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Global Classical Solutions to the Mixed Initial-boundary Value Problem for a Class of Quasilinear Hyperbolic Systems of Balance Laws

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Abstract

It is proven that the mixed initial-boundary value problem for a class of quasilinear hyperbolic systems of balance laws with general nonlinear boundary conditions in the half space $|\{(t,x)\,|\,t\geq 0, x\geq 0\}$ admits a unique global C1 solution $u = u(t,x)$ with small C1 norm, provided that each characteristic with positive velocity is weakly linearly degenerate. This result is also applied to the flow equations of a model class of fluids with viscosity induced by fading memory.

MSC: 35L45; 35L50; 35Q72.

Keywords: Mixed initial-boundary value problem; Global classical solution; Quasilinear hyperbolic; systems of balance laws; Weakly linearly degenerate characteristics

Introduction and Main Result

Consider the following quasilinear hyperbolic system of balance laws in one space dimension:

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}+Lu=0 \tag{1.1}$$

where L>0 is a constant; $u=(u^1,\ldots,u^n)^T$ is the unknown vector function of (t, x), f(u) is a given C^3 vector function of u.

It is assumed that system (1.1) is strictly hyperbolic, i.e., for any given u on the domain under consideration, the Jacobian $A(u)=\nabla f(u)$ has n real distinct eigenvalues.

$$\lambda_1(u)<\lambda_2(u)<\ldots<\lambda_n(u) \tag{1.2}$$

Let $l_i(u)=(l_{i1}(u),\ldots l_{in}(u))$ (resp. $r_i(u)=(r_i(u),\ldots,r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda i(u)(i=1,\ldots,n)$

$$l_i(u)A(u)=\lambda_i(u)l_i(u)\ \ (\text{resp.}\ A(u)r_i(u)=\lambda_i(u)r_i(u)) \tag{1.3}$$

then we have

$$\det|l_{ij}(u)|\neq 0\ (\text{Equivalently, }\det|l_{ij}(u)|\neq 0). \tag{1.4}$$

Without loss of generality, we may assume that on the domain under consideration

$$l_i(u)r_j(u)\equiv\delta_{ij}(i,j=1,\ldots,n) \tag{1.5}$$

And

$$r_i^T(u)r_i(u)\equiv 1\quad (i=1,\ldots,n) \tag{1.6}$$

Where δ_{ij} stands for the Kronecker's symbol.

Clearly, all $\lambda_i(u), l_{ij}(u)$ and $r_{ij}(u)(i,j=1,\ldots,n)$ (1.7)

have the same regularity as A(u), i.e., C^2 regularity.

We assume that on the domain under consideration, each characteristic with positive velocity is weakly linearly degenerate and the eigenvalues of $A(u)=\nabla f(u)$ (1.8)

satisfy the non-characteristic condition.

$$\lambda r(u)<0<\lambda s(u) \tag{1.9}$$

$$(r=1,\ldots,m;\ s=m+1,\ldots,n) \tag{1.10}$$

We are concerned with the existence and uniqueness of global C^1 solutions to the mixed initial-boundary value problem for system (1.1) in the half space

$$D=\{(t,x)\,|\,t\geq 0, x\geq 0\} \tag{1.11}$$

with the initial condition:

$$t=0: u=\varphi(x)(x\geq 0) \tag{1.12}$$

and the nonlinear boundary condition:

$$x=0: v_s=G_s(\alpha(t), v1\ldots vm)+h_s(t), s=m+1,\ldots,n(t\geq 0) \tag{1.13}$$

Where

$$v_i=l_i(u)u(i=1,\ldots,n) \tag{1.14}$$

And

$$\alpha(t)=(\alpha_1(t),\ldots,\alpha_k(t))$$

Here, $\varphi=(\varphi_1,\ldots,\varphi_n)^T,\alpha,G_s$ and $h_s(s=m+1,\ldots,n)$ are all C^1 functions with respect to their arguments, which satisfy the conditions of C^1 compatibility at the point (0; 0). Also, we assume that there exists a constant μ >0 such that

$$\theta\triangleq\max\{\sup_{x\geq 0}(1+x)l+\mu(|\varphi(x)|+|\varphi'(x)|\sup_{t\geq 0}(1+t)l+\mu(|\varphi(t)|+|h(t)|+|\alpha(t)|+|\alpha'(t)|+|h'(t)|)\}<+\infty \tag{1.15}$$

in which

$$h(t)=(h_{m+1}(t),\ldots,h_n(t))$$

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Without loss of generality, we assume that

$$G_s(\alpha(t),...,0) \equiv 0 (s = m+1,...,n) \tag{1.16}$$

For the special case where (1.1) is a quasilinear hyperbolic system of conservation laws, i.e., L=0, such kinds of problems have been extensively studied (for instance, [1-8] and the references therein). In particular, Li and Wang proved the existence and uniqueness of global C^1 solutions to the mixed initial boundary value problem for first order quasilinear hyperbolic systems with general nonlinear boundary conditions in the half space $\{(t,x \mid t \geq 0, x \geq 0|)\}$. On the other hand, for quasilinear hyperbolic systems of balance laws, many results on the existence of global solutions have also been obtained by Liu, et al., (for instance, see [8-14] and the references therein), and some methods have been established. So the following question arises naturally: when can we obtain the existence and uniqueness of semi-global C1 solutions for quasilinear hyperbolic systems of balance laws? It is well known that for first-order quasilinear hyperbolic systems of balance laws, generically speaking, the classical solution exists only locally in time and the singularity will appear in a finite time even if the data are sufficiently smooth and small [15-20]. However, in some cases global existence in time of classical solutions can be obtained. In this paper, we will generalize the results in [21] to a nonhomogeneous quasilinear hyperbolic system, the analysis relies on a careful study of the interaction of the nonhomogeneous term. Our main results can be stated as follows:

Theorem 1.1. Suppose that the non-characteristic condition (1.10) holds and system (1.1) is strictly hyperbolic. Suppose furthermore that for j = m + 1,…., n; each j-characteristic field with positive velocity is weakly linearly degenerate. Suppose finally that $\varphi, \alpha, G_s, h_s (s = m+1,...,n)$ are all C^1 functions with respect to their arguments, satisfying (1.15)-(1.16) and the conditions of C^1 compatibility at the point (0; 0). Then there exists a sufficiently small $\theta_0 > 0$ such that for any given $\theta \in [0, \theta_0]$ the mixed initial-boundary value problem (1.1) and (1.12)-(1.13) admits a unique global C1 solution u=u(t; x) in the half space $\{(t,x) \mid t \geq 0, x \geq 0\}$.

The rest of this paper is organized as follows. In Section 2, we give the main tools of the proof that is several formulas on the decomposition of waves for system (1.1). Then, the main result will be proved in Section 3. Finally, an application is given in Section 4 [22].

Decomposition of Waves

Suppose that on the domain under consideration, system (1.1) is strictly hyperbolic and (1.2)-(1.6) hold.

Suppose that $A(u) \in C^k$ where k is an integer ¸ 1. By Lemma 2.5 in [23], there exists an invertible C^{k+1} transformation $u = u(\tilde{u})(u(0) = 0)$ such that in $\tilde{u}$ -space for each i = 1,…, n, the ith characteristic trajectory passing through $\tilde{u} = 0$ coincides with the $\tilde{u}_i$ -axis at least for $|\tilde{u}_i|$ small, namely,

$$\tilde{r}_i(\tilde{u}_i e_i) \equiv ei, \forall |\tilde{u}i| \text{ small } (i = 1,…., n); \tag{2.1}$$

Where

$$e_i = (0,...,0,\overset{(i)}{1},0,...,0)^T$$

This transformation is called the normalized transformation, and the corresponding unknown variables $\tilde{u} = (\tilde{u}1,...,\tilde{u}n)^T$ are called the normalized variables or normalized coordinates [24-28].

Let

$$wi = li(u)ux \ (i = 1,…, n);$$

where

$$li(u) = (li1(u); : : : ; lin(u))$$

denotes the ith left eigenvector.

By (1.5), it follows from (1.14) and (2.2) that

$$u = \sum_{k=1}^{n} v_k r_k(u) \tag{2.3}$$

And

$$u_x = \sum_{k=1}^{n} w_k r_k(u) \tag{2.4}$$

Let

$$\frac{d}{d_i t} = \frac{\partial}{\partial t} + \lambda_i(u)\frac{\partial}{\partial x} \tag{2.5}$$

be the directional derivative along the ith characteristic. Our aim in this section is to prove several formulas on the decomposition of waves for system (1.1), which will play an important role in our discussion.

Lemma 2.1.

$$\frac{d(e^{Lt}w_i)}{d_i t} = \sum_{j,k=1}^{n} e^{Lt}\gamma_{ijk}(u) w_j w_k + \sum_{j,k=1}^{n} e^{Lt}\tilde{\gamma}_{ijk}(u) v_j w_k (i = 1,...,n) \tag{2.6}$$

Where

$$\gamma_{ijk}(u) = (\lambda_k(u) - \lambda_j(u) r_j^T(u) \nabla l_i(u) r_k(u) - \nabla\lambda_i(u) r_j(u)\delta_{ik}) \tag{2.7}$$

$$\tilde{\gamma}_{ijk}(u) = -L r_j^T(u) \nabla l_i(u) r_k(u) \tag{2.8}$$

Hence, we have

$$\tilde{\gamma}_{ijk}(u) \equiv 0, \forall j \neq i (i, j = 1,…., n) \tag{2.9}$$

$$\gamma_{iii}(u) = -\nabla\lambda_i(u) r_i(u) (i = 1,…., n) \tag{2.10}$$

Moreover, in the normalized coordinates,

$$\tilde{\gamma}_{ijj}(u_j e_j) \equiv 0, \quad \forall |u_j| \text{ small}, \forall i, j; \tag{2.11}$$

while, when the ith characteristic $\lambda_i(u)$ is weakly linearly degenerate, in the normalized coordinates,

$$\gamma_{iii}(u_i e_i) \equiv 0, \forall |u_i| \text{ small}, \forall i. \tag{2.12}$$

Lemma 2.2.

$$\frac{d(e^{Lt}v_i)}{d_i t} = \sum_{j,k=1}^{n} e^{Lt}\beta_{ijk}(u) v_j w_k + \sum_{j,k=1}^{n} e^{Lt}\tilde{\beta}_{ijk}(u) v_j v_k (i = 1,…., n) \tag{2.13}$$

Where

$$\beta_{ijk}(u) = (\lambda_i(u)) r_j^T(u) \nabla li(u) r_k(u) \tag{2.14}$$

$$\tilde{\beta}_{ijk}(u) = -L r_j^T(u) \nabla li(u) r_k(u) \tag{2.15}$$

Thus, we have

$$\tilde{\beta}_{ijk}(u) \equiv 0, \qquad \forall i, j (i, j = 1,...,n) \tag{2.16}$$

Moreover, by (2.1), in the normalized coordinates we have

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$$\beta_{ijk}(u_j e_j) \equiv 0, \qquad \forall |u_j| \text{ small}, \forall i, j \tag{2.17}$$

And

$$\tilde{\beta}_{ijk}(u_j e_j) \equiv 0, \qquad \forall |u_j| \text{ small}, \forall i, j \tag{2.18}$$

The proofs of Lemmas 2.1-2.2 can be found in [29].

Similarly, we have

Lemma 2.3. In the normalized coordinates, it follows that

$$\frac{d(e^{Lt}u_i)}{d_i t} = \sum_{j,k=1}^{n} e^{Lt} p_{ijk}(u) u_j w_k (i = 1, ..., n) \tag{2.19}$$

Where

$$p_{ijj}(u) = 0, \ \forall i, j \tag{2.20}$$

And

$$p_{ijj}(u) = (\lambda_k(u)) \int_0^1 \frac{\partial r_{ki}}{\partial u_j}(Tu1, ..., Tu_{k-1,} u_{k+1}, ..., Tu_n) dT \tag{2.21}$$

Hence,

$$p_{ijj}(u) \equiv 0, \quad \forall i, j \tag{2.22}$$

Proof. By (1.1), (2.4) and (2.5), it is easy to see that

$$\frac{du}{d_i t} = \sum_{k=1}^{n} (\lambda_i(u) - \lambda_k(u)) w_k r_k(u) - Lu \tag{2.23}$$

On the other hand, we have

$$u_i(t, x) = u^T(t, x) e_i \tag{2.24}$$

Thus, noting (2.1), in the normalized coordinates, it follows from (2.23)-(2.24) that

$$\frac{du_i}{d_i t} = \sum_{k=1}^{n} (\lambda_i(u) - \lambda_k(u)) w_k r_k^T(u) e_i - Lu_i = \sum_{k=1}^{n} (\lambda_i(u) - \lambda_k(u)) w_k [r_k^T(u) - r_k^T(u_k e_k)] e_i - Lu_i \tag{2.25}$$

By Hadamard's formula, we have

$$r_k^T(u) - r_k^T(u_k e_k) = \int_0^1 \sum_{j \neq k} \frac{\partial r_k^T}{\partial u_j}(Tu_1, ... Tu_{k-1}, u_k, Tu_{k+1}, ..., Tu_n) u_j ds \tag{2.26}$$

Therefore, from (2.25)-(2.26) we immediately get (2.19)-(2.22). The proof of Lemma 2.3 is finished.

