

## Friction Head Loss in Center-Pivot Laterals with the Lateral Divided into Several Reaches

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### Abstract

This paper presents efficient closed form expressions, based on the discrete outflow distribution, to compute the friction head loss and the friction correction factor in center-pivot laterals divided into several reaches. Since such formulas cannot be found in the literature, we develop these expressions based on the discharge required by each sprinkler and each reach to irrigate an annular area.

These expressions can be used in center-pivot laterals with and without end gun sprinkler and with and without closed outlets at the beginning of the lateral. Moreover, these expressions also allow to compute the head loss between the last outlet and the end gun sprinkler usually neglected by another investigators.

These expressions were developed for center-pivot laterals with: 1) a single diameter and with constant spacing between sprinklers along the lateral and 2) a single diameter and a lateral divided into several reaches where the spacing between outlets is constant but the spacing is different in each reach.

This apparently complex method is easy to implement computationally and easy to understand. It can be applied using the Hazen-Williams, Scobey or Darcy-Weisbach friction loss equations.

An application using the Hazen-Williams equation to compute the friction head loss for a center-pivot divided into three reaches is presented. The results obtained with the method of the discrete outflow distribution are compared with the method of the continuous outflow distribution.

**Keywords:** Fluid mechanics; Center-pivot systems; Friction correction factor; Friction head loss; Laterals with several reaches

### Nomenclature

$A$ =annulus area;  
 $c_1$ =constant in (5);  
 $c_2$ =constant;  
 $D$ =internal diameter of lateral;  
 $d_{eg}$ =distance of the end gun sprinkler from last sprinkler;  
 $E_a$ =irrigation application efficiency;  
 $F_{cf}$ =Friction correction factor;  
 $h_{M-}$ maximum irrigation depth;  
 $h_{r(1-n)}$ =friction head loss between the beginning of segment 1 and the end segment n;  
 $N_{dw}$ =number of irrigation days per week;  
 $n$ =number of open outlets / segments;  
 $Q_{ad}(i)$ =flow rate in the pipe at the segment i;  
 $Q_{cd}(s)$ =flow rate in the pipe at the coordinate s;  
 $Q_e$ =discharge of end gun sprinkler;  
 $Q_{ei}$ =discharge at the beginning or end of the reach i;  
 $Q_0$ =total inflow rate in lateral;  
 $q_j$ =discharge of sprinkler j;  
 $R_{ir}$ = radius of the irrigated basic circle;  
 $R_{big}$ = big radius of the irrigated annulus area;  
 $R_{les}$ = less radius of the irrigated annulus area;

$r$ \_variable of integration with respect to distance;  
 $r_j$ =distance from pivot to sprinkler at point j;  
 $r_0$ =radius of dry area;  
 $S_f$ =friction slope;  
 $s_{sp}$ =spacing between sprinklers;  
 $T_{mr}$ =minimal time period required for one revolution of the system;  
 $\alpha$ = exponent of velocity  
 $\beta$ =roughness factor;  
 $\gamma$ =exponent of diameter;  
 $\lambda$ = coefficient in (29) and (40);  
 $\xi$ = coefficient in (13);  
 $\rho$ =integration (auxiliary) variable;  
 $\psi$ =coefficient in (10) represent the ratio between  $Q_e$  and  $Q_0$ .

### Introduction

Center-pivot irrigation systems are designed to irrigate big surfaces.

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This irrigation system is widely used in United States, Canada, European countries, etc. Moreover, the rate of increase in the area irrigated by this irrigation system is highest among all existing irrigation systems.

Since the area irrigated by this irrigation system is usually a large surface, some attention must be paid to irrigation uniformity. High irrigation uniformity requires an appropriate selection of the sprinkler discharge along the lateral. Moreover, to avoid problems associated with large nozzles, it is typical for the lateral to be divided into several reaches where the spacing between sprinklers is different in each reach.

Choosing the sprinklers along the lateral requires good knowledge of the head distribution in the lateral which, in turn, requires good knowledge of the friction head loss along the lateral.

Many contributions have been given by several investigators to improve the center-pivot sprinkler irrigation systems.

Kincaid and Heermann [1] and Chu and Moe [2] presented methods for computing the pressure distribution along the laterals operating on level fields.

Reddy and Apolayo [3] derived a friction correction factor ( $F_{cf}$ ) for center-pivot laterals as a function of the number of outlets using the Hazen-Williams equation to calculate the friction head loss.

Scaloppi and Allen [4] used a differential approach to solve hydraulic problems of various types which simulate the pressure head distribution along the lateral and the friction head loss.

Anwar [5] developed an expression to the friction correction factor for center-pivots without end gun sprinkler. Later, Anwar [6] presented the first explicit expression to the friction correction factor for center-pivots with end gun.

Valiantzas and Dercas [7] developed analytical equations for direct hydraulic analysis of a multidiameter center-pivot lateral with and without an end gun.

All the above references assumed equal spacing between outlets and between the first open outlet and the pivot. Usually, the distance from the pivot to the first open outlet is longer than the distance between outlets due to the presence of several closed outlets. Moreover, when the center-pivot lateral has an end gun, the friction head loss in the distance between the last sprinkler and the end gun is neglected by several investigators.

Tabuada [8] developed closed form expressions and analytical expressions (the last based on the Hypergeometric function) characterizing the friction head loss of center-pivot laterals with and without closed outlets at the beginning of the center-pivot lateral and with and without end gun sprinkler where he considered friction head loss also in the distance between the last outlet and the end gun sprinkler.

For a center-pivot lateral divided into several reaches the existing literature does not present expressions to solve this problem.

In this paper, we propose general expressions, based on the discrete outflow distribution, for the computation of the friction head loss and to obtain the friction correction factors in center-pivot laterals with and without closed outlets at the beginning of the center-pivot lateral and with and without end gun sprinkler.

These expressions are based on the discharge required by each sprinkler and each reach to irrigate an annular area.

The friction head loss based on the discrete outflow distribution is compared with the values obtained based on the continuous outflow distribution.

## Sprinkler discharge

The center-pivots can use different kinds of sprinklers. The choice of sprinklers is based on the soil infiltration rate and the shape of the water application rate profile.

Sprinklers can be installed using equidistant or variable spacing. In the case of equidistant spacing, the water flow per sprinkler increases from the pivot to the end of the lateral. When variable spacing is used, the distance between sprinklers decreases from the pivot to the end of the lateral while the water flow per sprinkler is kept constant.

To avoid problems associated with large nozzles, sometimes the lateral is divided into three or four reaches, and a different uniform spacing is used in each reach [9].

