Figure 6: The relationship between the predicted average bread temperature and the predicted average weight loss of bread at the average oven temperature.
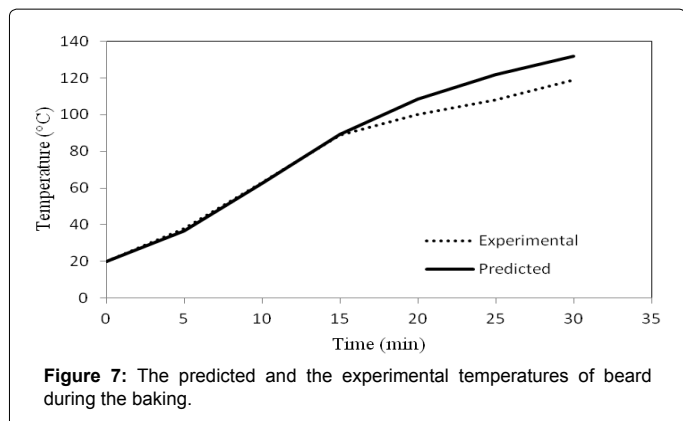
Figure 7: The predicted and the experimental temperatures of beard during the baking.
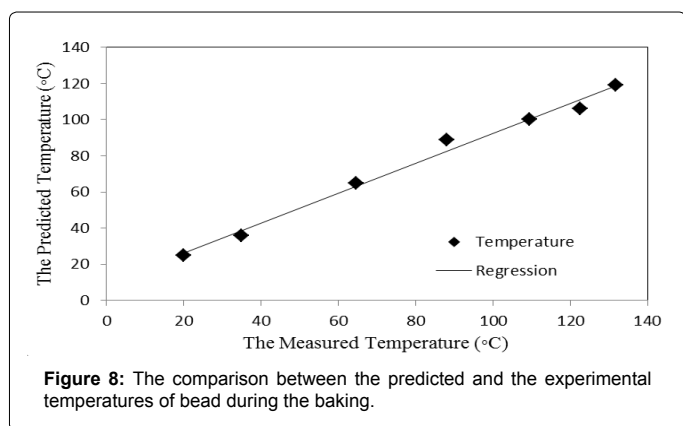

Figure 8: The comparison between the predicted and the experimental temperatures of bead during the baking.
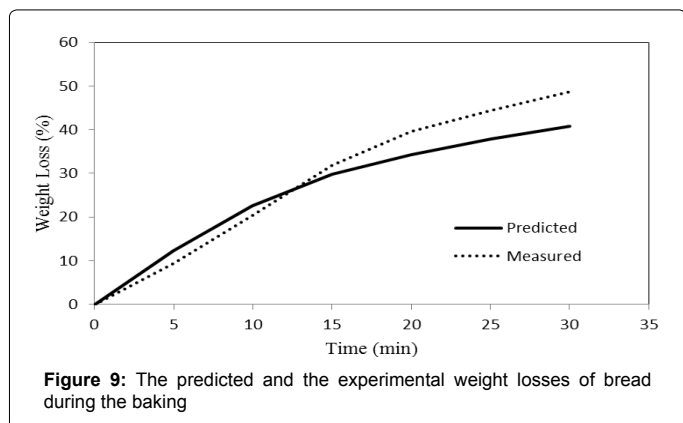

Figure 9: The predicted and the experimental weight losses of bread during the baking

bread temperature and the predicted average weight loss of bread at the average oven temperature. The average weight loss of bread increases with increasing average bread temperature, the average weight loss increased linearly during the temperature range of 20-100°C, then increment. It indicates that when the average bread temperature increased from 20.00 to 137.03°C, the average weight loss of bread increased from 0.00 to 39.38% at the average oven temperature.

The relationship between the predicted average bread temperature and the predicted average weight loss of bread at average oven temperature is shown in figure 6. This relationship is expressed by the following equation:

$$WL_P = 20.47 \ln T_P - 61.3226 \qquad\qquad R^2 = 0.99 \qquad (21)$$

Model validation

The model was validated using the data published by Purlis and Salvadori [7].

Temperature

Figure 7 shows the predicted and the experimental temperatures of bread during the baking. It could be seen that, the temperature by the model was in a reasonable agreement with those measured, where; it ranged 20.00 to 131.69°C theoretically while it was from 20.00 to 119.00°C experimentally during the baking at 200°C oven temperature.

Regression analysis was carried out to find a relationship between the predicted and experimental bread temperatures and the most suitable form is shown as follows (Figure 8):

$$T_P = 1.1456T_M - 9.4539 \qquad\qquad R^2 = 0.99 \qquad (22)$$

Weight loss

Figure 9 shows the predicted and the experimental weight losses of bread during the baking. It could be seen that, the weight loss by the model was in a reasonable agreement with those measured during the period between 0-15 min, where it ranged from 0.00 to 33.16% then there were variations between the predicted and experimental data, where the model results were lower than those of the experimental, where it ranged from 33.16 to 49.79% theoretically while it was from 0.00 to 48.69% experimentally during the baking at 200°C oven temperature in the period between 15-30 min, this underestimation of bread weight loss is probably due to not considering the evaporation-condensation contribution in this process.