For any given $y \geq 0$ on the existence domain of C^1 solution, let $x = \tilde{x}_i(t, y)$ be the ith characteristic passing through point $(\frac{y}{a}; y)$(a > 0, constant):

$$\begin{cases} \dfrac{d\tilde{x}_i(t,y)}{dt} = \lambda_i(u(t, \tilde{x}_i(t,y))) \\ \tilde{x}i(\frac{y}{a}, y) = y \end{cases} \tag{2.27}$$

Lemma 2.4. Let $qi(t; x)$ be defined by $q_i(t, \tilde{x}_i(t, y)) = w_i(t, \tilde{x}_i(t, y)) \frac{\partial \tilde{x}_i(t, y)}{\partial y}$

Then along the ith characteristic

$x = \tilde{x}_i(t, y)$ we have

$$\frac{d(e^{Lt} q_i)}{d_i t} = \sum_{j,k=1}^{n} e^{Lt} \Gamma_{ijk}(u) \frac{\partial x_i(t, y)}{\partial y} w_j w_k + \sum_{j,k=1}^{n} e^{Lt} \tilde{\gamma} ijk(u) \frac{\partial \tilde{x}_i(t, y)}{\partial y} v_j w_k \tag{2.28}$$

Where $\tilde{\gamma} ijk(u)$ is given by (2.8) and

$$\Gamma_{ijk}(u) = (\lambda_k(u) - \lambda_j(u)) r_j^T(u) \nabla l_i(u) r_k(u) \tag{2.29}$$

Hence,

$$\Gamma_{ijk}(u) = 0 \qquad \forall i, j \tag{2.30}$$

Proof. Differentiating the first equation of (2.27) with respect to *y* gives

$$\frac{d}{dt}\left(\frac{\partial \tilde{x}_i(t, y)}{\partial y}\right) = \nabla \lambda_i(u(t, \tilde{x}_i(t, y))) \frac{\partial u}{\partial x}(t, \tilde{x}_i(t, y)) \frac{\partial \tilde{x}_i(t, y)}{\partial y} \tag{2.31}$$

Then, noting (2.6), it follows from (2.31) that

$$\begin{gathered} \frac{d(e^{Lt} q_i)}{d_i t} = \frac{d(e^{Lt} w_i)}{d_i t} \frac{\partial \tilde{x}_i(t, y)}{\partial y} + e^{Lt} w_i \frac{d}{d_i t}\left(\frac{\partial \tilde{x}_i(t, y)}{\partial y}\right) = \\ \left(\sum_{j,k=1}^{n} e^{Lt} [\gamma_{ijk}(u) w_j w_k + \tilde{\gamma}_{ijk}(u) w_j w_k] + e^{Lt} w_i \nabla \lambda_i(u) u_x\right) \frac{\partial \tilde{x}_i(t, y)}{\partial y} \end{gathered} \tag{2.32}$$

Thus, from (2.4), (2.7) and (2.32), we immediately get (2.28)-(2.30). The proof of Lemma 2.4 is finished.

Similarly, noting (2.4), by (2.13) and (2.31), we have

Lemma 2.5. Let $pi(t; x)$ be defined by $p_i(t, \tilde{x}_i(t, y)) = v_i(t, \tilde{x}_i(t, y)) \frac{\partial \tilde{x}_i(t, y)}{\partial y}$ Then along the ith

Characteristic $x = \tilde{x}_i(t, y)$ we have

$$\frac{d(e^{Lt} p_i)}{d_i t} = \sum_{j,k=1}^{n} e^{Lt} B_{ijk}(u) \frac{\partial \tilde{x}_i(t, y)}{\partial y} v_j w_k + \sum_{j,k=1}^{n} e^{Lt} \tilde{\beta}_{ijk}(u) \frac{\partial \tilde{x}_i(t, y)}{\partial y} v_j v_k \tag{2.33}$$

Where $\tilde{\beta}_{ijk}(u)$ is given by (2.15) and

$$\tilde{\beta}_{ijk}(u) = \tilde{\beta}_{ijk}(u) + \nabla \lambda_i(u) r_k(u) \delta_{ij} \tag{2.34}$$

By (2.16), it is easy to see that

$$B_{iji}(u) \equiv 0 \qquad \forall i \neq j (i, j = 1, ..., n) \tag{2.35}$$

$$B_{iji}(u) \equiv \nabla \lambda_i(u) r_i(u) \qquad \forall i (i = 1, ..., n) \tag{2.36}$$

Moreover, by (2.17), in the normalized coordinates we have

$$B_{iji}(u_j e_j) \equiv 0 \qquad \forall |uj| \text{ small}, \ \forall j \neq i \tag{2.37}$$

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while, when the ith characteristic $\lambda_i(u)$ is weakly linearly degenerate, in the normalized coordinates,

$$B_{iii}(u_i e_i) \equiv 0 \quad \forall |u_i| \text{ small } \forall i \tag{2.38}$$

Lemma 2.6. Let $zi(t; x)$ be defined by $Z_i(t, \tilde{x}_i(t, y)) = u_i(t, \tilde{x}_i(t, y)) \frac{\partial \tilde{x}_i(t, y)}{\partial y}$

Then along the ith characteristic

$x = \tilde{x}_i(t, y)$ we have

$$\frac{d(e^{Lt} Z_i)}{dit} = \sum_{j,k=1}^{n} e^{Lt} F_{ijk}(u) \frac{\partial \tilde{x}_i(t, y)}{\partial y} u_j w_k \tag{2.39}$$

Where

$$F_{ijk}(u) = p_{ijk}(u) + \nabla \lambda_i(u) r_k(u) \delta_{ij} \tag{2.40}$$

By (2.20) and (2.22), it is easy to see that

$$F_{ijj}(u) \equiv 0, \qquad \forall i \neq j (i, j = 1, ..., n) \tag{2.41}$$

$$F_{iji}(u) \equiv 0, \qquad \forall i \neq j (i, j = 1, ..., n) \tag{2.42}$$

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And

$$F_{iji}(u) = \nabla\lambda_i(u) r_i(u) \qquad \forall i(i=1,\dots,n) \tag{2.43}$$

Proof. Differentiating the first equation of (2.27) with respect to *y* gives

$$\frac{d}{dt}\left(\frac{\partial \tilde{x}_i(t,y)}{\partial y}\right) = \nabla\lambda_i(u(t,\tilde{x}_i(t,y)))\frac{\partial u}{\partial x}(t,\tilde{x}_i,(t,y))\frac{\partial \tilde{x}_i(t,y)}{\partial y} \tag{2.44}$$

Then, noting (2.19), it follows from (2.44) that

$$\begin{aligned}&\frac{d(e^{Lt}Z_i)}{d_i t} = \frac{d(e^{Lt}Z_i)}{d_i t}\frac{\partial \tilde{x}_i(t,y)}{\partial y} + e^{Lt}Z_i\frac{d}{d_i t}\left(\frac{\partial \tilde{x}_i(t,y)}{\partial y}\right) = \\ &\left(\sum_{j,k=1}^{n} e^{Lt}p_{ijk}(u)u_j w_k + e^{Lt}Z_i\nabla\lambda_i(u)u_x\right)\frac{\partial \tilde{x}_i(t,y)}{\partial y}\end{aligned} \tag{2.45}$$

Thus, from (2.4), (2.20)-(2.22) and (2.45), we immediately get (2.39)-(2.43). The proof of Lemma 2.6 is finished.

Proof of Theorem 1.1

By the existence and uniqueness of a local *C*1 solution for quasilinear hyperbolic systems [22], there exists *T*0>0 such that the mixed initial-boundary value problem (1.1) and (1.12)-(1.13) admits a unique C^1 solution $u=u(t, x)$ on the domain

$$D(T_0) \overset{\text{def}}{=} \{(t,x) \mid 0 \le t \le T_o, x \ge 0\} \tag{3.1}$$

Thus, in order to prove Theorem 1.1 it suffices to establish a uniform a priori estimate for the C^0 norm of u and u_x on any given domain of existence of the C^1 solution u = u(t; x).

Noting (1.2) and (1.10), we have

$$\lambda_1(0) < \dots < \lambda_m(0) < \lambda_{m+1}(0) < \dots < \lambda_n(0) \tag{3.2}$$

Thus, there exist sufficiently small positive constants δ and δ_0 such that

$$\lambda_{i+1}(u) - \lambda_i(v) \ge 4\delta_0 \qquad \forall |u|,|v| \le \delta(i-1,\dots,n-1) \tag{3.3}$$

$$\lambda_i(u) - \lambda_i(v) \le \frac{\delta_0}{2} \qquad \forall |u|,|v| \le \delta(i-1,\dots,n) \tag{3.4}$$

And

$$|\lambda_i(0)| \ge \delta_0 \quad (i=1,\dots,n) \tag{3.5}$$

For the time being it is supposed that on the domain of existence of the C^1 solution $u = u(t; x)$ to the mixed initial-boundary value problem (1.1) and (1.12)-(1.13), we have

$$|u(t,x)| \le \delta \tag{3.6}$$

At the end of the proof of Lemma 3.3, we will explain that this hypothesis is reasonable. Thus, in order to prove Theorem 1.1, we only need to establish a uniform a priori estimate for the piecewise C^0 norm of *v* and *w* defined by (1.14) and (2.1) on the domain of existence of the C^1 solution $u=u(t; x)$.

For any fixed $T > 0$, let

$$D_+^T = \{(t,x) \mid 0 \le t \le T, x \ge (\lambda_n(0)+\delta_0)t\} \tag{3.7}$$

$$D_-^T = \{(t,x) \mid 0 \le t \le T, 0 \le x \le (\lambda_{m+1}(0)+\delta_0)t\} \tag{3.8}$$

$$D^T = \{(t,x) \mid 0 \le t \le T, (\lambda_{m+1}(0)-\delta_0)t \le x \le (\lambda_n(0)+\delta_0)t\} \tag{3.9}$$

and for $i = m + 1; : : : ; n;$ let

$$\begin{aligned}D^T = \{(t,x) \mid 0 \le t \le T, -[\delta_0 + n(\lambda_i(0)-\lambda_{m+1}(0))]t \\ \le x - \lambda_i(0)t \le [\delta_0 + n(\lambda_n(0))]t\}\end{aligned} \tag{3.10}$$

where *n*>0 is suitably small (Figure 1).

Noting that *n* > 0 is small, by (3.3), it is easy to see that

$$D_i^T \cap D_j^T = \varnothing, \quad \forall_i \ne j \tag{3.11}$$

And

$$\bigcup_{i=m+1}^{n} D_i^T \subset D^T \tag{3.12}$$

By the definitions of D_i^T and D^T , it is easy to get the following lemma. Lemma 3.1. For each *i* = *m* + 1,…, *n*, on the domain D^T / D_i^T we have

$$ct \le |x - \lambda_i(0)t| \le Ct, \; cx \le |x - \lambda_i(0)t| \le Cx \tag{3.13}$$

where *c* and *C* are positive constants independent of *T*.

Let

$$V(D_+^T) = \max_{i=1,\dots,n} \|(1+x)^{1+\mu}{}_{vi}(t,x)\|_{L\infty(D_+^T)} \tag{3.14}$$

$$W(D_+^T) = \max_{i=1,\dots,n} \|(1+t)^{1+\mu}{}_{wi}(t,x)\|_{L\infty(D_+^T)} \tag{3.15}$$

$$V(D_-^T) = \max_{i=1,\dots,n} \|(1+t)^{1+\mu}{}_{wi}(t,x)\|_{L\infty(D_-^T)} \tag{3.16}$$

$$W(D_-^T) = \max_{i=1,\dots,n} \|(1+t)^{1+\mu}{}_{wi}(t,x)\|_{L\infty(D_-^T)} \tag{3.17}$$

$$\begin{aligned}V_\infty^c(T) = \max\{&\max_{r=1,\dots,m}\sup_{(t,x)\in D^T}(1+T)^{1+\mu}|V_r(t,x)| \\ &\max_{s=m+1,\dots,n}\sup_{(t,x)\in D^T\setminus D_S^T}(1+|x-\lambda_s(0)t|)^{1+\mu}|v_s(t,x)|\}\end{aligned} \tag{3.18}$$

$$\begin{aligned}W_\infty^c(T) = \max\{&\max_{r=1,\dots,m}\sup_{(t,x)\in D^T}(1+T)^{1+\mu}|W_r(t,x)| \\ &\max_{s=m+1,\dots,n}\sup_{(t,x)\in D^T\setminus D_S^T}(1+|x-\lambda_s(0)t|)^{1+\mu}|W_s(t,x)|\}\end{aligned} \tag{3.19}$$

$$\begin{aligned}U_\infty^c(T) = \max\{&\max_{r=1,\dots,m}\sup_{(t,x)\in D^T}(1+t)^{1+\mu}|u_r(t,x)| \\ &\max_{s=m+1,\dots,n}\sup_{(t,x)\in D^T\setminus D_S^T}(1+|x-\lambda_s(0)t|)^{1+\mu}|W_s(t,x)|\}\end{aligned} \tag{3.20}$$

$$\tilde{V}_1(T) = \max_{i=m+1,\dots,n}\max_{j\ne 1}\sup_{\tilde{c}_j}\int_{\tilde{C}_j}|v_i(t,x)|\,dt \tag{3.21}$$

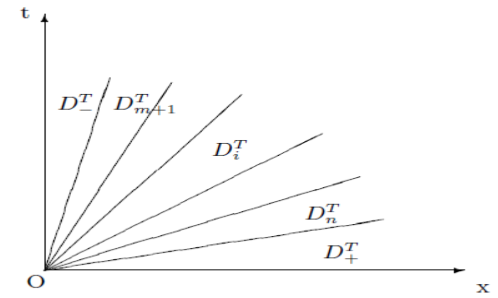

Figure 1: Where *n*>0 is suitably small.

$$\tilde{W}_1(T) = \max_{i=m+1,\ldots,n} \max_{j\neq 1} \sup_{\tilde{c}_j} \int_{\tilde{C}_j} |w_i(t,x)|\,dt \tag{3.22}$$

$$\tilde{U}_1(T) = \max_{i=m+1,\ldots,n} \max_{j\neq 1} \sup_{\tilde{c}_j} \int_{\tilde{C}_j} |U_i(t,x)|\,dt \tag{3.23}$$

Where $\tilde{C}_j$ denotes any given jth characteristic in

$D_i^T (j \neq i, i = m+1,..,n)$

$$V1(T) = \max_{i=m+,\ldots,n} \sup_{0\le t\le T} \int_{D_i^T(t)} |v_i(t,x)|\,dx \tag{3.24}$$

$$W_1(T) = \max_{i=m+,\ldots,n} \sup_{0\le t\le T} \int_{D_i^T(t)} |w_i(t,x)|\,dx \tag{3.25}$$

$$U_1(T) = \max_{i=m+,\ldots,n} \sup_{0\le t\le T} \int_{D_i^T(t)} |u_i(t,x)|\,dx \tag{3.26}$$

Where $D_i^T(t)(t \ge 0)$ denotes the t-section of D_i^T

$$D_i^T(t) = \{(t,x) \mid T = t, (t,x) \in D_i^T\} \tag{3.27}$$

$$V\infty(T) = \max_{i=1,\ldots,n} \sup_{0\le t\le T\le x\ge 0} |v_i(t,x)| \tag{3.28}$$

And

$$V_\infty(T) = \max_{i=1,\ldots,n} \sup_{0\le t\le T\le x\ge 0} |w_i(t,x)| \tag{3.29}$$

Clearly, $V_\infty(T)$ is equivalent to

$$U_\infty(T) = \max_{i=1,\ldots,n} \sup_{0\le t\le T\le x\ge 0} |U_i(t,x)| \tag{3.30}$$

In the present situation, similar to the corresponding result in [24,30-33], we have

Lemma 3.2. Suppose that in a neighborhood of *u*=0; $A(u) \in C^2$ system (1.1) is strictly hyperbolic and (1.10) holds. Suppose furthermore that $\varphi(x)$ satisfies (1.15). Then there exists a sufficiently small $\theta_0 > 0$ such that for any fixed $\theta_0 \in [0, \theta_0]$ on any given existence domain $\{(t,x) \mid 0 \le t \le T, x \ge 0\}$ of the C^1 solution $u = u(t; x)$ to the mixed initial-boundary value problem (1.1) and (1.12)-(1.13), we have the following uniform a priori estimates:

$$V(D_+^T), W(D_+^T) \le k_1 \theta \tag{3.31}$$

where here and henceforth, *ki*(*i*=1; 2,….) are positive constants independent of θ and *T*.