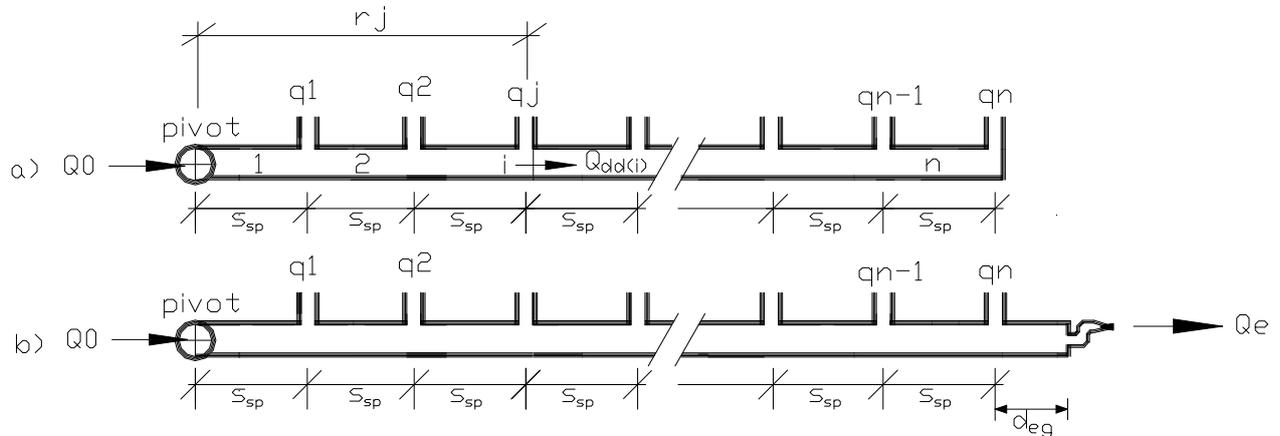
Tabuada [8] showed that the discharge ( $q_j$ ) for each sprinkler located at the point  $j$  where the radius of the center-pivot for this point is  $r_j$  (Figure 1), can be obtained by:

$$q_j = \frac{2\pi}{T_{mr}} \frac{7}{N_{dw}} \frac{1}{E_a} r_j s_{sp} h_{Max} = \frac{14\pi}{T_{mr} N_{dw} E_a} r_j s_{sp} h_{Max} \quad (1)$$

Where  $2\pi$  radians is the angle described by the center-pivot,  $T_{mr}$  is the minimal time period required for one revolution of the system,  $N_{dw}$  is the number of irrigation days per week,  $E_a$  is the irrigation application efficiency,  $s_{sp}$  is the spacing between sprinklers, and  $h_{Max}$  represents the maximum irrigation depth.

If the irrigation interval is greater than seven days then  $7/N_{dw} = 1$ .

Equation (1) is an improved expression and is similar to an equation proposed by Heermann and Hein [10] and later by Mohamoud et al. [11].



**Figure 1:** Schematic diagram of a lateral divided into  $n$  segments where is shown the spacing between sprinklers ( $S_{sp}$ ), outflow along the lateral ( $q_1, \dots, q_n$ ), inflow ( $Q_0$ ) and outflow ( $Q_e$ ) that represents the discharge of the end gun, the flow rate  $[Q_{dd}(i)]$  into segment  $i$  (in this case  $i = 4$ ) and the distance between the last sprinkler and the end gun sprinkler ( $d_{eg}$ ): without an end gun; with an end gun.

### End-gun discharge

Many center-pivots are designed with an end gun at the end the lateral to extend the irrigated area, therefore the discharge of an end gun sprinkler ( $Q_e$ ) is the discharge required to irrigate an annular area at the end of the lateral.

The discharge of the end gun sprinkler ( $Q_e$ ) is a fraction ( $\psi$ ) of the total system flow rate ( $Q_0$ ). They can be related as follows:

$$Q_e = \psi Q_0 \text{ with } \psi = [0;1] \quad (2)$$

In the Figure 1 is shown a schematic diagram of a lateral for the discrete outflow.

### Flow rate at any point along a lateral

In order to calculate the friction head loss we need to know the flow rate at any point along the lateral. This can be done by the discrete outflow distribution (DOD) or continuous outflow distribution (COD).

**Discrete approach: 1) with single diameter and constant spacing along the lateral:** For a discrete distribution of the outflow, the flow inside every segment  $i$  (Figure 1), in a lateral with  $n$  segments (or  $n$  open outlets), is given by [8]:

$$Q_{dd(i)} = Q_0 - c_1 s_{sp} \sum_{j=1}^{i-1} r_j \quad (3)$$

With  $Q_{dd(1)} = Q_0$  and:

$$Q_0 = Q_e + c_1 s_{sp} \sum_{j=1}^n r_j = \psi Q_0 + c_1 s_{sp} \sum_{j=1}^n r_j = \frac{1}{1-\psi} c_1 s_{sp}^2 \sum_{j=1}^n (\xi + j - 1) = \quad (4)$$

$$- \frac{1}{1-\psi} c_1 s_{sp}^2 \left[ n\xi + \frac{(n-1)^2 + (n-1)}{2} \right]$$

where  $j$  represents the number of open outlets and  $i$  the number of segments on the pipe until the section with coordinates  $S = r_j$  and

$$\xi = \frac{r_1}{s_{sp}} \text{ and:}$$

$$c_1 = \frac{2\pi}{T_{mr}} \frac{7}{N_{dw}} \frac{1}{E_a} h_M = \frac{14\pi}{T_{mr} N_{dw} E_a} h_{Max} \quad (5)$$

Also for the various segments, Tabuada [8] presented the following equations:

$$- Q_{dd(1)} = Q_0$$

$$- Q_{dd(2)} = Q_0 \left[ \frac{Q_{dd(2)}}{Q_0} \right] = Q_0 \left[ \frac{Q_0 - c_1 s_{sp} r_1}{Q_0} \right] = Q_0 \left[ 1 - \frac{c_1 s_{sp} r_1}{Q_0} \right]$$

$$- Q_{r(3)} = Q_0 \left[ 1 - \frac{c_1 s_{sp} (2r_1 + s_{sp})}{Q_0} \right]$$

.....