The variations between the predicted and the experimental are shown in figure 10. The relationship between the predicted and measured temperatures is expressed by the following equation:

$$WL_P = 1.2308WL_M - 3.5117 \qquad\qquad R^2 = 0.97 \quad (23)$$

Conclusions

A mathematical model for the baking process was developed successively according to heat balance and mass balance to study the bread temperature and water content at different oven temperatures. Also the model was validated using an experimental data from [7]. The most important results obtained can be summarized as follows:

- The model showed that the predicted bread temperature increases with increasing oven temperature, where, the oven temperature increased from 180 to 220°C, the temperature of the crust of bread increased from 112.73 to 168.49°C.
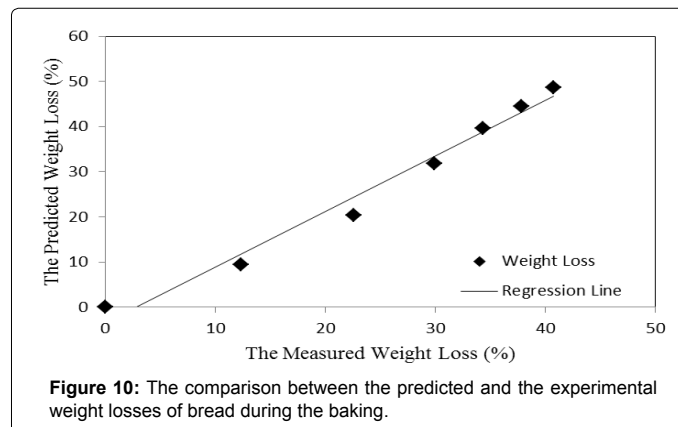

Figure 10: The comparison between the predicted and the experimental weight losses of bread during the baking.

- The model showed that the temperature increases with increasing time, where, it increased from 20 to 112.73°C when the time increased from 0 to 30 min at oven temperature 180°C. On the other hand, the temperature increased from 20 to 168.49°C at oven temperature 220°C.

- The model showed that the weight loss increases with increasing oven temperature, where, the oven temperature increased from 180 to 220°C, the weight loss of bread increased from 22.40 to 52.46%. The model showed that the weight loss increases with increasing time, this increment starts to decline after 18-20 min, until it reach equilibrium after 30 min. It increased from 0 to 22.40% when the time increased from 0 to 30 min at oven temperature 180°C. On the other hand, the weight loss increased from 0 to 52.46% at oven temperature 220°C.

- The predicted bread temperature was in a reasonable agreement with the experimental temperature with a coefficient of determination of 0.98. The bread temperature ranged from 20.00 to 131.69°C theoretically while it was from 25.00 to 119.00°C experimentally during the baking at 200°C oven temperature.

- The predicted weight loss of bread was in a reasonable agreement with the experimental weight loss of bread with a coefficient of determination of 0.97. The weight loss of bread ranged from 0.00 to 40.79% theoretically while it was from 0.00 to 48.69% experimentally during the baking at 200°C oven temperature.

Nomenclature

ρ bread density, kg m^{-3} C_p specific heat of bread, J kg^{-1} K^{-1} T temperature of bread at as a function of time, K t time, s r bread radius, m k bread thermal conductivity, W m^{-1} K^{-1} h heat transfer coefficient, W m^{-2} K^{-1} T_s surface temperature, K T_∞ ambient temperature, K ε emissivity, dimensionless σ Stefan-Boltzmann constant, dimensionless W water content of bread, kg kg^{-1} D water diffusion coefficient, m^2 s^{-1} k_g corrected mass transfer coefficient, kg Pa^{-1} m^2 s^{-1} p_s water vapour pressure of the air at the bread surface, Pa P_∞ water vapour pressure of the air at the oven ambient, Pa	ρ_s density of solid (dough/crumb and crust), kg m^{-3} p_{sat} water vapour pressure at saturation, Pa a_w water activity, dimensionless RH relative humidity, % λ_v latent heat of evaporation, J kg^{-1} δ delta-type function, dimensionless k_g^* mass transfer coefficient, kg pa^{-1} m^{-2} s^{-1} M_{air} molecular mass of air, g mol^{-1} M_w molecular mass of water, g mol^{-1} Sc Schmidt number, dimensionless Pr Prandlt number, dimensionless v kinematic viscosity, m^2 s^{-1} α thermal diffusivity, m^2 s^{-1} T_P predicted bread temperature, °C T_M measured bread temperature, °C T_o oven temperature, °C WL_P predicted weight loss, % WL_M measured weight loss, %

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