Proof. We first estimate $W(D_+^T)$

(i) For *i* = 1,…,*m*, let $\xi = x_i(s,y)$ be the ith characteristic passing through any fixed point $(t,x) \in D_+^T$ and intersecting the *x*-axis at a point (0; *y*). Noting (3.6), by (3.3)-(3.4), it is easy to see that the whole characteristic

$\xi = x_i(s,y)(0 \le s \le t)$ is included in D_+^T

Noting (3.6), by (3.4) we get

$$y + (\lambda_i(0) - \delta_0/2)s \le x_i(s,y) \le y \quad \forall s \in [o,t] \tag{3.32}$$

By (3.4), it is easy to see that

$$s \le t \le t_0 \tag{3.33}$$

where *t*0 denotes the t-coordinate of the intersection point of the straight line $x = (\lambda_n(0) + \delta_0)t$ with the straight line $x = y + (\lambda_i(0) + \delta_0/2)t$ passing through the point (0; *y*). Clearly,

$$t_0 = \frac{y}{\lambda_n(0) - \lambda_i(0) + \delta_0/2} \tag{3.34}$$

Therefore it follows from (3.32)-(3.34) that

$$\frac{\lambda_n(0)}{\lambda_n(0) - \lambda_i(0) + \delta_0/2} y \le x_i(s,y) \le y, \quad \forall s \in [0,t] \tag{3.35}$$

By integrating (2.6) along this ith characteristic, we have

$$w_i(t,x) = e^{-Lt} w_i(o,y) + \int_0^t e^{-L(t-s)} [\sum_{j,k=1}^n \gamma_{ijk}(u) w_j w_k$$

$$+ \sum_{j,k=1}^n \tilde{\gamma}_{ijk}(u) v_j w_k](s, x_i(s,y))\,ds \tag{3.36}$$

Thus, noting (3.6) and the fact that $L > 0$, using(3.33)-(3.35), it follows from (3.36) that

$$\begin{aligned}(1+x)^{1+\mu} |w_i(t,x)| &\le C_1\{(1+y)^{1+\mu} |w_i(0,y)| + (1+y)^{-(1+\mu)} t[(W(D_+^T)) + W(D_+^T)V(D_+^T)]\} \\ &\le C_2\{\theta + (1+y)^{-\mu} W(D_+^T)[W(D_+^T) + V(D_+^T)]\} \\ &\le C_2\{\theta + W(D_+^T)[W(D_+^T) + V(D_+^T)]\}\end{aligned} \tag{3.37}$$

where here and henceforth, *Ci*(*i* = 1; 2,….) will denote positive constants independent of and *T*.

(ii) For *i* = *m* + 1; : : : ; *n*; let $\xi = x_i(s,y)$ be the ith characteristic passing through any fixed point $(t,x) \in D_+^T$ and intersecting the *x*-axis at a point (0; *y*). Noting (3.6), by (3.3)-(3.4), it is easy to see that the whole characteristic $\xi = x_i(s,y)(0 \le s \le t)$ is included in D_+^T

Noting (3.6), by (3.4) we get

$$y \le x_i(s,y) \le y + \lambda(\lambda_i(0) + \delta_0/2)s \quad \forall s \in [0,t] \tag{3.38}$$

By (3.4), it is easy to see that

$$s \le t \le t_0 \tag{3.39}$$

where *t*0 denotes the t-coordinate of the intersection point of the straight line $x = (\lambda_n(0) + \delta_0)t$ with the straight line $x = y + (\lambda_i(0) + \delta_0/2)t$ passing through the point (0; *y*). Clearly,

$$t_0 = \frac{y}{\lambda_n(0) - \lambda_i(0) + \delta_0/2} \tag{3.40}$$

Therefore it follows from (3.38)-(3.40) that

$$y \le x_i(s,y) = \frac{\lambda_n(0) + \delta_0}{\lambda_n(0) - \lambda_i(0) + \delta_0/2} y, \forall s \in [0,t] \tag{3.41}$$

Then, similar to (3.37), we have

$$(1+x)^{1+\mu} |w_i(t,x)| \le C_3\{\theta + w(D_+^T)[W(D_+^T) + V(D_+^T)]\} \tag{3.42}$$

Combining (3.37) and (3.42), we obtain

$$W(D_+^T) \le C_4\{\theta + W(D_+^T)[W(D_+^T) + V(D_+^T)]\} \tag{3.43}$$

Similarly, we have

$$V(D_+^T) \le C_4\{\theta + V(D_+^T)[W(D_+^T) + V(D_+^T)]\} \tag{3.44}$$

By (3.43) and (3.44), it is easy to prove that for $\mu > 0$ suitably small, there exists a positive constant *k*1 independent of θ and *T*, such that for any fixed $T_0 (0 < T_0 \le T)$ if

$$W(D_+^{T_0}), V(D_+^{T_0}) \le 2k_1\theta \tag{3.45}$$

Then

$$W(D_+^{T_0}), V(D_+^{T_0}) \le k_1\theta \tag{3.46}$$

Hence, noting (1.15), by continuity we immediately get (3.31). The proof of Lemma 3.2 is finished. 2 Lemma 3.3. Under the assumptions of Lemma 3.2, suppose furthermore that system (1.1) is weakly linearly degenerate. Then in the normalized coordinates there exists a sufficiently small $\theta_0 > 0$ such that for any fixed $\theta \in [0, \theta_0]$ on any given existence domain $\{(t,x) \mid 0 \le t \le T, x \ge 0\}$ of the *C*1 solution *u*=*u*(*t; x*) to the mixed initial-boundary value problem (1.1) and (1.12)-(1.13), we have the following uniform a priori estimates:

$$W(D_-^T) \le k_2\,\theta \tag{3.47}$$

$$V(D_-^T) \le k_3\,\theta \tag{3.48}$$

$$U_\infty^c(T) \le k_4\,\theta \tag{3.49}$$

$$W_\infty^c(T), V_\infty^c(T) \le k_5\,\theta \tag{3.50}$$

$$\tilde{W}_1(T), W_1(T), \tilde{V}_1(T), V_1(T), \tilde{U}_1(T), U_1(T) \le k_6\,\theta \tag{3.51}$$

$$U_\infty(T), V_\infty(T) \le k_7\,\theta \tag{3.52}$$

And

$$W\infty(T) \le k_8\,\theta \tag{3.53}$$

Proof. We first estimate $W(D_-^T)$

For *j* = 1*; : : : ;m*, passing through any fixed point $(t,x) \in D_-^T$ we draw the jth characteristic $C_j : \xi = \xi_j(s,t,x)$ which must intersect the boundary $x = (\lambda_n(0)+\delta_0)\,t$ of D^T at a point (*t*0*; y*).

Proposition 3.1. On this jth characteristic $C_j : \xi = \xi_j(s,t,x)$ it follows that

$$t \ge t_0 \ge \frac{-\lambda_j(0) - \frac{\delta_0}{2}}{\lambda_n(0) - \lambda_j(0) + \frac{\delta_0}{2}}\,t \tag{3.54}$$

Proof. Noting (3.4), it is easy to see that

$$x - (\lambda_j(0) + \frac{\delta_0}{2})\,t \le y - (\lambda_j(0) + \frac{\delta_0}{2})\,t_0 \tag{3.55}$$

On the other hand, from (3.8), we have

$$x \ge 0 \tag{3.56}$$

Since

$$y = (\lambda_n(0) + \delta_0)\,t_0 \tag{3.57}$$

we conclude from (3.55)-(3.57) that

$$t_0 \ge \frac{-\lambda_j(0) - \frac{\delta_0}{2}}{\lambda_n(0)\lambda_j - \frac{\delta_0}{2}}\,t \tag{3.58}$$

Noting the fact that $t \ge t_0$ we immediately get (3.54).

By integrating (2.6) along $\xi = \xi_j(s;t,x)$ and noting (2.9) and (2.11), we have

$$\begin{aligned} w_j(t,x) &= e{-}L^{(t-t_0)} w_j(t_0,y) \\ &+ \int_{t_0}^{t} e^{-L(t-s)} \Big(\sum_{i,l=1}^{m} + \sum_{\substack{i\in\{1,\dots,m\}\\ l\notin\{1,\dots,m\}}} + \sum_{\substack{l\in\{1,\dots,m\}\\ i\notin\{1,\dots,m\}}} + \sum_{\substack{i,l=m+1\\ i\ne l}}^{n}\Big)\gamma_{jil}(u)\,w_i w_l(s,\xi_j(s;t,x))\,ds \\ &+ \int_{t_0}^{t} e^{-L(t-s)} \Big(\sum_{i,l=1}^{m} + \sum_{\substack{i\in\{1,\dots,m\}\\ l\notin\{1,\dots,m\}}} + \sum_{\substack{l\in\{1,\dots,m\}\\ i\notin\{1,\dots,m\}}} + \sum_{\substack{i,l=m+1\\ i\ne l}}^{n}\Big)\tilde{\gamma}_{jil}(u)\,w_i w_l(s,\xi_j(s;t,x))\,ds \\ &+ \int_{t_0}^{t} e^{-L(t-s)} \Big[\sum_{l=m+1}^{n} \tilde{\gamma}_{jll}(u) - \tilde{\gamma}_{jll}(u_l e_l))\,v_l w_l\Big](s,\xi_j(s;t,x))\,ds \end{aligned} \tag{3.59}$$

By using Lemma 3.2 and noting (3.54) and (3.57), it is easy to see that

$$|\,w_j(t_0,y)| \le k_1\,\theta(1+y)^{-(1+\mu)} \le C_6\theta(1+t_0)^{-(1+\mu)} \le C_7\theta(1+t)^{-(1+\mu)} \tag{3.60}$$

By Hadamard's formula, we have

$$\tilde{\gamma}_{jll}(u) - \tilde{\gamma}_{jll}(u_l e_l) = \int_0^1 \sum_{\substack{k=1\\ k\ne l}}^{n} \frac{\partial \tilde{\gamma}_{jll}}{\partial u_k}(tu_1,\dots,Tu_{l-1},\dots,Tu_n)\,u_k\,d_T \tag{3.61}$$

Thus, noting the fact that *L*>0, and using (3.13) and (3.54), we obtain from (3.59) that

$$\begin{aligned} &(1+t)^{1+\mu}\,|\,w_j(t,x)| \le C_8\{\theta + (W(D_-^T))^2 + W(D_-^T)\,V(D_-^T) \\ &+ W_\infty^c(T)[\tilde{W}_1(T) + W_\infty^c(T) + \tilde{V}_1(T) + V_\infty^c(T) + \tilde{U}_1(T)\,V_\infty(T) + U_\infty^c(T)\,V_\infty(T)] \\ &+ \tilde{W}_1(T)[V_\infty^c(T) + U_\infty^c(T)\,V_\infty(T)]\} \end{aligned} \tag{3.62}$$

Similar to Lemma 3.2 in [21], differentiating the nonlinear boundary condition (1.13) with respect to *t*, we get

$$\begin{aligned} x = 0 : \frac{\partial v_s}{\partial t} &= \sum_{r=1}^{m} \frac{\partial G_s}{\partial v_r}(\alpha(t), v_1,..,v_m)\frac{\partial v_r}{\partial t} \\ &+ \sum_{i=1}^{k} \frac{\partial G_s}{\partial \alpha_r}(\alpha(t), v_1,\dots,v_m)\alpha_i'(t) + h_s'(t)(s = m+1,\dots,n) \end{aligned} \tag{3.63}$$

By (1.1), (1.3) and (2.4), it is easy to see that

$$\frac{\partial vi}{\partial t} = \frac{\partial}{\partial t}(l_i(u)\,u) = -\lambda_i(u)\,w_i + \sum_{k=1}^{n} a_{ik}(u)\,w_k - Lu^T\nabla l_i(u)\,u\,(i = 1,\dots,n) \tag{3.64}$$

Where

$$a_{ik}(u) = -\lambda_k(u)\,r_k^T(u)\nabla\,l_i(u)\,u \tag{3.65}$$

Therefore it follows from (3.63)-(3.65) that

$$x = 0 : (I_{n-m} - B_1(u))\begin{pmatrix} w_{m+1} \\ \vdots \\ w_n \end{pmatrix} = B_2\begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix} - \vec{B}_3 \tag{3.66}$$

where *B*1(*u*) is a matrix whose elements are all *C*1 functions of *u*, which satisfy

*In-m - B*1(*u*) is invertible; for sufficiently small |u|

*B*2 is an (*n - m*) × *m* matrix independent of *wi* (*i* = 1,…, *n*); (3.67)

$$\vec{B}_3 = \Big(\frac{\sum_{i=1}^{k} \frac{\partial G_s}{\partial \alpha_i}(\alpha(t), v_1,\dots,v_m)\alpha_i'(t) + h_s'(t) + F_s(t,u)\,u}{\lambda_s(u)}\Big)_{m+1\le s\le n} \tag{3.68}$$

in which *Fs*(*s* = *m* + 1,…,*n*) are continuous functions of *t* and *u*.