$$Q_{dd(i)} = Q_0 \left\{ 1 - \frac{c_1 s_{sp} \left[ (i-1)r_1 + \sum_{j=1}^{i-1} (j-1)s_{sp} \right]}{Q_0} \right\} = Q_0 \left\{ 1 - \frac{c_1 s_{sp}^2 \left[ (i-1)\xi + \left[ \frac{(i-2) + (i-2)^2}{2} \right] \right]}{Q_0} \right\}$$

for  $i \geq 2$

or

$$Q_{dd(i)} = Q_0 \left\{ 1 - \frac{c_1 s_{sp}^2 \left[ (i-1)\xi + \left[ \frac{(i-2) + (i-2)^2}{2} \right] \right]}{\frac{1}{1-\psi} c_1 s_{sp}^2 \left[ n\xi + \frac{(n-1)^2 + (n-1)}{2} \right]} \right\} = Q_0 \left\{ 1 - \frac{2(i-1)\xi + i^2 - 3i + 2}{\frac{1}{1-\psi} (2n\xi + n^2 - n)} \right\} \quad (6)$$

At the beginning of the lateral there is, usually, one or two (sometimes three) closed outlets, therefore the first segment ( $r_1$ , Figure 2) is longer than the remaining segments. This creates a dry area. This segment ( $r_1$ ) is correlated with the distance between sprinklers,  $s_{sp}$ , that is:

$$\frac{r_1}{s_{sp}} = \xi \Rightarrow r_1 = \xi s_{sp} \quad (7)$$

where  $\xi$  is the coefficient that must be multiplied by  $s_{sp}$  to obtain  $r_1$ .

**2) With single diameter but with the lateral divided into several reaches where the spacing between sprinklers is different but in each reach the spacing is kept constant:** Consider now a pipe of length  $R$  (Figure 3) divided into three reaches (only) with an inflow of  $Q_0$  at the inlet



Figure 2: Photo of the center-pivot beginning showing three closed outlets.

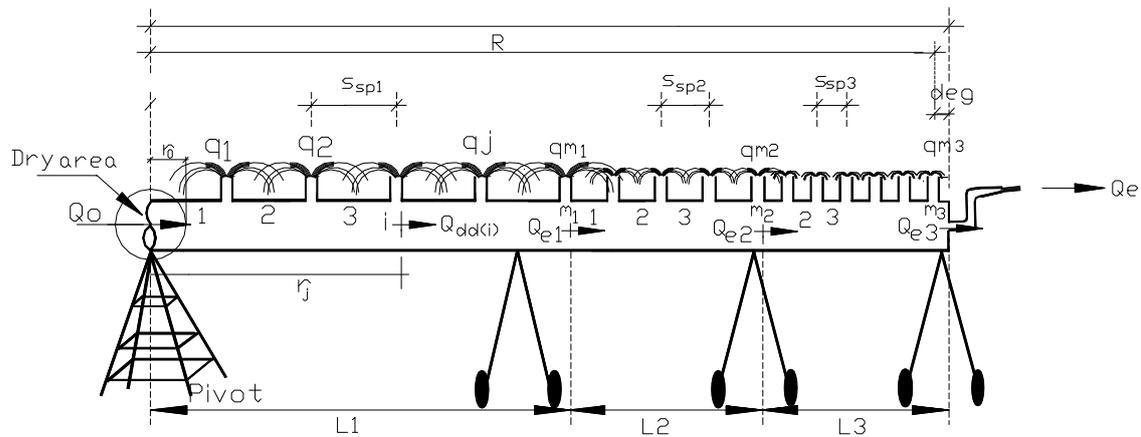


Figure 3: Representation of the center-pivot lateral with three reaches ( $L_1, L_2$  and  $L_3$ ) where each  $L_i$  has a different spacing ( $s_{sp}$ ).

and an outflow of  $Q_e$  at the outlet and where the open outlets are represented by  $m_1, m_2$  and  $m_3$  in each reach. Using the same procedure used to obtain equation (7) we have for this case:

$$\left\{ \begin{array}{l} \frac{r_{11}}{s_{sp1}} = \xi_1 \Rightarrow r_{11} = \xi_1 s_{sp1} \\ \frac{r_{12}}{s_{sp2}} = \xi_2 \Rightarrow r_{12} = \xi_2 s_{sp2} \\ \frac{r_{13}}{s_{sp3}} = \xi_3 \Rightarrow r_{13} = \xi_3 s_{sp3} \end{array} \right. \text{ and } \left\{ \begin{array}{l} \frac{s_{sp2}}{s_{sp1}} = \xi_4 \Rightarrow s_{sp2} = \xi_4 s_{sp1} \\ \frac{s_{sp3}}{s_{sp1}} = \xi_5 \Rightarrow s_{sp3} = \xi_5 s_{sp1} \end{array} \right. \quad (8)$$

The flow inside every segment in a center-pivot lateral with three reaches, can be obtained, for each reach by:

$$Q_{dd(i)} = Q_0 - c_1 \sum_{l=1}^{m=3} s_{spl} \sum_{j=1}^{i-1} r_{jl} \quad (9 a)$$

$$\text{with } Q_{dd_1(1)} = Q_0 \text{ (for first reach)}$$

$$Q_{dd_2(1)} = Q_{e1} \text{ (for second reach)}$$

$$Q_{dd_3(1)} = Q_{e2} \text{ (ford third reach)}$$

$$\text{and } Q_0 = Q_{e1} + c_1 s_{sp1} \sum_{j=1}^n r_{j1} \text{ (for first reach)} \quad (9 b)$$

$$Q_{e1} = Q_{e2} + c_1 s_{sp2} \sum_{j=1}^n r_{j2} \text{ (for second reach)} \quad (9 c)$$

$$Q_{e2} = Q_{e3} + c_1 s_{sp3} \sum_{j=1}^n r_{j3} \text{ (ford third reach)} \quad (9 d)$$

where  $Q_0, Q_{e1}$  and  $Q_{e2}$  represent the inflow at the inlet of each reach and  $Q_{e1}, Q_{e2}$  and  $Q_{e3}$  represent also the outflow at the outlet of each reach. They are related between themselves, that is:

$$Q_{e1} = \psi_1 Q_0, Q_{e2} = \psi_2 Q_{e1} \text{ and } Q_{e3} = \psi_3 Q_{e2} \quad (10)$$

where  $Q_e = Q_{e3}$  in this case.