Thus, noting (3.6), for ∂ > 0 small enough, by (3.66)-(3.68) we easily get

$$x=0: ws=\sum_{j=1}^{m} f_{sj}(t,u)w_j+\sum_{i=1}^{k} f_{si}(t,u)\alpha_i'(t)+\sum_{l=m+1}^{n}\bar{f}_{sl}(t,u)h_l'(t)+f_s(t,u)u(s=m+1,\dots,n) \tag{3.69}$$

Where $f_s, f_{sj}, \bar{f}_{si}$ and $\bar{f}_{sl}$ are continuous functions of *t* and *u*.

For *j* = *m*+1; : : : ; *n*, passing through any fixed point $(t,x)\in D_-^T$ we draw the jth characteristic $c_j:\xi=\xi_j(s;t,x)$ which must intersect the t-axis at a point (*t*0; 0). Then, we have

Proposition 3.2. On this jth characteristic $c_j:\xi=\xi_j(s;t,x)$ it follows that

$$t\ge t_0\ge \frac{\lambda_j(0)-\lambda_{m+1}(0)+\frac{\delta_0}{2}}{\lambda_j(0)-\frac{\delta_0}{2}}t \tag{3.70}$$

Proof. Noting (3.4), it is easy to see that

$$x-(\lambda_j(0)-\frac{\delta_0}{2})t\ge -(\lambda_j(0)-\frac{\delta_0}{2})t_0 \tag{3.71}$$

On the other hand, by (3.8), we have

$$x\le(\lambda_{m+1}(0)-\delta_0)t \tag{3.72}$$

Therefore, it follows from (3.71)-(3.72) that

$$t_0\ge \frac{\lambda_j(0)-\lambda_{m+1}(0)+\frac{\delta_0}{2}}{\lambda_j(0)-\frac{\delta_0}{2}}t \tag{3.73}$$

Noting the fact that $t\ge t_0$ we immediately get (3.70).

By integrating (2.6) along $c_j:\xi=\xi_j(s;t,x)$ we have

$$w_j(t,x)=e^{-L(t-t0)}wj(t_0,0)+\int_{t_0}^{t}e^{-L(t-s)}\sum_{k,l=1}^{n}[\gamma_{jkl}(u)w_kw_l]+\bar{\gamma}_{jkl}(u)v_kw_l](s,\xi_j(s;t,x))ds \tag{3.74}$$

By (3.69), we have

$$w_j(t_0,0)=\sum_{r=1}^{m} f_{jr}(t_0,u(t_0,0))w_r(t_0,0)+\sum_{i=1}^{k}\bar{f}_{ji}(t_0,u(t_0,0))\alpha_i'(t_0,0)+\sum_{l=m+1}^{n}\bar{f}_{jl}(t_0,u(t_0,0))h_l'(t_0)+f_j(t_0,u(t_0,0))u(t_0,0) \tag{3.75}$$

By employing the same arguments as in (i), we can obtain

$$\begin{aligned}(1+t_0)^{1+\mu}|wr(t_0,0)|\le c_9\{\theta+(W(D_-^T))^2+W(D_-^T)V(D_-^T)\\+W_\infty^c(T)[\bar{W}_1(T)+W_\infty^c(T)+\bar{V}_1(T)+V_\infty^c(T)+\bar{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]\\+\bar{W}_1(T)[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.76}$$

Thus, noting (1.15), (3.6) and (3.70), it follows from (3.75) and (3.76) that

$$\begin{aligned}(1+t_0)^{1+\mu}|w_j(t_0,0)|\le c_{10}(1+t_0)^{1+\mu}|w_j(t_0,0)|\\\le c_{11}\{\theta+(V(D_-^T))^2+W(D_-^T)W(D_-^T)\\+W_\infty^c(T)[\bar{W}_1(T)+W_\infty^c(T)+\bar{V}_1(T)+V_\infty^c(T)+\bar{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]\\+\bar{W}_1(T)[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.77}$$

Hence, noting the fact that *L*>0, we obtain from (3.74) that

$$\begin{aligned}(1+t)^{1+\mu}|wj(t,x)|\le C12\{\theta+V(D_-^T)+(W(D_-^T))^2+W(D_-^T)V(D_-^T)+W_\infty^c(T)\\[\bar{W}_1(T)+W_\infty^c(T)+\bar{V}_1(T)+V_\infty^c(T)+\bar{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]\\+\bar{W}_1(T)[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.78}$$

Combining (3.62) with (3.78), we get

$$\begin{aligned}W(D_-^T)\le C_{13}\{\theta+V(D_-^T)+(W(D_-^T))^2+W(D_-^T)V(D_-^T)+W_\infty^c(T)\\[\bar{W}_1(T)+W_\infty^c(T)+\bar{V}_1(T)+V_\infty^c(T)+\bar{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]\\+\bar{W}_1(T)[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.79}$$

We next estimate $\bar{W}_1(T)$

Let

$\bar{C}_j: x=x_j(t)(t_1\le t\le t_2)$

be any given jth characteristic in $D_i^T(j\ne i, i=m+1,\dots,n)$ By (3.4), the whole ith characteristic *x*=*xi*(*t*) passing through *O*(0; 0) is included in D_i^T Let (*t*0; *xj*(*t*0)) be the intersection point of this characteristic with $\bar{C}_j$ Passing through any given point (*t*; *xj*(*t*)) on $\bar{C}_j$ we draw the ith characteristic $\xi=\bar{x}_i(s,y)$ which intersects one of the boundaries of *DT* , say $x=(\lambda n(0)+\delta_0)t$ (resp. $x=(\lambda_{m+1}(0)-\delta_0)t$) at a point $A_y(y/(\lambda_n(0)+\delta_0),y)$ (resp. $B_y(y/(\lambda_{m+1}(0)-\delta_0)y)$ if $t_0\le t\le t_2$(resp. $t_1\le t\le t_0$) Clearly, we have

$$\tilde{x}_i(t,y)=x_j(t) \tag{3.80}$$

which gives a one-to-one correspondence *t* = *t*(*y*) between the segment $\overline{OA_{y2}}$ | (resp. $\overline{B_{y1}O}$) and $\tilde{C}_j(t_0\le t\le t_2)$(resp $\tilde{C}_j(t_1\le t\le t_0)$). Thus, the integral on $\tilde{C}_j$ with respect to *t* can be reduced to the integral with respect to *y*. Differentiating (3.80) with respect to *t* gives

$$dt=\frac{1}{\lambda_j(u(t,\tilde{x}_i(t,y)))-\lambda_i(u(t,\tilde{x}_i(t,y)))}\frac{\partial\tilde{x}_i(t,y)}{\partial y}dy \tag{3.81}$$

in which *t*=*t*(*y*). Then, noting (3.3) and (3.6), it is easy to see that in order to estimate

$$\begin{aligned}\int_{\tilde{C}_j}|w_i(t,x)|dt=\int_{t_1}^{t_0}|w_i(t,x_j(t))|dt+\int_{t_0}^{t_2}|w_i(t,x_j(t))|dt\\=\int_{t_1}^{t_0}|w_i(t,\tilde{x}_j(t))|dt+\int_{t_0}^{t_2}|w_i(t,\tilde{x}_j(t))|dt\end{aligned} \tag{3.82}$$

it suffices to estimate

$$\int_0^{y_1}|q_i(t,\tilde{x}_i(t,y))|_{t=t(y)}dy \text{ and } \int_0^{y_2}|q_i(t,\tilde{x}_i(t,y))|_{t=t(y)}dy \tag{3.83}$$

We now estimate $\int_0^{y_2}|q_i(t,\tilde{x}_i(t,y))|_{t=t(y)}dy$

By integrating (2.28) along $\xi=\tilde{x}_i(s,y)$ and noting (2.30) and the fact that $\tilde{x}_i(y/\lambda_n(0)+\delta_0),y)=y$ we obtain

$$\begin{aligned}q_i(t,\tilde{x}_i(t,y))|_{t=t(y)}=e^{-L(t(y)-\frac{y}{\lambda_n(0)+\delta_0})}w_i(\frac{y}{\lambda n(0)+\delta_0},y)\{1-\frac{\lambda_i(u(y/(\lambda_n(0)+\delta_0),y)}{\lambda_n(0)+\delta_0}\}\\+\int_{ty/(\lambda n(0)+\delta 0)}^{t(y)}e^{-L(t(y)-s)\frac{\partial\tilde{x}i(s,y)}{\partial y}}(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\j\ne l}}^{n})T_{ijk}(u)w_iw_k(s,\tilde{x}_i(s,y))ds\\+\int_{ty/(\lambda n(0)+\delta 0)}^{t(y)}e^{-L(t(y)-s)\frac{\partial\tilde{x}i(s,y)}{\partial y}}(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\j\ne l}}^{n})\tilde{\gamma}_{ijk}(u)w_iw_k(s,\tilde{x}_i(s,y))ds\\+\int_{ty/(\lambda n(0)+\delta 0)}^{t(y)}e^{-L(t(y)-s)\frac{\partial\tilde{x}i(s,y)}{\partial y}}[\sum_{j=m+1}^{n}\tilde{\gamma}_{ijj}(u)v_jw_j](s,\tilde{x}_i(s,y))ds\end{aligned} \tag{3.84}$$

By Hadamard's formula and (2.11), we have

$$\tilde{\gamma}_{ijj}(\mathrm{u}) = \tilde{\gamma}_{ijj}(\mathrm{u}) - \tilde{\gamma}_{ijj}(\mathrm{u_j e_j}) = \int_0^1 \sum_{l \neq j} \frac{\partial \tilde{\gamma}_{ijj}}{\partial u_l}(\mathrm{Tu_1, ..., Tu_{j-1}, u_j, Tu_{j+1}, ..., Tu_n})\,\mathrm{u_l d_T} \quad (3.85)$$

Noting (3.6), (3.11), (3.13) and the fact that $L > 0$ and $\frac{\tilde{\partial}_{xi}(\mathrm{s,y})}{\partial y} > 0$ obtain from (3.84) and (3.85) that

$$\begin{aligned} q_i(\mathrm{t}, \tilde{\mathrm{x}}_i(\mathrm{t,y}))\,|\,\mathrm{t = t(y)} \le &|\,\mathrm{w_i}(\frac{y}{\lambda_n(0)+\partial_0}\mathrm{y})| + \mathrm{C_{15}}\{[(\mathrm{W}^c_\infty(\mathrm{T}))^2 + \mathrm{W}^c_\infty(\mathrm{T})\mathrm{V}^c_\infty(\mathrm{T}) + \mathrm{W}^c_\infty(\mathrm{T})\mathrm{U}^c_\infty(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})] \\ &\times \int_{y/(\lambda n(0)+\delta 0)}^{t(y)} (1+\mathrm{s})^{-(1+\mu)}(1+|\tilde{\mathrm{x}}_i(\mathrm{s,y})|)^{-(1+\mu)} \frac{\partial \tilde{x}_i(\mathrm{s,y})}{\partial y} ds \\ &+ [\mathrm{W}^c_\infty(\mathrm{T}) + V^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})] \\ &\times \sum_{k=1}^n \int_{(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y})) \in \mathrm{D}^T_k} (1+\mathrm{s})^{-(1-\mu)}\,|\,\mathrm{w_k}(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y}))|\,\frac{\partial \tilde{x}_i(\mathrm{s,y})}{\partial y} ds \end{aligned}$$

$$\begin{aligned} &+ W^c_\infty(\mathrm{T}) \sum_{k=1}^n \int_{(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y})) \in \mathrm{D}^T_j} (1+\mathrm{s})^{-(1-\mu)}\,|\,\mathrm{v}_j(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y}))|\,\frac{\partial \tilde{x}_i(\mathrm{s,y})}{\partial y} ds \\ &+ W^c_\infty(\mathrm{T}) \sum_{k=1}^n \int_{(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y})) \in \mathrm{D}^T_j} (1+\mathrm{s})^{-(1-\mu)}\,|\,\mathrm{u}_l(\mathrm{s}, \tilde{\mathrm{x}}_i(\mathrm{s,y}))|\,\frac{\partial \tilde{x}_i(\mathrm{s,y})}{\partial y} ds \end{aligned} \quad (3.86)$$

Noting that the transformation $\left\{\begin{smallmatrix} x = \tilde{x}_i(\mathrm{s,y}) \\ t = s \end{smallmatrix}\right.$ gives the area element

$$dtdx = \frac{\partial \tilde{x}_i(\mathrm{s,y})}{\partial y} dsdy \quad (3.87)$$

by Lemma 3.2, it easily follows from (3.86) that

$$\begin{aligned} \int_0^{y_2} |\,\mathrm{q_i}(\mathrm{t}, \tilde{\mathrm{x}}_i(\mathrm{t,y}))|_{t=t(\mathrm{y})} dy \le C_{16}\{\theta + W^c_\infty(\mathrm{T})[\mathrm{W_1(T)} + W^c_\infty(\mathrm{T}) + W^c_\infty(\mathrm{T}) + V_1(\mathrm{T}) + V^c_\infty(\mathrm{T}) + \mathrm{U_1(T)V_\infty(T)}] \\ + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}] + \mathrm{W_1(T)}] + \mathrm{W_1(T)}[\mathrm{V}^c_\infty(\mathrm{T}) + V^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}]] \end{aligned} \quad (3.88)$$

Similarly, we have

$$\begin{aligned} \int_0^{y_2} |\,\mathrm{q_i}(\mathrm{t}, \tilde{\mathrm{x}}_i(\mathrm{t,y}))|_{t=t(\mathrm{y})} dy \le C_{17}\{\theta + W^c_\infty(\mathrm{T})[\mathrm{W_1(T)} + W^c_\infty(\mathrm{T}) + W^c_\infty(\mathrm{T}) + V_1(\mathrm{T}) + V^c_\infty(\mathrm{T}) + \mathrm{U_1(T)V_\infty(T)}] \\ + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}] + \mathrm{W_1(T)}] + \mathrm{W_1(T)}[\mathrm{V}^c_\infty(\mathrm{T}) + V^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}]] \end{aligned} \quad (3.89)$$

Thus, we obtain

$$\begin{aligned} \tilde{W}_1(\mathrm{T}) \le C_{17}\{\theta + W(\mathrm{D}^T_-) + \mathrm{W}^c_\infty(\mathrm{T})[\mathrm{W_1(T)} + W^c_\infty(\mathrm{T}) + W^c_\infty(\mathrm{T}) + V_1(\mathrm{T}) + V^c_\infty(\mathrm{T}) + \mathrm{U_1(T)V_\infty(T)}] \\ + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}] + \mathrm{W_1(T)}] + \mathrm{W_1(T)}[\mathrm{V}^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T}) + \mathrm{V_\infty(T)}]] \end{aligned} \quad (3.90)$$

We next estimate $W1(T)$.