In this study we assumed that:

- 1)- the irrigated area by each sprinkler consists of all the points whose distance to the pivot is between  $r_i - s_{sp}/2$  and  $r_i + s_{sp}/2$  (Figure 4);
- 2)- the irrigated area by each reach is also an annulus;

$$3)- \text{ the length of the basic irrigated circle for a lateral without an end gun sprinkler is } R_{ir} = \left[ \sum_{i=1}^{m=3} (\xi_i + m_i - 1) s_{spi} \right] + s_{spm} / 2 ;$$

4)- there is no overlap between the areas irrigated by each sprinkler;

$$5)- \text{ the distance between the last sprinkler of reach } i \text{ and the first sprinkler of reach } i+1 \text{ is } r_{1(i+1)} = \frac{s_{spi}}{2} + \frac{s_{spi+1}}{2}.$$

So it is easier, for this case, to compute the discharge of each sprinkler and of each reach as:

$$Q_i = A_i c_2 \quad (11)$$

with  $A = \pi(R_{big}^2 - R_{les}^2)$ ,  $c_2 = h_{Max} \frac{7}{N_{dw}} \frac{1}{E_a} \frac{1}{T_{mr}}$  and where  $R_{big}$  and  $R_{les}$  represent the larger and the smaller radius of the annulus.

For the first reach, for example,  $R_{les} = r_0$

The irrigated area for each reach is given by:

$$A_1 = \pi(RR1^2 - r_0^2) \quad (12 a)$$

$$A_2 = \pi(RR2^2 - RR1^2) \quad (12 b)$$

$$A_3 = \pi(RR3^2 - RR2^2) \quad (12 c)$$

for the first, second, and third, respectively.

Considering the assumptions referred above, the equations (9 b), (9 c) and (9 d) can be rewritten as:

$$Q_0 = \pi s_{sp1}^2 c_2 \left[ (\xi_1 + m_1 - 1 + 0.5)^2 - (\xi_1 - 1 + 0.5)^2 \right] \frac{1}{1 - \psi_1} \quad (13 a)$$

$$Q_{e1} = \pi s_{sp1}^2 c_2 \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1 + 0.5) \xi_4 \right]^2 - \left[ \xi_1 + m_1 - 1 + 0.5 \right]^2 \right\} \frac{1}{1 - \psi_2} \quad (13 b)$$

$$Q_{e2} = s_{sp1}^2 c_2 \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (m_3 - 1 + 0.5) \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1 + 0.5) \xi_4 \right]^2 \right\} \frac{1}{1 - \psi_3} \quad (13 c)$$

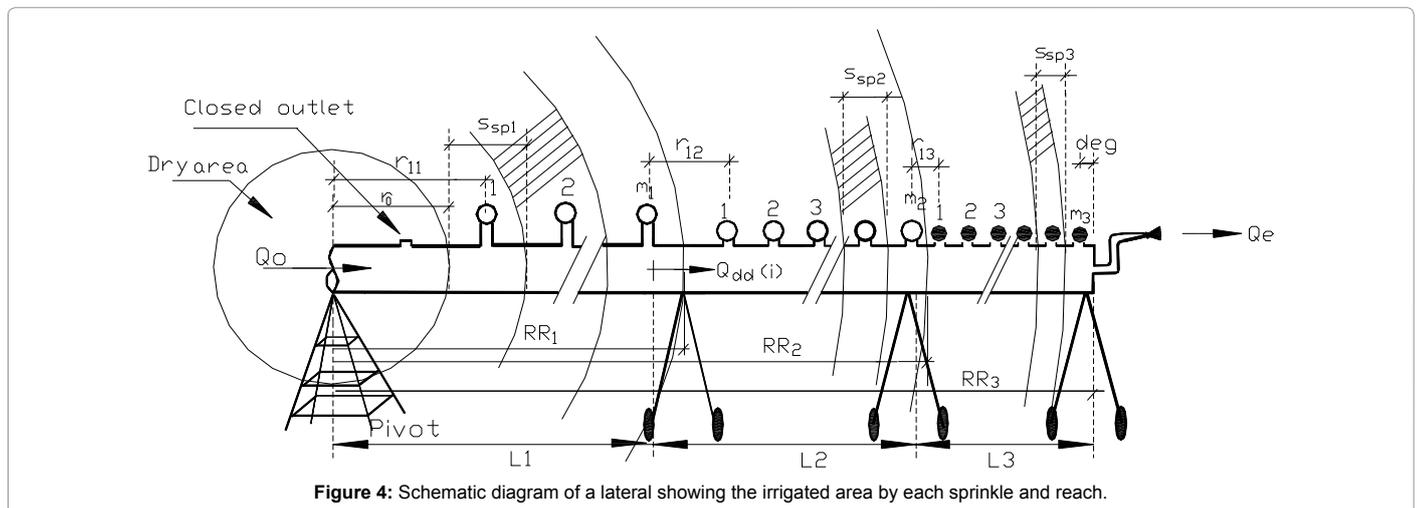


Figure 4: Schematic diagram of a lateral showing the irrigated area by each sprinkle and reach.

Hence, we can obtain the friction head loss and the friction correction factor for each reach. The internal flow in each segment can be obtained by:

a) For the first reach:

$$\begin{aligned}
 & - Q_{dd_1(1)} = Q_0 \\
 & - Q_{dd_1(2)} = Q_0 \left[ \frac{Q_{dd_1(2)}}{Q_0} \right] = Q_0 \left\{ \frac{Q_0 - \pi c_2 [(r_{11} + 0.5s_{sp1})^2 - (r_{11} - 0.5s_{sp1})^2]}{Q_0} \right\} \quad (14)
 \end{aligned}$$

Substituting (13 a) and (8) in (14) this becomes:

$$\begin{aligned}
 & - Q_{dd_1(2)} = Q_0 \left\{ 1 - \frac{\pi s_{sp1}^2 c_2 [(\xi_1 + 0.5)^2 - (\xi_1 - 0.5)^2]}{\pi s_{sp1}^2 c_2 [(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2] \frac{1}{1 - \psi_1}} \right\} \\
 & - Q_{dd_1(3)} = Q_0 \left\{ 1 - \frac{[(\xi_1 + 1 + 0.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1 - \psi_1} [(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\} \\
 & - \dots \\
 & - Q_{dd_1(i)} = Q_0 \left\{ 1 - \frac{[(\xi_1 + i - 1.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1 - \psi_1} [(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\} \text{ for } (2 \leq i \leq m_1) \quad (15)
 \end{aligned}$$

b) For the second reach:

$$\begin{aligned}
 & - Q_{dd_2(1)} = Q_{e1} \\
 & - Q_{dd_2(2)} = Q_{e1} \left[ \frac{Q_{dd_2(2)}}{Q_{e1}} \right] = Q_{e1} \left\{ \frac{Q_{e1} - \pi c_2 [(r_{11} + m_1 s_{sp1} - s_{sp1} + r_{12} + 0.5s_{sp2})^2 - (r_{11} + m_1 s_{sp1} - s_{sp1} + r_{12} - 0.5s_{sp2})^2]}{Q_{e1}} \right\} \quad (16)
 \end{aligned}$$