For $i = m+1, \ldots, n$, passing through any given point $(t; x) \in D^T_i(\mathrm{t})$ we draw the ith characteristic $\xi = \tilde{x}_i(\mathrm{s,y})$ which intersects one of the boundaries of DT, say $x = (\lambda_n(0)+\delta_0)\mathrm{t}$ (resp. $\mathrm{x} = (\lambda_{m+1}(0)-\delta_0)\mathrm{t}$ at a point $A_y(\mathrm{y}/(\lambda_n(0)+\delta_0), \mathrm{y})$ (resp. $\mathrm{B_y}(\mathrm{y}/\lambda_{m+1}(0)-\delta_0)\mathrm{y})$ Clearly, we have $x = \tilde{x}_i(\mathrm{t,y})$ Therefore we obtain

$$\int_{D^T_i(\mathrm{t})} |\,\mathrm{w_i(t,x)}\,| dx = \int_0^{y_1} |\,\mathrm{q_i}(\mathrm{t}, \tilde{\mathrm{x}}_i(\mathrm{t,y}))\,\mathrm{dy} + \int_0^{y_2} |\,\mathrm{q_i}(\mathrm{t}, \tilde{\mathrm{x}}_i(\mathrm{t,y}))\,\mathrm{dy} \quad (3.91)$$

where $y1$ and $y2$ are shown in Figure 2.

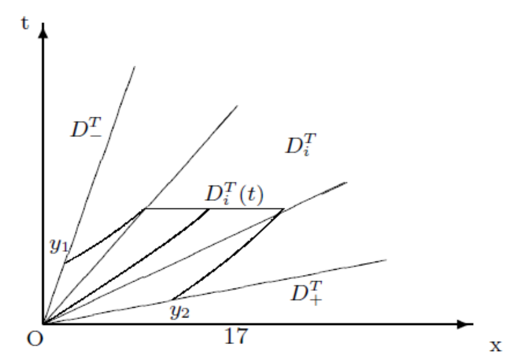

Figure 2: Where $y1$ and $y2$ are shown in figure.

Similar to (3.90), it follows from (3.91) that

$$\begin{aligned} W_1(\mathrm{T}) \le C_{19}\{\theta + W(\mathrm{D}^T_-) + \mathrm{W}^c_\infty(\mathrm{T})[\mathrm{W_1(T)} + W^c_\infty(\mathrm{T}) + W^c_\infty(\mathrm{T}) + V_1(\mathrm{T}) + V^c_\infty(\mathrm{T}) + \mathrm{U_1(T)V_\infty(T)}] \\ + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}] + \mathrm{W_1(T)}] + \mathrm{W_1(T)}[\mathrm{V}^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T}) + \mathrm{V_\infty(T)}]] \end{aligned} \quad (3.92)$$

We next estimate $\mathrm{W}^c_\infty(\mathrm{T})$

(i) For $r = 1; : : : ;m$, passing through any fixed point $(\mathrm{t,x}) \in \mathrm{D}^T$ we draw the rth characteristic $C_r : \xi = \xi_r(\mathrm{s;t,x})$ which must intersect the boundary $x = (\lambda_n(0)+\delta_0)\,\mathrm{t}$ of D^T at a point $(t0; y)$. Then, we have

Proposition 3.3. On this rth characteristic $C_r : \xi = \xi_r(\mathrm{s;t,x})$ it follows that

$$t \ge t_0 \ge \frac{\lambda_{m+1}(0) - \lambda_r(0) - \frac{3\delta_0}{2}}{\lambda_n(0) - \lambda_r(0) - \frac{\delta_0}{2}} t \quad (3.93)$$

Proof. By (3.4), it is easy to see that

$$x - (\lambda_r(0) + \frac{\delta_0}{2})\mathrm{t} \le \mathrm{y} - (\lambda_r(0) + \frac{\delta_0}{2})\mathrm{t_0} \quad (3.94)$$

On the other hand, from (3.9), we have

$$x \ge (\lambda_{m+1}(0) - \delta_0)\,\mathrm{t} \quad (3.95)$$

Since

$$y = (\lambda_n(0) + \delta_0)\,\mathrm{t_0} \quad (3.96)$$

we conclude from (3.94)-(3.96) that

$$t_0 \ge \frac{\lambda_{m+1}(0) - \lambda_r(0) - \frac{3\delta_0}{2}}{\lambda_n(0) - \lambda_r(0) - \frac{\delta_0}{2}} t \quad (3.97)$$

Noting the fact that $t \ge t_0$ we immediately get (3.93).

By integrating (2.6) along $\xi = \xi_r(\mathrm{s;t,x})$ and noting (2.9) and (2.11), we have

$$\begin{aligned} w_r(\mathrm{t,x}) = &\,\mathrm{e}^{-\mathrm{L(t-t_0)}}\mathrm{w_r(t_0,y)} \\ &+ \int_{t_0}^t e^{-L(\mathrm{t-s})}\Big(\sum_{\mathrm{i,k=1}}^m + \sum_{\substack{i \in \{1,...,m\} \\ l \notin \{1,...,m\}}} + \sum_{\substack{i \in \{1,...,m\} \\ l \notin \{1,...,m\}}} + \sum_{\substack{\mathrm{i,l}=m+1 \\ i \neq l}}^n\Big)\gamma_{ijk}(\mathrm{u})\,\mathrm{w_i}\mathrm{w}_k(\mathrm{s}, \xi_\mathrm{i}(\mathrm{s,t,x}))\,\mathrm{ds} \\ &+ \int_{t_0}^t e^{-L(\mathrm{t-s})}\Big(\sum_{\mathrm{i,k=1}}^m + \sum_{\substack{i \in \{1,...,m\} \\ l \notin \{1,...,m\}}} + \sum_{\substack{i \in \{1,...,m\} \\ l \notin \{1,...,m\}}} + \sum_{\substack{\mathrm{i,l}=m+1 \\ i \neq l}}^n\Big)\gamma_{ijk}(\mathrm{u})\,\mathrm{v_i}\mathrm{w}_k(\mathrm{s}, \xi_\mathrm{i}(\mathrm{s,t,x}))\,\mathrm{ds} \\ &+ \int_{t_0}^t e^{-L(\mathrm{t-s})}\Big[\sum_{l=m+1}^n \tilde{\gamma}_{rll}(\mathrm{u}) - \tilde{\gamma}_{rll}(\mathrm{u_1 e_1}))\,\mathrm{v_1 w_1}\Big](\mathrm{s}, \tilde{\mathrm{x}}_\mathrm{i}(\mathrm{s,y}))\,\mathrm{ds} \end{aligned} \quad (3.98)$$

By using Lemma 3.2 and noting (3.93) and (3.96), it is easy to see that

$$|\,\mathrm{w_r(t_0,y)}\,| \le \mathrm{k_1}\,\theta(1+\mathrm{y})^{-(1+\mu)} \le \mathrm{C_{20}}\,\theta(1+\mathrm{t})^{-(1+\mu)} \le \mathrm{C_{21}}\,\theta(1+\mathrm{t})^{-(1+\mu)} \quad (3.99)$$

By Hadamard's formula, we have

$$\tilde{\gamma}_{ru}(u) - \tilde{\gamma}_{ru}(\mathrm{u}_l \mathrm{e}_l) = \int_0^1 \sum_{k \neq l} \frac{\partial \tilde{\gamma}_r u}{\partial u_k}(\mathrm{TU_1, .., TU_{l-1}, U_1}, TU_{l+1}, ..., TU_n) U_k dT. \quad (3.100)$$

Thus, noting (3.93) and the fact that $L > 0$, we obtain from (3.98) that

$$\begin{aligned} (1+\mathrm{t})^{1+\mu}\,|\,\mathrm{w_r(t,x)}\,| \le \mathrm{C_{22}}\{\theta + W^c_\infty(\mathrm{T})[\tilde{\mathrm{W}}_1(\mathrm{T}) + \mathrm{W}^c_\infty(\mathrm{T}) + \tilde{\mathrm{V}}_1(\mathrm{T}) + \mathrm{V}^c_\infty(\mathrm{T}) + \tilde{\mathrm{U}}_1(\mathrm{T})\mathrm{V_\infty(T)} \\ + \mathrm{U}^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}] + \tilde{\mathrm{W}}_1(\mathrm{T})[\mathrm{V}^c_\infty(\mathrm{T}) + U^c_\infty(\mathrm{T})\mathrm{V_\infty(T)}]\} \end{aligned} \quad (3.101)$$

(ii) For $i = m + 1; : : : ; n$, for any fixed point $(\mathrm{t,x}) \in \mathrm{D}^T$ but $(\mathrm{t,x}) \notin \mathrm{D}^T_i$ by the definition of D^T_i for fixing the idea we may assume that

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$$x-\lambda_i(0)t > [\delta_0+\eta(\lambda_n(0)-\lambda_i(0))]t \tag{3.102}$$

which implies $i < n$. Let $\xi=\xi_i(s;t,x)$ be the ith characteristic passing through $(t; x)$, which intersects the boundary $x=(\lambda_n(0)+\delta_0)t$ of D^T at a point $(t0; y)$ (Figure 3).

Recalling (3.4), it is easy to see that

$$x-(\lambda_i(0)+\frac{\delta_0}{2})t \le y-(\lambda_i(0)+\frac{\delta_0}{2})t_0 \tag{3.103}$$

Since

$$y=(\lambda_n(0)+\delta_0)t_0, \tag{3.104}$$

recalling (3.102) and the fact that $t \ge t_0$ it follows from (3.103) that

$$t \ge t_0 \ge \eta t. \tag{3.105}$$

By integrating (2.6) along $\xi=\xi_i(s;t,x)$ and noting (2.9) and (2.11), we have

$$\begin{aligned} w_j(t,x) &= e{-}L^{(t-t_0)}w_j(t_0,y) \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{i,l=1}^m+\sum_{\substack{i\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\in\{1,...,m\}\\ i\notin\{1,...,m\}}}+\sum_{\substack{i,l=m+1\\ i\ne l}}^n\Big)\gamma_{jil}(u)\,w_j w_l(s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{i,l=1}^m+\sum_{\substack{i\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\in\{1,...,m\}\\ i\notin\{1,...,m\}}}+\sum_{\substack{i,l=m+1\\ i\ne l}}^n\Big)\tilde{\gamma}_{jil}(u)\,v_j w_1(s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big[\sum_{l=m+1}^n \tilde{\gamma}_{jil}(u)-\tilde{\gamma}_{jil}(u_1e_1))\,v_1w_1\Big](s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\tilde{\gamma}_{jil}(u)\,w_j w_l(s,\xi_j(s;t,x))ds \end{aligned} \tag{3.106}$$

By Lemma 3.2 and observing (3.104)-(3.105), it is easy to see that

$$|w_i(t_0,y)| \le k_1\theta(1+y)^{-(1+\mu)} \le C_{23}\,\theta(1+t_0)^{-(1+\mu)} \le C_{24}\theta(1+t)^{-(1+\mu)} \tag{3.107}$$

By Hadamard's formula, we have

$$\tilde{\gamma}_{ill}(u)-\tilde{\gamma}_{ill}(u_le_l)=\int_0^1\sum_{k\ne 1}\frac{\partial\tilde{\gamma}_{ill}}{\partial u_k}(Tu_1,...,Tu_{l-1},u_l,Tu_{l+1},....,Tu_n)u_k dT \tag{3.108}$$

Thus, recalling (3.13) and (3.105), and noting the fact that $L>0$, it follows from (3.106) that

$$\begin{aligned}(1+|x-\lambda_i(0)t|)^{1+\mu}|w_i(t,x)| \le C_{25}\{&\theta+W_\infty^c(T)[\tilde{W}(T)+W_\infty^c(T)+\tilde{V}_1(T)+V_\infty^c(T)+\tilde{U}_1(T)V_\infty(T)\\ &+U_\infty^c(T)V_\infty(T)]+\tilde{W}_1(T)[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.109}$$

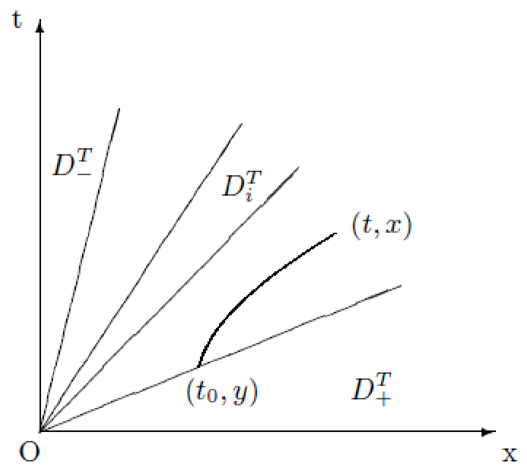

Figure 3: At a point $(t0; y)$.

Next, we assume that

$$x-\lambda_i(0)t < -[\delta_0+\eta(\lambda_i(0)-\lambda_{m+1}(0))]t \tag{3.110}$$

which implies $i > m + 1$. Let $\xi=\xi_i(s;t,x)$ $(t; x)$, which intersects the boundary $x=(\lambda_{m+1}(0)-\delta_0)t$ of D^T at a point $(t0; y)$.