Substituting (13 b) and (8) in (16) this becomes:

$$\begin{aligned}
 & - Q_{dd_2(2)} = Q_{e1} \left\{ 1 - \frac{\pi s_{sp1}^2 c_2 [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\pi s_{sp1}^2 c_2 \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2] \right\} \frac{1}{1 - \psi_2}} \right\} \\
 & - Q_{dd_2(3)} = Q_{e1} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + 1.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1 - \psi_2} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2] \right\}} \right\} \\
 & - \dots
 \end{aligned}$$

$$- Q_{dd_2(i)} = Q_{e1} \left\{ 1 - \frac{\left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (i - 1.5) \xi_4 \right]^2 - (\xi_1 + m_1 - 0.5)^2 \right\}}{\frac{1}{1 - \psi_1} \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 - (\xi_1 + m_1 - 0.5)^2 \right\}} \right\} \text{for } (2 \leq i \leq m_2) \quad (17)$$

c) For the third reach:

$$- Q_{dd_3(1)} = Q_{e2}$$

$$Q_{dd_3(2)} = Q_{e2} \left\{ 1 - \frac{\pi c_2 \left[ (r_{11} + m_1 s_1 - s_1 + r_{12} + m_2 s_2 - s_2 + r_{13} + 0.5 s_3)^2 - (r_{11} + m_1 s_1 - s_1 + r_{12} + m_2 s_2 - 0.5 s_2)^2 \right]}{Q_{e2}} \right\} \quad (18)$$

Substituting (13 c) and (8) in (18) we have:

$$Q_{dd_3(2)} = Q_{e1} \left\{ 1 - \frac{\pi s_{sp1}^2 c_2 \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + 0.5 \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\}}{\pi s_{sp1}^2 c_2 \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (m_3 - 0.5) \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\} \frac{1}{1 - \psi_3}} \right\}$$

$$Q_{dd_3(3)} = Q_{e2} \left\{ 1 - \frac{\left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + 1.5 \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\}}{\frac{1}{1 - \psi_3} \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (m_3 - 0.5) \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\}} \right\}$$

- .....

$$Q_{dd_3(i)} = Q_{e2} \left\{ 1 - \frac{\left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (i - 1.5) \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\}}{\frac{1}{1 - \psi_3} \left\{ \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (m_3 - 0.5) \xi_5 \right]^2 - \left[ \xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4 \right]^2 \right\}} \right\}$$

for  $(2 \leq i \leq m_3)$  (19)

On the other hand, the lateral length with a single diameter and constant spacing along the lateral is related with (7) and also with  $n$  segments through the expression:

$$R = r_1 + (n - 1) s_{sp} = s(\xi + n - 1) \Rightarrow s_{sp} = \frac{R}{\xi + n - 1} \quad (20)$$

In the case of a lateral divided into three reaches, the lateral length ( $R = L_1 + L_2 + L_3$ ) is related with (8) and with  $m_1$ ,  $m_2$  and  $m_3$  which represent the number of the open outlets in each reach, that is:

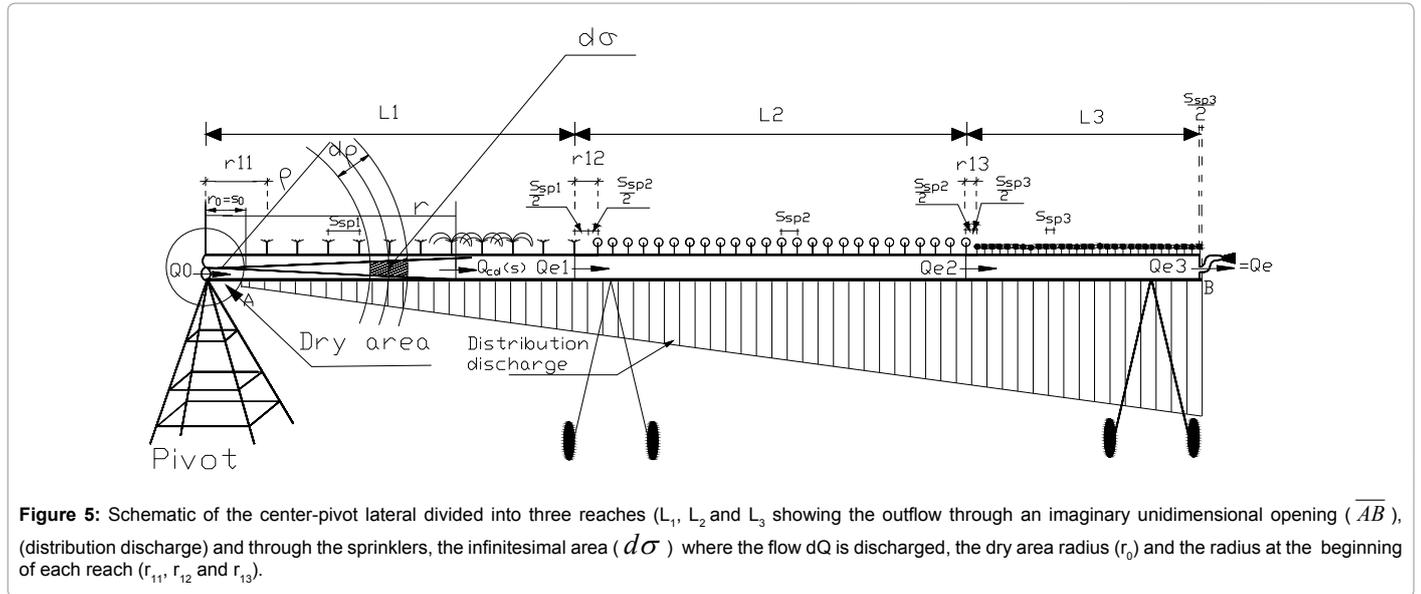
$$s_{sp} = \frac{L_2}{\xi_2 + m_2 - 1} \text{ and } s_{sp3} = \frac{L_3}{\xi_3 + m_3 - 1} \quad (21)$$

**Continuous approach: 1) with single diameter and constant spacing along the lateral:** For a continuous outflow distribution (COD) and considering all details of the center-pivot laterals, Tabuada [8] developed analytical expressions to compute the friction head loss. These expressions are based on the Hypergeometric function.

For this situation (Figure 5) the outflow  $dQ$  on an infinitesimal irrigated area ( $da$ ) at the distance  $\rho$  from the pivot point is given by:

$$dQ = q = c_1 \rho d\rho \quad (22)$$

where  $d\rho$  is the infinitesimal spacing and  $c_1$  is given by equation (5).



**Figure 5:** Schematic of the center-pivot lateral divided into three reaches ( $L_1$ ,  $L_2$  and  $L_3$  showing the outflow through an imaginary unidimensional opening ( $\overline{AB}$ ), (distribution discharge) and through the sprinklers, the infinitesimal area ( $d\sigma$ ) where the flow  $dQ$  is discharged, the dry area radius ( $r_0$ ) and the radius at the beginning of each reach ( $r_{11}$ ,  $r_{12}$  and  $r_{13}$ ).

For a center-pivot lateral with closed outlets at the beginning of the lateral, the discharge [ $Q_{cd}(s)$ ] in the main line at distance  $r$  can be given by:

$$Q_{cd}(s) = Q_0 - \int_{r_0}^r c_1 \rho d\rho = Q_0 - \frac{c_1}{2}(r^2 - r_0^2) \quad (23)$$

$$\text{with } Q_0 = Q_e + \frac{c_1}{2}(R_{cd}^2 - r_0^2) \quad (24)$$

Substituting (2) in (24) and after (24) in (23) this becomes:

$$Q_{cd}(s) = Q_0 \left[ 1 - (1 - \psi) \frac{r^2 - r_0^2}{R_{cd}^2 - r_0^2} \right] \quad r_0 \leq r \leq R_{cd} \quad (25)$$

**ii) With single diameter but with the lateral divided into several reaches where the spacing between sprinklers is different but in each reach the spacing is kept constant:** For this situation, since we are assuming no overlap between the area irrigated by sprinkler  $i$  and the area irrigated by sprinkler  $i+1$ , equation (25) can still be used.

**Friction head loss:** In this study the friction head loss at any segment along a lateral is calculated by the expression:

$$S_f = \beta D^\gamma Q^\alpha(s) \quad (26)$$

In (26),  $\beta$  is the roughness factor, which we assume to be independent of  $s$ ,  $D$  represents the internal diameter of the main line, and  $\gamma$  and  $\alpha$  are exponents obtained from the formula used to compute  $S_f$ .

**Discrete approach: i) with single diameter and constant spacing along the lateral:** For a single diameter, taking into account (6) and (26), the head loss can be computed segment by segment [8] as:

$$\text{- segment 1 } h_{r(1)} = \beta D^\gamma Q_0^\alpha r_1 = \beta D^\gamma Q_0^\alpha \xi s_{sp}$$

.....

$$\text{- segment } i \quad h_{r(i)} = \beta D^\gamma Q_0^\alpha s_{sp} \left[ 1 - \frac{2(i-1)\xi + i^2 - 3i + 2}{1 - \psi} \frac{1}{(2n\xi + n^2 - n)} \right]^\alpha \quad (27)$$

So the total friction head loss between the beginning of segment 1 and the end of segment  $n$ , is given by:

$$h_{r(1-n)} = \sum_{i=1}^n S_{fi} [r_1 + (n-1)s_{sp}] = \beta D^\gamma Q_0^\alpha S_{sp} \left\{ \xi + \sum_{i=2}^n \left[ 1 - \frac{2(i-1)\xi + i^2 - 3i + 2}{\frac{1}{1-\psi}(2n\xi + n^2 - n)} \right]^\alpha \right\} \quad (28)$$

If  $\psi \neq 0$ , in equation (28), then the center-pivot lateral has an end gun sprinkler and then we have to consider the friction head loss in the distance between the last sprinkler and the end gun ( $d_{eg}$ , Figure 4) given by  $\beta D^\gamma Q_c^\alpha d_{eg} = \beta D^\gamma \psi^\alpha Q_0^\alpha \lambda s_{sp}$  and equation (28) becomes:

$$h_{Rgs} = \beta D^\gamma Q_0^\alpha R \left\{ \frac{1}{\xi + n - 1} \left\{ \xi + \sum_{i=2}^n \left[ 1 - \frac{2(i-1)\xi + i^2 - 3i + 2}{\frac{1}{1-\psi}(2n\xi + n^2 - n)} \right]^\alpha + \lambda \psi^\alpha \right\} \right\} \quad (29)$$

where  $d_{eg} = \lambda s_{sp} = \lambda \frac{R}{(\xi + n - 1)}$ .

**ii) With single diameter but with the lateral divided into several reaches where the spacing between sprinklers is different but in each reach the spacing is kept constant:**

Following a similar procedure as before we have:

a) **First reach:**

- segment 1  $h_r(1) = \beta D^\gamma Q_0^\alpha r_{11} = \beta D^\gamma Q_0^\alpha \xi_1 s_{sp1}$

- segment 2  $h_r(2) = \beta D^\gamma Q_0^\alpha s_{sp1} \left\{ 1 - \frac{[(\xi_1 + i - 1.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1-\psi_1}[(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\}^\alpha$

.....

- segment i  $h_r(i) = \beta D^\gamma Q_0^\alpha s_{sp1} \left\{ 1 - \frac{[(\xi_1 + i - 1.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1-\psi_1}[(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\}^\alpha$  (for  $2 \leq i \leq m_1$ )

So the total friction head loss between the beginning of segment 1 and the end of segment  $m_1$ , is obtained by:

$$h_r(L_1) = \beta D^\gamma Q_0^\alpha s_{sp1} \left\{ \xi_1 + \sum_{i=2}^{m_1} \left\{ 1 - \frac{[(\xi_1 + i - 1.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1-\psi_1}[(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\}^\alpha \right\} \quad (30)$$

For the friction correction factor, after substituting (21) in (30) we have:

$$F_{fc}^1 = \frac{1}{\xi + m_1 - 1} \left\{ \xi_1 + \sum_{i=2}^{m_1} \left\{ 1 - \frac{[(\xi_1 + i - 1.5)^2 - (\xi_1 - 0.5)^2]}{\frac{1}{1-\psi_1}[(\xi_1 + m_1 - 0.5)^2 - (\xi_1 - 0.5)^2]} \right\}^\alpha \right\} = F_{cf}(\alpha, m_1, \xi_1, \psi_1) \quad (31)$$

**b) Second reach:**

- segment 1  $h_r(1) = \beta D^\gamma Q_{e1}^\alpha r_{12} = \beta D^\gamma Q_0^\alpha \xi_2 s_{sp2}$

- segment 2

$$h_r(2) = \beta D^\gamma Q_{e1}^\alpha s_{sp2} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1-\psi_2} [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + m_2 \xi_4 - 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]} \right\}^\alpha$$

.....