Recalling (3.4), it is easy to see that

$$x-(\lambda_i(0)-\frac{\delta_0}{2})t \ge y-(\lambda_i(0)-\frac{\delta_0}{2})t_0 \tag{3.111}$$

Since

$$y=(\lambda_{m+1}(0)-\delta_0)t_0 \tag{3.112}$$

noting (3.110) and the fact that $t \ge t_0 nt$ it follows from (3.111) that

$$t \ge t_0 \ge \eta t \tag{3.113}$$

By integrating (2.6) along $\xi=\xi_i(s;t,x)$ and noting (2.9) and (2.11), we have

$$\begin{aligned} w_i(t,x) &= e^{-L(t-t_0)}w_i(t_0,y) \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{j,l=1}^m+\sum_{\substack{j\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\notin\{1,...,m\}\\ j\notin\{1,...,m\}}}+\sum_{\substack{j,l=m+1\\ j\ne l}}^n\Big)\gamma_{ijl}(u)\,w_j w_1(s,\xi_i(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{j,l=1}^m+\sum_{\substack{l\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\in\{1,...,m\}\\ j\notin\{1,...,m\}}}+\sum_{\substack{j,l=m+1\\ j\ne l}}^n\Big)\tilde{\gamma}_{ijl}(u)v_jw_l(s,\xi_i(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big[\sum_{l=m+1}^n(\tilde{\gamma}_{ill}(u)-\tilde{\gamma}_{ill}(u_le_l))v_lw_l\Big](s,\xi_i(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\gamma_{iii}(u)w_iw_i(s,\xi_i(s;t,x))ds \end{aligned} \tag{3.114}$$

By (3.113), it is easy to see that

$$|w_i(t_0,y)| \le C_{26}W(D_-^T)(1+t_0)^{-(1+\mu)} \le C_{27}W(D_-^T)(1+t)^{-(1+\mu)} \tag{3.115}$$

Thus, using (3.13), (3.108) and (3.113), and noting the fact that $L > 0$, it follows from (3.114) that

$$\begin{aligned}(1+|x-\lambda_i(0)t|)^{1+\mu}|w_i(t,x)| \le C_{28}\{&W(D_-^T)+W_\infty^c(T)[\tilde{W}_1(T)+W_\infty^c(T)+\tilde{V}_1(T)+V_\infty^c(T)\\ &+\tilde{U}_1(T)V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]+\tilde{W}_1(T)[V_\infty^c(T)\\ &+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.116}$$

Combining (3.101) and (3.109), (3.116), we obtain

$$\begin{aligned}W_\infty^c(T) \le C_{29}\{&\theta+W(D_-^T)+W_\infty^c(T)[\tilde{W}_1(T)+W_\infty^c(T)+\tilde{V}_1(T)\\ &+V_\infty^c(T)+\tilde{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]+\tilde{W}_1(T)\\ &[V_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\}\end{aligned} \tag{3.117}$$

We next estimate $V(D_-^T)$ (i) For $j = 1; : : : ;m$, for any fixed point $(t,x)\in D_-^T$ similar to (3.59), by integrating (2.13) along $\xi=\xi_i(s;t,x)$ and noting (2.17)-(2.18), we have

$$\begin{aligned} v_j(t,x) &= e^{-L(t-t_0)}v_j(t_0,y) \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{i,l=1}^m+\sum_{\substack{i\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\in\{1,...,m\}\\ i\notin\{1,...,m\}}}+\sum_{\substack{i,l=m+1\\ i\ne 1}}^n\Big)\beta_{jil}(u)v_iw_l(s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big(\sum_{i,l=1}^m+\sum_{\substack{i\in\{1,...,m\}\\ l\notin\{1,...,m\}}}+\sum_{\substack{l\in\{1,...,m\}\\ i\notin\{1,...,m\}}}+\sum_{\substack{i,l=m+1\\ i\ne 1}}^n\Big)\tilde{\beta}_{jil}(u)v_i\,v_1(s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big[\sum_{l=m+1}^n(\beta_{jll}(u)-\beta_{jll}(u_1e_1))v_lw_l\Big](s,\xi_j(s;t,x))ds \\ &+\int_{t_0}^t e^{-L(t-s)}\Big[\sum_{l=m+1}^n(\tilde{\beta}_{jll}(u)-\tilde{\beta}_{jll}(u_1e_1))v_l^2\Big](s,\xi_j(s;t,x))ds \end{aligned} \tag{3.118}$$

By using Lemma 3.2 and noting (3.54) and (3.57), it is easy to see that

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$$|v_j(t_0,y)|\le k_1\theta(1+y)^{-(1+\mu)}\le C_{30}\theta(1+t0)^{-(1+\mu)}\le C_{31}\theta(1+t)^{-(1+\mu)} \quad (3.119)$$

By Hadamard's formula, we have

$$\beta_{jll}(u)-\beta_{jll}(u_le_l)=\int_0^1\sum_{\substack{k=1\\k\ne 1}}^{n}\frac{\partial\beta_{jll}}{\partial u_k}(\mathrm{Tu}_1,\dots,\mathrm{Tu}_{l-1},\mathrm{u}_1,\mathrm{Tu}_{l+1},\dots,\mathrm{Tu}_n)\,\mathrm{u_k dT} \quad (3.120)$$

And

$$\tilde\beta_{jll}(u)-\tilde\beta_{jll}(u_le_l)=\int_0^1\sum_{\substack{k=1\\k\ne 1}}^{n}\frac{\partial\tilde\beta_{jll}}{\partial u_k}(\mathrm{Tu}_1,\dots,\mathrm{Tu}_{l-1},\mathrm{u}_1,\mathrm{Tu}_{l+1},\dots,\mathrm{Tu}_n)\,\mathrm{u_k dT} \quad (3.121)$$

Thus, noting the fact that $L>0$, and using (3.13) and (3.54), we obtain from (3.118) that

$$\begin{aligned}&(1+t)^{1+\mu}|v_j(t,x)|\le C_{32}\{\theta+(\mathrm{V}(D_-^T))^2+W(D_-^T)V(D_-^T)\\&+V_\infty^c(T)[\tilde W_1(T)+W_\infty^c(T)+\tilde V_1(T)+V_\infty^c(T)+\tilde U_1(T)V_\infty(T)+U_\infty^c(\mathrm{T})\mathrm{V}_\infty(T)]\\&+U_\infty^c(T)V_\infty(T)[W_\infty^c(T)+\tilde W_1(T)+\tilde V_1(T)]+W_\infty^c(T)[\tilde V_1(T)+\tilde U_1(T)V_\infty(\mathrm{T})]\}\end{aligned} \quad (3.122)$$

For $j = m + 1; : : : ; n$, for any fixed point $(t; x) \in D_-^T$ similar to (3.74), we have

$$v_j(t,x)=e^{-L(t-t_0)}v_j(t_0,0)+\int_{t_0}^t e^{-L(t-s)}\sum_{k,l=1}^{n}[\beta_{jkl}(u)v_kw_l \quad (3.123)$$
$$+\tilde\beta_{jkl}(\mathrm{u})\mathrm{v_k v_l}](\mathrm{s},\xi_j(\mathrm{s;t,x}))\,\mathrm{ds}$$

Noting (1.16), by (1.13), it is easy to see that

$$v_j(t_0,0)=\sum_{r=1}^{m}g_{jr}(t_0)v_r(t_0,0)+h_j(t_0) \quad (3.124)$$

Where

$$g_{jr}(t_0)=\int_0^1\frac{\partial G_j}{\partial v_r}(\alpha(\mathrm{t_0}),\mathrm{Tv_1(t_0,0)},\dots,\mathrm{tv_m(t_0,0)})\,\mathrm{dT} \quad (3.125)$$

By employing the same arguments as in (i), we can obtain

$$\begin{aligned}&(1+t_0)^{1+\mu}|v_r(t_0,0)|\le C_{33}\{\theta+(\mathrm{V}(D_-^T))^2+W(D_-^T)V(D_-^T)\\&+V_\infty^c(T)[\tilde W_1(T)+W_\infty^c(T)+\tilde V_1(T)+V_\infty^c(T)+\tilde U_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]\\&+\mathrm{U}_\infty^c(T)V_\infty(T)[W_\infty^c(T)+\tilde W_1(T)+\tilde V_1(T)]+W_\infty^c(T)[\tilde V_1(T)+\tilde U_1(T)V_\infty(T)]\}\end{aligned} \quad (3.126)$$

Thus, noting (1.15), (3.6) and (3.70), it follows from (3.124)-(3.126) that

$$\begin{aligned}&(1+t)^{1+\mu}|v_j(t_0,0)|\le C_{34}(1+t_0)^{1+\mu}|v_j(t_0,0)|\le C_{35}\{\theta\\&+(V(D_-^T))^2+W(D_-^T)V(D_-^T)+V_\infty^c(T)[\tilde W_1(T)+W_\infty^c(T)+\tilde V_1(T)+V_\infty^c(T)\\&+\tilde U_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]+U_\infty^c(T)V_\infty(T)[W_\infty^c(\mathrm{T})+\tilde{\mathrm{W}}_1(T)+\tilde V_1(T)]+\\&W_\infty^c(T)[\tilde V_1(T)+\tilde{\mathrm{U}}_1(T)V_\infty(T)]\}\end{aligned} \quad (3.127)$$

Hence, noting the fact that $L > 0$, we obtain from (3.123) that

$$\begin{aligned}&(1+t)^{1+\mu}|v_j(t,x)|\le C_{36}\{\theta+(V(D_-^T))^2+W(D_-^T)V(D_-^T)+V_\infty^c(T)[\tilde W_1(T)+W_\infty^c(T)+\tilde V_1(T)+V_\infty^c(T)\\&+\tilde U_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]+U_\infty^c(T)V_\infty(T)[W_\infty^c(\mathrm{T})+\tilde{\mathrm{W}}_1(T)+\tilde V_1(T)]+\\&W_\infty^c(T)[\tilde V_1(T)+\tilde{\mathrm{U}}_1(T)V_\infty(T)]\}\end{aligned} \quad (3.128)$$

Combining (3.122) and (3.128), we get

$$\begin{aligned}&V(D_-^T)\le C_{37}\{\theta+(V(D_-^T))^2+W(D_-^T)V(D_-^T)+V_\infty^c(T)[\tilde{\mathrm{W}}_1(T)+W_\infty^c(T)+\tilde V_1(T)+V_\infty^c(T)\\&+\tilde U_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)]+U_\infty^c(\mathrm{T})\mathrm{V}_\infty(T)[W_\infty^c(T)+\tilde W_1(T)+\tilde V_1(T)]+W_\infty^c(\mathrm{T})[\tilde{\mathrm{V}}_1(T)+\\&\tilde U_1(T)V_\infty(T)]\}\end{aligned} \quad (3.129)$$

We next estimate $\tilde V_1(\mathrm{T})$ and $V1(T)$.

For i=m + 1,…, n, for any given jth characteristic $\tilde C_j\ in\ D_i^T(\mathrm{j}\ne\mathrm{i})$ as in the proof of (3.90), in order to estimate $\tilde V_1(\mathrm{T})$ it suffices to estimate

$$\int_0^{y1}|p_i(t,\tilde x_i(t,y))|_{t=t(y)}\,dy \quad and \quad \int_0^{y2}|p_i(t,\tilde x_i(t,y))|_{t=t(y)}\,dy \quad (3.130)$$

By integrating (2.33) along $\xi=\tilde x_i(\mathrm{s,y})$ similar to (3.84), we have

$$\begin{aligned}&P_i(\mathrm{t},\tilde{\mathrm{x}}_i(\mathrm{t,y}))|_{\mathrm{t=t(y)}}=e^{-L(\mathrm{t(y)}-\frac{y}{\lambda_n(0)+\delta_0})}v_i(\frac{y}{\lambda_n(0)+\delta_0},\mathrm{y})\{1-\frac{\lambda_i(\mathrm{u(y/(\lambda_n(0)+\delta_0),y)}}{\lambda_n(0)+\delta_0}\}\\&+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)}e^{-L(\mathrm{t(y)-s})\frac{\partial\tilde x_i(\mathrm{s,y})}{\partial y}}(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\j\ne k}}^{n})B_{ijk}(\mathrm{u})\,\mathrm{v_j}\,w_k(\mathrm{s},\tilde{\mathrm{x}}_i(\mathrm{s,y}))\,\mathrm{ds}\\&+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)}e-L(\mathrm{t(y)-s})\frac{\partial\tilde x_i(\mathrm{s,y})}{\partial y}[\sum_{j=m+1}^{n}B_{ijj}(\mathrm{u})\,\mathrm{v_jw_j}](\mathrm{s},\tilde{\mathrm{x}}_i(\mathrm{s,y}))ds\\&+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)}e^{-L(\mathrm{t(y)-s})\frac{\partial\tilde x_i(\mathrm{s,y})}{\partial y}}(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\j\ne k}}^{n})B_{ijk}(\mathrm{u})\,\mathrm{v_j}\,w_k(\mathrm{s},\tilde{\mathrm{x}}_i(\mathrm{s,y}))\,\mathrm{ds}\\&+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)}e^{-L(\mathrm{t(y)-s})\frac{\partial\tilde x_i(\mathrm{s,y})}{\partial y}}[\sum_{j=m+1}^{n}\tilde B_{ijj}(\mathrm{u})\,\mathrm{v_jw_j}](\mathrm{s},\tilde{\mathrm{x}}_i(\mathrm{s,y}))\,\mathrm{ds}\end{aligned} \quad (3.131)$$

Noting that $\lambda_i(u)(i=m+1; : : : ; n)$ are weakly linearly degenerate, by (2.37) and (2.38), we have

$$B_{ijj}(\mathrm{u_je_j})\equiv 0,\forall\,\mathrm{j} \quad (3.132)$$

By Hadamard's formula, and noting (2.18) and (3.132), we have $Bijj(u) = Bijj(u)$ ¡ $Bijj(ujej)$

$$=\int_0^1\sum_{l\ne j}\frac{\partial B_{ijj}}{\partial u_l}(T\,\mathrm{u}_1,\dots,\mathrm{Tu}_{j-1},\mathrm{u}_j,\mathrm{Tu}_{j+1},\dots,\mathrm{Tu}_n)u_1d_T \quad (3.133)$$

And

$$\begin{aligned}\tilde\beta_{ijj}(\mathrm{u})&=\tilde\beta_{ijj}(\mathrm{u})-\tilde\beta_{ijj}(\mathrm{u_je_j})\\&=\int_0^1\sum_{l\ne j}\frac{\partial\tilde\beta_{ijj}}{\partial u_l}(\mathrm{Tu}_1,\dots,\mathrm{Tu}_{j-1},\mathrm{uj},\mathrm{Tu}_{j+1},\dots,\mathrm{Tu}_n)u_1d_T\end{aligned} \quad (3.134)$$

Then, using Lemma 3.2, similar to (3.88), it follows from (3.131) that

$$\begin{aligned}&\int_0^{y2}|\mathrm{p_i(t,\tilde x_i(t,y))}|_{t=t(y)}dy\le C_{38}\{\theta+\mathrm{W}_\infty^c(\mathrm{T})[\mathrm{V_1(T)}+\mathrm{U_1(T)V_\infty(T)}]\\&+\mathrm{V}_\infty^c(\mathrm{T})[\mathrm{W_1(T)}+\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{V_1(T)}+\mathrm{V}_\infty^c(\mathrm{T})+\mathrm{U_1(T)V_\infty(T)}+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})]\\&+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})[\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{W_1(T)}+\mathrm{V_1(T)}]\}\end{aligned} \quad (3.135)$$