-segment i

$$h_r(i) = \beta D^\gamma Q_{e1}^\alpha s_{sp2} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (i-1.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1-\psi_2} [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + m_2 \xi_4 - 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]} \right\}^\alpha$$

(for  $2 \leq i \leq m_2$ )

The total friction head loss, in the second reach, between the beginning of segment 1 (Figure 3) and the end of segment  $m_2$  is given by:

$$h_r(L_2) = \beta D^\gamma Q_{e1}^\alpha s_{sp2} \left\{ \xi_2 + \sum_{i=2}^{m_2} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (i-1.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1-\psi_2} [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + m_2 \xi_4 - 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]} \right\}^\alpha \right\} \quad (32)$$

And for the friction correction factor, after substituting (21) in (32) we have:

$$F_{fc}^2 = \frac{1}{\xi_2 + m_2 - 1} \left\{ \xi_2 + \sum_{i=2}^{m_2} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (i-1.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1-\psi_2} [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + m_2 \xi_4 - 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]} \right\}^\alpha \right\} = F_{cf}(\alpha, m_1, m_2, \xi_1, \xi_2, \psi_2) \quad (33)$$

for inflow inlet equal to  $Q_{e1}$ , or:

$$F_{fc}^2 = \frac{1}{\xi_2 + m_2 - 1} \left\{ \xi_2 + \sum_{i=2}^{m_2} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (i-1.5) \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]}{\frac{1}{1-\psi_2} [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + m_2 \xi_4 - 0.5 \xi_4)^2 - (\xi_1 + m_1 - 0.5)^2]} \right\}^\alpha \right\} \psi_1^\alpha \quad (34)$$

for inflow inlet equal to  $Q_0$ .

**c) Third reach:**

- segment 1  $h_r(1) = \beta D^\gamma Q_{e2}^\alpha r_{13} = \beta D^\gamma Q_0^\alpha \xi_3 s_{sp3}$

- segment 2

$$h_r(2) = \beta D^\gamma Q_{e2}^\alpha s_{sp3} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + 0.5 \xi_5)^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4]^2]}{\frac{1}{1-\psi_3} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1) \xi_4 + \xi_3 \xi_5 + (m_3 - 0.5) \xi_5)^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5) \xi_4]^2 \right\}} \right\}^\alpha$$

.....

- segment i

$$h_r(i) = \beta D^\gamma Q_{e2}^\alpha S_{sp3} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (i - 1.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2]}{\frac{1}{1 - \psi_3} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (m_3 - 0.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2 \right\}} \right\}^\alpha \quad (\text{for } 2 \leq i \leq m_3)$$

The total friction head loss, in the third reach considering the friction head loss in the distance  $d_{eg}$  also, is given by:

$$h_r(L_3) = \beta D^\gamma Q_{e2}^\alpha S_{sp3} \left\{ \xi_3 + \sum_{i=2}^{m_3} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (i - 1.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2]}{\frac{1}{1 - \psi_3} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (m_3 - 0.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2 \right\}} \right\}^\alpha + \lambda \psi_3^\alpha \right\} \quad (35)$$

For the friction correction factor, after substituting (21) in (35) we have:

$$F_{fc}^3 = \frac{1}{\xi_3 + m_3 - 1} \left\{ \xi_3 + \sum_{i=2}^{m_3} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (i - 1.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2]}{\frac{1}{1 - \psi_3} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (m_3 - 0.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2 \right\}} \right\}^\alpha + \lambda \psi_3^\alpha \right\} \quad (36)$$

$$F_{fc}^3 = F_{cf}(\alpha, m_1, m_2, m_3, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \psi_3, \lambda)$$

for inflow inlet equal to  $Q_{e2}$  or:

$$F_{fc}^3 = \frac{1}{\xi_3 + m_3 - 1} \left\{ \xi_3 + \sum_{i=2}^{m_3} \left\{ 1 - \frac{[(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (i - 1.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2]}{\frac{1}{1 - \psi_3} \left\{ [(\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 1)\xi_4 + \xi_3 \xi_5 + (m_3 - 0.5)\xi_5]^2 - [\xi_1 + m_1 - 1 + \xi_2 \xi_4 + (m_2 - 0.5)\xi_4]^2 \right\}} \right\}^\alpha + \lambda \psi_3^\alpha \right\} \psi_1^\alpha \psi_2^\alpha \quad (37)$$

for inflow inlet equal to  $Q_0$ ,

Taking into account the inflow at the inlet in each reach ( $Q_0$ ,  $Q_{e1}$  and  $Q_{e2}$ ) or the inflow at the inlet in the first reach ( $Q_0$ ), the friction head loss in each reach can be obtained by:

$$h_r(L_1) = \beta D^\gamma Q_0^\alpha L_1 F_{cf}^1 \quad (38 a)$$

$$h_r(L_2) = \beta D^\gamma Q_{e1}^\alpha L_2 F_{cf}^2 = \beta D^\gamma Q_0^\alpha L_2 F_{cf}^2 \psi_1^\alpha \quad (38 b)$$

$$h_r(L_3) = \beta D^\gamma Q_{e2}^\alpha L_3 F_{cf}^3 = \beta D^\gamma Q_0^\alpha L_3 F_{cf}^3 \psi_1^\alpha \psi_2^\alpha \quad (38 c)$$

**Continuous approach: i) - with single diameter and constant spacing along the lateral:** For this situation the flow rate in the main line is given by equation (25). So the friction head loss along a lateral between section  $r_0$  and  $r = S$ , is given by the integration of the equation (25), that is:

$$h_r(s) = \beta D^\gamma Q_0^\alpha \int_{r_0=s_0}^s \left[ 1 - (1 - \psi) \frac{r^2 - r_0^2}{R_{cd}^2 - r_0^2} \right]^\alpha dr \quad (39)$$

The solution of (39) depends on the  $\alpha$  exponent value,  $\alpha=2$ , as in the Darcy-Weisbach or Chézy equation, or  $\alpha=1.852$ , as in the Hazen-Williams equation.

In this study we only consider the case of  $\alpha$ =non integer real value since when  $\alpha$  is equal to 2 the integration of the equation (39) yields an analytical expression.