Similarly, we have

$$\begin{aligned}&\int_0^{y2}|\mathrm{p_i(t,\tilde x_i(t,y))}|_{t=t(y)}dy\le C_{39}\{\mathrm{V}(\mathrm{D}_-^T)+\mathrm{W}_\infty^c(\mathrm{T})[\mathrm{V_1(T)}+\mathrm{U_1(T)V_\infty(T)}]\\&+\mathrm{V}_\infty^c(\mathrm{T})[\mathrm{W_1(T)}+\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{V_1(T)}+\mathrm{V}_\infty^c(\mathrm{T})+\mathrm{U_1(T)V_\infty(T)}+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})]\\&+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})[\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{W_1(T)}+\mathrm{V_1(T)}]\}\end{aligned} \quad (3.136)$$

Thus, we obtain

$$\begin{aligned}&\tilde V_1\le C_{40}\{\mathrm{V}(\mathrm{D}_-^T)+\mathrm{W}_\infty^c(\mathrm{T})[\mathrm{V_1(T)}+\mathrm{U_1(T)V_\infty(T)}]\\&+\mathrm{V}_\infty^c(\mathrm{T})[\mathrm{W_1(T)}+\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{V_1(T)}+\mathrm{V}_\infty^c(\mathrm{T})+\mathrm{U_1(T)V_\infty(T)}+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})]\\&+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})[\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{W_1(T)}+\mathrm{V_1(T)}]\}\end{aligned} \quad (3.137)$$

Similarly, we have

$$\begin{aligned}&V_1\le C_{40}\{\mathrm{V}(\mathrm{D}_-^T)+\mathrm{W}_\infty^c(\mathrm{T})[\mathrm{V_1(T)}+\mathrm{U_1(T)V_\infty(T)}]\\&+\mathrm{V}_\infty^c(\mathrm{T})[\mathrm{W_1(T)}+\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{V_1(T)}+\mathrm{V}_\infty^c(\mathrm{T})+\mathrm{U_1(T)V_\infty(T)}+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})]\\&+\mathrm{U}_\infty^c(\mathrm{T})\mathrm{V}_\infty(\mathrm{T})[\mathrm{W}_\infty^c(\mathrm{T})+\mathrm{W_1(T)}+\mathrm{V_1(T)}]\}\end{aligned} \quad (3.138)$$

We next estimate $V_\infty^c(\mathrm{T})$

(i) For r=1; : : : ;m, for any fixed point $(t; x) \in DT$, noting (2.17) and (2.18), similar to (3.98), we

Have

$$Vr(t,x)=e^{-L(t-t_0)}Vr(t_0,y) + \int_{t_0}^{t} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m} + \sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}} + \sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}} + \sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)B_{ijk}(u)v_j w_k(s,\tilde{x}_i(s,y))ds + \int_{t_0}^{t} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m} + \sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}} + \sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}} + \sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)\tilde{B}_{ijk}(u)v_j w_k(s,\tilde{x}_i(s,y))ds + \int_{t_0}^{t} e^{-L(t-s)}\Big[\sum_{l=m+1}^{n}(\beta_{rll}(u)-\tilde{\beta}_{rll}(u_1e_1))v_l^2\Big](s,\xi r(s;t,x))ds + \int_{t_0}^{t} e^{-L(t-s)}\Big[\sum_{l=m+1}^{n}(\tilde{\beta}_{rll}(u)-\tilde{\beta}_{rll}(u_1e_1))v_l^2\Big](s,\xi r(s;t,x))ds \quad (3.139)$$

By using Lemma 3.2 and noting (3.93) and (3.96), it is easy to see that

$$|Vr(t_0,y)|\le K_1\theta(1+y)^{-(1+\mu)} \le C_{42}\theta(1+t_0)^{-(1+\mu)} \le C_{43}\theta(1+t)^{-(1+\mu)} \quad (3.140)$$

By Hadamard's formula, we have

$$\beta_{rll}(u)-\beta_{rll}(u_1e_1)=\int_0^1\sum_{k\neq l}\frac{\partial\beta_{rll}}{\partial u_k}(Tu_1,\dots,Tu_{l-1},u_1,Tu_{l+1},\dots,Tu_n)u_k d_T \quad (3.141)$$

And

$$\tilde{\beta}_{rll}(u)-\tilde{\beta}_{rll}(u_1e_1)=\int_0^1\sum_{k\neq l}\frac{\partial\tilde{\beta}_{rll}}{\partial u_k}(Tu_1,\dots,Tu_{l-1},u_1,Tu_{l+1},\dots,Tu_n)u_k d_T \quad (3.142)$$

Thus, noting (3.93) and the fact that $L>0$, we obtain from (3.139) that

$$(1+t)^{1+\mu}|v_r(t,x)|\le C_{44}\{\theta + W_\infty^c(T)[\tilde{V}_1(T)+\tilde{U}_1(T)V_\infty(T)] + V_\infty^c(T)[\tilde{W}_1(T)+W_\infty^c(T)+\tilde{V}_1(T)+V_\infty^c(T)+\tilde{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)] + U_\infty^c(T)V_\infty(T)[W_\infty^c(T)+\tilde{W}_1(T)+\tilde{V}_1(T)]\} \quad (3.143)$$

(ii) For $i=m+1;::::n$, for any fixed point $(t,x)\in D^T$ *but* $(t,x)\notin D_i^T$ similar to (3.116), we have

$$(1+|x-\lambda_i(0)t|)^{1+\mu}|v_i(t,x)|\le C_{45}\{V(D_-^T)+W_\infty^c(T)[\tilde{V}_1(T)+\tilde{U}_1(T)V_\infty(T)] + V_\infty^c(T)[\tilde{W}_1(T)+W_\infty^c(T)+\tilde{V}_1(T)+V_\infty^c(T)+\tilde{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)] + U_\infty^c(T)V_\infty(T)[W_\infty^c(T)+\tilde{W}_1(T)+\tilde{V}_1(T)]\} \quad (3.144)$$

Then, it follows from (3.143) and (3.144) that

$$V_\infty^c(T)\le C_{46}\{V(D_-^T)+W_\infty^c(T)[\tilde{V}_1(T)+\tilde{U}_1(T)V_\infty(T)] + V_\infty^c(T)[\tilde{W}_1(T)+W_\infty^c(T)+\tilde{V}_1(T)+V_\infty^c(T)+\tilde{U}_1(T)V_\infty(T)+U_\infty^c(T)V_\infty(T)] + U_\infty^c(T)V_\infty(T)[W_\infty^c(T)+\tilde{W}_1(T)+\tilde{V}_1(T)]\} \quad (3.145)$$

We next estimate $\tilde{U}_1$ and $U1(T)$.

For $i=m+1;::::n$, for any given jth characteristic $\tilde{C}_j$ $In\ D_i^T$ $(j\neq i)$ as in the proof of (3.90), in order to estimate $\tilde{U}_1$ (T) it suffices to estimate

$$\int_0^{y_1}|Z_i(t,\tilde{x}_i(t,y))|_{t=t(y)}dy \text{ and } \int_0^{y_2}|Z_i(t,\tilde{x}_i(t,y))|_{t=t(y)}dy \quad (3.146)$$

By integrating (2.39) along $\xi=\tilde{x}_i(s,y)$ noting (2.41), similar to (3.84), we have

$$Z_i(t,\tilde{x}_i(t,y))|_{t=t(y)} = e^{-L(t(y)-\frac{y}{\lambda_n(0)+\delta_0})}u_i\Big(\frac{y}{\lambda_n(0)+\delta_0},y\Big)\Big\{1-\frac{\lambda_i(u(y/(\lambda_n(0)+\delta_0),y)}{\lambda_n(0)+\delta_0}\Big\} + \int_{y/(\lambda n(0)+\delta_0)}^{t(y)} e^{-L(t(y)-s)\frac{\partial\tilde{x}_i(s,y)}{\partial y}}\Big(\sum_{j,k=l}^{m}+\sum_{\substack{j\in\{1...m\}\\ k\notin\{1...m\}}}+\sum_{\substack{k\in\{1...m\}\\ j\notin\{1...m\}}}+\sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)F_{ijk}(u)u_j w_k(s,\tilde{x}_i(s,y))ds + \int_{y/(\lambda n(0)+\delta_0)}^{t(y)} e^{-L(t(y)-s)}\frac{\partial\tilde{x}_i(s,y)}{\partial y}F_{iii}(u)u_i w_i(s,\tilde{x}_i(s,y))ds \quad (3.147)$$

Since $\lambda i(u)$ is weakly linearly degenerate and $u=(u1;::::un)T$ are normalized coordinates, by (2.43),

we have

$$F_{iii}(u_ie_i)\equiv 0,\forall|u_i| \quad (3.148)$$

Then, using Hadamard's formula, we have

$$F_{iii}(u)=F_{iii}(u)-F_{iii}(u_ie_i) = \int_0^1\sum_{l\neq i}\frac{\partial F_{iii}}{\partial u_l}(Tu_1,\dots,Tu_{i-1},u_i,Tu_{i+1},\dots,Tu_n)u_l d_T \quad (3.149)$$

Hence, noting (3.6), (3.11), (3.13) and the fact that $L>0$ *and* $\dfrac{\partial\tilde{x}_i(s,y)}{\partial y}>0$ we obtain from (3.147) and (3.149) that

$$z_i(t,\tilde{x}_i(t,y))|_{t=t(y)}\le|u_i(\frac{y}{\lambda_n(0)+\delta_0},y)|+C_{47}\{[W_\infty^c(T)U_\infty^c(T)+W_\infty^c(T)U_\infty^c(T)V_\infty(T)] \times\int_{y/(\lambda n(0)+\delta_0)}^{t(y)}(1+s)^{-(1+\mu)}(1+|\tilde{x}_i(s,y)|)^{-(1+\mu)}\frac{\partial\tilde{x}_i(s,y)}{\partial y}ds + [U_\infty^c(T)+U_\infty^c(T)V_\infty(T)]\sum_{k=1}^{n}\int_{(s,\tilde{x}_i(s,y))\in D_k^T}(1+s)^{-(1+\mu)}|w_k(s,\tilde{x}_i(s,y))|\frac{\partial\tilde{x}_i(s,y)}{\partial y} + [W_\infty^c(T)+W_\infty^c(T)V_\infty(T)]\sum_{k=1}^{n}\int_{(s,\tilde{x}_i(s,y))\in D_k^T}(1+s)^{-(1+\mu)}|w_k(s,\tilde{x}_i(s,y))|\frac{\partial\tilde{x}_i(s,y)}{\partial y}\} \quad (3.150)$$

By Lemma 3.2, similar to (3.88), it follows from (3.150) that

$$\int_0^{y2}|Zi(t,\tilde{x}_i(t,y))|_{t=t(y)}\le C_{48}\{\theta+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) + [W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \quad (3.151)$$

Similarly, we have

$$\int_0^{y2}|Zi(t,\tilde{x}_i(t,y))|_{t=t(y)}\le C_{49}\{V(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) + [W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \quad (3.152)$$

Thus, we obtain

$$\tilde{U}_1(T)\le C_{50}\{\theta+V(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) + [W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \quad (3.153)$$

Similarly, we have

$$U_1(T)\le C_{51}\{\theta+V(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) + [W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \quad (3.154)$$

We next estimate U_∞^c

(i) For $r=1;::::m$, for any fixed point $(t,x)\in D^T$ noting (2.19) and (2.20), similar to (3.98), we have

$$u_r(t,x)=e^{-L(t-t_0)}u_r(t_0,y) + \int_{t_0}^{t}e^{-L(t-s)}\Big(\sum_{i,l=l}^{m}+\sum_{\substack{i\in\{1,\dots,m\}\\ l\notin\{1,\dots,m\}}}+\sum_{\substack{l\in\{1,\dots,m\}\\ i\notin\{1,\dots,m\}}}+\sum_{\substack{l,i=m+1\\ i\neq k}}^{n}\Big)\rho_{ril}(u)u_jw_k(s,\tilde{x}_i(s,y))ds \quad (3.155)$$

By using Lemma 3.2 and noting (3.93) and (3.96), it is easy to see that

$$|u_r(t_0,y)|\le C_{52}\theta(1+y)^{-(1+\mu)}\le C_{53}\theta(1+t_0)^{-(1+\mu)}\le C_{54}\theta(1+t)^{-(1+\mu)} \quad (3.156)$$

Thus, noting (3.93) and the fact that $L>0$, we obtain from (3.155) that

$$(1+t)^{1+\mu}|u_r(t,x)|\le C_{55}\{\theta+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)\tilde{W}_1(T)=W_\infty^c(T)\tilde{U}_1(T)\} \quad (3.157)$$

(ii) For $i=m+1;::::n$, for any fixed point $(t,x)\in D^T$ but $(t,x)\notin D_i^T$ similar to (3.116), we have

$$(1+|x-\lambda_i(0)t|)^{1+\mu}|u_r(t,x)|\le C_{56}\{\theta+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)\tilde{W}_1(T)=W_\infty^c(T)\tilde{U}_1(T)\} \quad (3.158)$$

Then, it follows from (3.157) and (3.158) that

$$U_\infty^c\le C_{57}\{\theta+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)\tilde{W}_1(T)=W_\infty^c(T)\tilde{U}_1(T)\} \quad (3.159)$$

We now estimate V_∞ (T)

For $i=m+1;::::n$, passing through any given point $(t,x)\in D^T$ we draw the ith characteristic $\xi=\xi_i(s;t,x)$ which intersects one of the boundaries of DT at one point. For fixing the idea, suppose that this characteristic intersects $x=(\lambda_n(0)+\delta_0)t$ at a point $(y/(\lambda_n(0)+\delta_0),y)$