Tabuada [8] studied the friction head loss for  $\alpha=2$  and for  $\alpha \neq$  non integer real value and for several cases: center-pivot laterals with and without an end gun sprinkler with and without close outlets at the beginning of the center-pivot lateral and he showed that for this situation,  $\alpha \neq$  non integer real value and for a center-pivot lateral with an end gun sprinkler, the friction head loss is given by:

$$h_r(s) = \beta D^\gamma Q_0^\alpha \left\{ s \left[ \frac{R_{cd}^2 - r_0^2 + (1 - \psi)r_0^2}{R_{cd}^2 - r_0^2} \right]^\alpha F \left[ 0.5, -\alpha; 1.5; \frac{(1 - \psi)s^2}{R_{cd}^2 - r_0^2 + (1 - \psi)r_0^2} \right] - \right.$$

$$-r_0 \left[ \frac{R_{cd}^2 - r_0^2 + (1-\psi)r_0^2}{R_{cd}^2 - r_0^2} \right]^\alpha F \left[ 0.5, -\alpha; 1.5; \frac{(1-\psi)r_0^2}{R_{cd}^2 - r_0^2 + (1-\psi)r_0^2} \right] + r_0 \left. \right\} (r_0 \leq s \leq R_{cd}) \quad (40)$$

where  $F \left[ 0.5, -\alpha; 1.5; \frac{(1-\psi)s^2}{R_{cd}^2 - r_0^2 + (1-\psi)r_0^2} \right]$  and  $F \left[ 0.5, -\alpha; 1.5; \frac{(1-\psi)r_0^2}{R_{cd}^2 - r_0^2 + (1-\psi)r_0^2} \right]$  represent the Hypergeometric function.

ii) **With single diameter but with the lateral divided into several reaches where the spacing between sprinklers is different but in each reach the spacing is kept constant:** It follows from assumption 4) that equation (40) can also be used in this case.

### Comparative Analyses

To illustrate the proposed methods we consider a center-pivot lateral with an end gun sprinkler and with the following characteristics:

- Length:  $L=L_1 + L_2 + L_3=460.5$  ( $L_1=144$  m,  $L_2=153$  m and  $L_3=163.5$  m) for DOD and with  $L=462$  m for the COD;
- Spacing:  $s_{sp1} = 12$  m,  $s_{sp2} = 6$  m and  $s_{sp3} = 3$  m and  $Q_0=109.29$  L/s,  $Q_{e1}=98.89$  L/s and  $Q_{e2}=67.66$  L/s;
- Distance between the last sprinkler of the reach  $i$  and the first sprinkler of the reach  $i+1$ :  $r_{11}=r_0 + s_{sp1}/2=12$  m (at the beginning of the lateral, with  $r_0=6$  m),  $r_{12}=s_{sp1}/2 + s_{sp2}/2=9$  m and  $r_{13}=s_{sp2}/2 + s_{sp3}/2=4.5$  m. This is based on the assumption that the irrigated area by each sprinkler is an annulus with a width equal to the distance  $s_{sp}$ ;
- With an end gun sprinkler ( $Q_{e3}=10.53$  L/s) and for  $h_{Max} = 8$  mm,  $T_{mr} = 22$  h,  $N_{dw} = 6$  days and  $E_a = 80\%$ ;
- $\alpha=1.852$ ,  $\beta=4.87$  and  $CH=135$  (assumed).

The friction head loss computed using equation (40) (COD) and using equations (30), (32) and (35) (DOD) is presented in the Figure 6. To obtain the values of the friction head loss using the Hypergeometric function we can use any mathematical software such as Mathematica.

Figure 6 shows that the difference of the friction head loss calculated with COD and with DOD is neglected.

Tabuada (2011) verified that to irrigate a similar area with a center-pivot without an end gun sprinkler the difference between the values of the friction head loss obtained for a center-pivot with end gun and without end gun sprinkler is small. This is verified when the discharge of the end gun sprinkler is similar to the discharge distributed by  $m$  sprinklers ( $q_{n+1}, \dots, q_{n+m}$ ) to irrigate a similar area. On the other hand to irrigate the similar area without end gun sprinkler it is necessary to increase the number of sprinklers (or outlets) on the lateral and therefore the friction correction factor ( $F_{cp}$ ) decreases as well as the friction head loss.

### Summary and Conclusions

This paper presents efficient closed form expressions to calculate the friction head loss and the friction correction factor using the discrete outflow distribution.

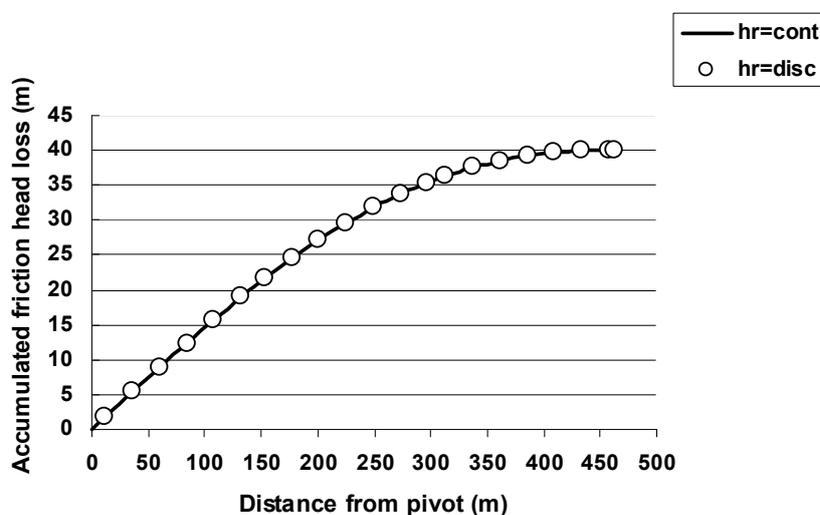


Figure 6: Friction head loss along the center-pivot lateral.

Friction factor expressions are developed using the inflow at the inlet in each reach and the inflow at the inlet in the first reach only.

The method developed in this paper is based on the discharge required by each sprinkler and each reach to irrigate an annular area.

This proposed procedure can be used in center-pivot laterals with and without end gun sprinkler and with and without closed outlets at the beginning of the lateral.

This apparently complex method is easy to implement computationally. After to arrive at a decision about the number of the reaches on the lateral and after to know  $\epsilon_s$  and  $m_i$  values, the computation of the friction head loss using equations (30), (32) and (35), for example, becomes at the solution of sums and products.

It can be applied using the Hazen-Williams, Scobey or Darcy-Weisbach friction loss equations.

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