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By integrating (2.13) along this characteristic and noting (2.16)-(2.18), we have

$$\begin{aligned} Vr(t,x) &= e^{-L(t-t_0)}Vr(t_0,y) \\ &+\int_{t_0}^{t} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)B_{ijk}(u)v_j w_k(s,\tilde{x}_i(s,y))ds \\ &+\int_{t_0}^{t} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)\tilde{B}_{ijk}(u)v_j w_k(s,\tilde{x}_i(s,y))ds \\ &+\int_{t_0}^{t} e^{-L(t-s)}\Big[\sum_{l=m+1}^{n}(\tilde{\beta}_{rll}(u)-\tilde{\beta}_{rll}(u_l e_l))v_l^2\Big](s,\xi r(s;t,x))ds \\ &+\int_{t_0}^{t} e^{-L(t-s)}\Big[\sum_{l=m+1}^{n}(\tilde{\beta}_{rll}(u)-\tilde{\beta}_{rll}(u_l e_l))v_l^2\Big](s,\xi r(s;t,x))ds \end{aligned} \quad (3.160)$$

Noting Lemma 3.1 and Lemma 3.2, and using Hadamard's formula, it follows from (3.160) that

$$\begin{aligned} |V_i(t,x)| \le C_{58}\{&\theta+V(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) \\ &+[W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \end{aligned} \quad (3.161)$$

On the other hand, for $i = m + 1; : : : ; n$, for any fixed point $(t,x)\notin D(T)=\{(t,x)\,|\,0\le t\le T, x\ge 0\}$ but $(t,x)\notin D_i^T$, $|v_i(t,x)|$ can be controlled by $V_\infty^c(T)$ or $V(D_\pm^T)$ Moreover, for $i = 1; : : : ;m$, for any fixed point $(t,x)\notin D(T)=\{(t,x)\,|\,0\le t\le T, x\ge 0\}$ $|v_i(t,x)|$ can be controlled by $V_\infty^c(T)$ or $V(D_\pm^T)$ as well. Thus, by using Lemma 3.2 again, we have

$$\begin{aligned} V_\infty \le C_{58}\{&\theta+V(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) \\ &+[W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \end{aligned} \quad (3.162)$$

We finally estimate $W_\infty(T)$

For $i = m+ 1; : : : ; n$, passing through any given point $(t,x)\in D^T$ similar to (3.160), noting (2.9), (2.11)-(2.12) and the fact that $\lambda_i(u)$ is weakly linearly degenerate, we have

$$\begin{aligned} w_i(t,x) &= e^{-L(t(y)-\frac{y}{\lambda_n(0)+\delta_0})}w_i\Big(\frac{y}{\lambda_n(0)+\delta_0},y\Big) \\ &+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)\gamma_{ijk}(u)w_j w_k(s,\xi_i(s;t,x))ds \\ &+\int_{y/(\lambda_n(0)+\delta_0)}^{t} e^{-L(t(y)-s)}[\gamma_{iii}(u)-\gamma_{iii}(u_i e_i))w_i^2](s,\xi_i(s;t,x))ds \\ &+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)} e^{-L(t-s)}\Big(\sum_{j,k=1}^{m}+\sum_{\substack{j\in\{1,\dots,m\}\\ k\notin\{1,\dots,m\}}}+\sum_{\substack{k\in\{1,\dots,m\}\\ j\notin\{1,\dots,m\}}}+\sum_{\substack{j,k=m+1\\ j\neq k}}^{n}\Big)\tilde{\gamma}_{ijk}(u)v_j w_k(s,\xi_i(s;t,x))ds \\ &+\int_{y/(\lambda_n(0)+\delta_0)}^{t(y)} e^{-L(t-s)}\Big[\sum_{j=m+1}^{n}\tilde{\gamma}_{ijj}(u)-\tilde{\gamma}_{iii}(u_i e_i))w_i^2\Big](s,\xi_i(s;t,x))ds \end{aligned} \quad (3.163)$$

Noting Lemma 3.1 and Lemma 3.2, and using Hadamard's formula, it follows from (3.163) that

$$\begin{aligned} |w_i(t,x)| \le C_{60}\{&\theta+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) \\ &+[W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \end{aligned} \quad (3.164)$$

Thus, by using the definitions of $W_\infty^c(T), W(D_+^T)$ and $W(D_-^T)$ and using Lemma 3.2, we have

$$\begin{aligned} W_\infty(T) \le C_{61}\{&\theta+W(D_-^T)+W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)W_1(T)+W_\infty^c(T)U_1(T) \\ &+[W_\infty^c(T)U_\infty^c(T)+U_\infty^c(T)+W_\infty^c(T)U_1(T)]V_\infty(T)\} \end{aligned} \quad (3.165)$$

We now prove (3.47)-(3.53).

Noting (1.15), evidently we have

$$W_\infty^c(0), V_\infty^c(0), U_\infty^c(0) \le C_{62}\theta \quad (3.166)$$

$$W_1(0)=V_1(0)=U_1(0)=\tilde{W}_1(0)=\tilde{V}_1(0)=\tilde{U}_1(0)=0 \quad (3.167)$$

$$W_\infty(0), V_\infty(0) \le C_{63}\theta \quad (3.168)$$

And

$$T=0: W(D_-^T), V(D_-^T) \le C_{64}\theta \quad (3.169)$$

Thus, by continuity there exist positive constants *k*2*; k*3*; k*4*; k*5*; k*6*; k*7 and *k*8 independent of *μ*, such that (3.47)-(3.53) hold at least for $0\le T\le T0$ whereT0 is a small positive number. Hence, in order to prove (3.47)-(3.53) it su±ces to show that we can choose *k*2*; k*3*; k*4*; k*5*; k*6*; k*7 and *k*8 in such a way that for any fixed $T_0 (0<T_0\le T)$.

$$W(D_-^{T_0}) \le 2k_2\theta \quad (3.170)$$

$$V(D_-^{T_0}) \le 2k_3\theta \quad (3.171)$$

$$U_\infty^c(T_0) \le 2k_4\theta \quad (3.172)$$

$$W_\infty^c(T_0), V_\infty^c(T_0) \le 2k_5\theta \quad (3.173)$$

$$\tilde{W}_1(T_0), W_1(T_0), \tilde{V}_1(T_0), V_1(T_0), \tilde{U}_1(T_0), U_1(T_0) \le 2k_6\theta \quad (3.174)$$

$$V_\infty(T_0), U_\infty(T_0) \le 2k_7\theta \quad (3.175)$$

$$W_\infty(T_0) \le 2k_8\theta \quad (3.176)$$

we have

$$W(D_-^{T_0}) \le k_2\theta \quad (3.177)$$

$$W(D_-^{T_0}) \le k_3\theta \quad (3.178)$$

$$W(D_-^{T_0}) \le k_4\theta \quad (3.179)$$

$$W_\infty^c(T_0), V_\infty^c(T_0) \le k_5\theta \quad (3.180)$$

$$\tilde{W}_1(T_0), W_1(T_0), \tilde{V}_1(T_0), V_1(T_0), \tilde{U}_1(T_0), U_1(T_0) \le k_6\theta \quad (3.181)$$

$$V_\infty(T_0), U_\infty(T_0) \le k_7\theta \quad (3.182)$$

$$W_\infty(T_0) \le k_8\theta \quad (3.183)$$

To this end, substituting (3.170)-(3.176) into the right-hand sides of (3.79), (3.90), (3.92), (3.117), (3.129), (3.137)-(3.138), (3.145), (3.153)-(3.154), (3.159), (3.162) and (3.165)(in which we take *T*=*T*0), it is easy to see that, when $\theta_0>0$ is suitably small, we have

$$W(D_-^{T_0}) \le 2C_{13}(1+k_3)\theta \quad (3.184)$$

$$\tilde{W}_1(D_-^{T_0}) \le 2C_{13}(1+k_2)\theta \quad (3.185)$$

$$W_1(T_0) \le 2C_{19}(1+k_2)\theta \quad (3.186)$$

$$W_\infty^c(T_0) \le 2C_{29}(1+k_2)\theta \quad (3.187)$$

$$V(D_-^{T_0}) \le 2C_{37}\theta \quad (3.188)$$

$$\tilde{V}_1(T_0) \le 2C_{40}(1+k_3)\theta \quad (3.189)$$

$$V_1(T_0) \le 2C_{41}(1+k_3)\theta \quad (3.190)$$

$$V_\infty^c(T_0) \le 2C_{46}(1+k_3)\theta \quad (3.191)$$

$$\tilde{U}_1(T_0) \le 2C_{50}(1+k_3)\theta \quad (3.192)$$

$$U_1(T_0) \le 2C_{51}(1+k_3)\theta \quad (3.193)$$

$$U_\infty^c(T_0) \le 2C_{57}(1+k_3)\theta \quad (3.194)$$

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$$V_\infty(T_0) \le 2C_{59}(1+k_3+k_5)\theta \quad (3.195)$$

$$W_\infty(T_0) \le 2C_{61}(1+k_2+k_5)\theta \quad (3.196)$$

Hence, if *k*3≥2*C*37; *k*2≥2*C*13(1+*k*3); *k*4≥2*C*57(1+*k*3); *k*5≥2 max*f**C*29(1+*k*2);*C*46(1+*k*3)*g*, *k*6 ≥2max*f**C*18(1+*k*2);*C*19(1+*k*2); *C*40 (1+*k*3);*C*41(1+*k*3);*C*50(1+*k*3);*C*51(1+*k*3)*g*, *k*7≥2*C*59(1+*k*3+*k*5) and *k*8≥2*C*61(1 + *k*2 + *k*5), then we get (3.177)-(3.183). This proves (3.47)-(3.53).

Finally, we observe that when *μ*0 > 0 is suitably small, by (3.52) we have

$$U_\infty(T) \le k_7\theta \le k_7\theta \le \frac{\delta}{2} \quad (3.197)$$

This implies the validity of hypothesis (3.6). The proof of Lemma 3.3 is finished.

Proof of Theorem 1.1. It su±ces to prove Theorem 1.1 in the normalized coordinates. Under the assumptions of Theorem 1.1, by (3.52) and (3.53), we know that there is a su±ciently small $\theta_0 > 0$ such that for any fixed $\theta \in (0,\theta_0]$ on any given domain of existence $D(T) = \{(t,x) \mid 0 \le t \le T, x \ge 0\}$ of the *C*1 solution *u* = *u*(*t; x*) to the mixed initial-boundary value problem (1.1) and (1.12)-(1.13), we have the following uniform a priori estimate for the *C*1 norm of the solution:

$$\| u(t,.) \| c^1 \triangleq \| u(t,.) \| c^0 + \| u_x(t,.) \| c^0 \le k_9\theta \quad (3.198)$$

Thus we immediately get the conclusion of Theorem 1.1. The proof of Theorem 1.1 is finished.

Application

Consider the following mixed initial-boundary value problem for the system of the flow equations of a model class of °uids with viscosity induced by fading memory (cf. [7]):

$$\begin{cases} wt - vx + w = 0 \\ vt - (\sigma(w))x + v = 0 \end{cases} \quad (4.1)$$

with the initial condition

$$t = 0 : w = w_0(x), v = \tilde{v}_0 + v_0(x)(x \ge 0) \quad (4.2)$$

and the boundary condition

$$x = 0 : v = h(t)(t \ge 0) \quad (4.3)$$

Here, *w* is the displacement gradient and *v* the velocity of a model class of fluids, the stress-strain function σ (*w*) is a suitably smooth function of *w* such that

$$\sigma'(0) > 0 \quad (4.4)$$

$\tilde{V}_0$ is a constant, $(W_0(x), V_o(x)) \in C^1$ and we assume that there exists a constant *μ* > 0 such that

$$\sup_{c\ge 0}\{(1+x)^{1+\mu}(| w_0(x) |) + | v_0(x) | + | v_0(x) | + | w_0'(x) | + | v_0'(x) |)\} < +\infty \quad (4.5)$$

In addition, we assume that $h(t) \in C^1$

$$\sup_{c\ge 0}\{(1+t)^{1+\mu}(| h(t) | + | h'(t) |)\} < +\infty \quad (4.6)$$

Moreover, the conditions of *C*1 compatibility are supposed to be satis¯ed at the point (0; 0).

Let

$$u = \begin{pmatrix} w \\ v \end{pmatrix} \quad (4.7)$$

By (4.4), it is easy to see that in a neighborhood of $u_0 = \begin{pmatrix} 0 \\ \tilde{v}_0 \end{pmatrix}$ system (4.1) is strictly hyperbolic and has the following two distinct real eigenvalues:

$$\lambda_1(u) = -\sqrt{\sigma'(w)} < 0 < \lambda_2(u) = \sqrt{\sigma'(w)} \quad (4.8)$$

The corresponding right eigenvectors are

$$r_1(u) // (\sqrt{\sigma'(w)})^T, r_2(u) // (1 - \sqrt{\sigma'(w)}))^T \quad (4.9)$$

It is easy to see that in a neighborhood of $u_0 = \begin{pmatrix} 0 \\ \tilde{v}_0 \end{pmatrix}$ all characteristics are linearly degenerate,

then weakly linearly degenerate, provided that

$$\sigma''(w) \equiv 0, \forall | w | \quad \text{small} \quad (4.10)$$

The corresponding left eigenvectors can be taken as

$$l_1(u) = (\sqrt{\sigma'(w)}, 1), l_2(u) = (-\sqrt{\sigma'(w)}, 1) \quad (4.11)$$

Let

vi = *li*(*u*)*u* (*i* = 1; 2): (4.12)

Then, the boundary condition (4.3) can be rewritten as

$$x = 0 : v_1 + v_2 = 2h(t) \triangleq H(t) \quad (4.13)$$

By Theorem 1.1 we get

Theorem 5.1. Suppose that (4.4) and (4.10) hold. Suppose furthermore that *w*0(*x*); *v*0(*x*) are all *C*1

functions with respect to their arguments, for which there is a constant $\mu > 0$ such that

$$\theta \triangleq \max\{\sup_{x\ge 0}\{(1+x)^{1+\mu}(| w_0(x) | + | v_0(x) | + | w_0'(x) | + | v_0'(x) |), \sup_{t\ge 0}(1+t)^{1+\mu}(| H(t) | + | H'(t) |)\} < +\infty \quad (4.14)$$

Suppose finally that $h(t) \in C^1$ satisfies (4.14) and that the conditions of *C*1 compatibility are satisfied at the point (0; 0). Then there is a sufficiently small $\theta_0 > 0$ such that for any given $\theta \in [0,\theta_0]$ the mixed initial-boundary value problem (4.1)-(4.3) admits a unique global *C*1 solution *u* = *u*(*t; x*) in the half space $\{(t,x) \mid t \ge 0, x \ge 0\}$.

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