

Fock Space of Level infinity and Characters of Vertex Operators

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Abstract

We present an extension of partition function of open Gromov Witten theory of CY 3-folds defined by the trace of an extended vertex operator and give a Representation theoretic interpretation of the trace of the vertex operator involved. Specifically, we consider a twist of the vertex operator with infinitely many Casimir operators. We extend the definition of the Fock space of level l , to let the level to be infinity. We prove a duality between gl_∞ and $a_\infty = gl_\infty$ of Howe type, which provides a decomposition of F_∞ into irreducible representations obtained from joint highest weight vector for gl_∞ and a_∞ . The decomposition of the Fock space F_∞ into highest weight representations provide a method to calculate and interpret the extended trace. Alternatively, the trace can be interpreted as the character of a representation of $gl(\infty|\infty)$ on the self-tensor product $F \otimes F$ of the ordinary Fock space. Thinking of the character as a super-trace we give another calculation as a result.

Introduction

The vertex operators provide a representation theoretic and the combinatorial framework for the generating series in string theory. A vertex operator naturally act on the Fock space of infinite dimension and its trace produces generating series which are crucial in quantum Physics. They provide a powerful tool to compute string theory amplitudes. The successive application of commutation rule of the boson fields interacting to the vertex operator produces product factors. The product formulas for the string partition functions are specially of interests. We follow a computation of the character of the infinite wedge representation (Fock space) in where a product formula is established for the character. Bloch-Okounkov prove that the associated character is quasimodular [1].

The symmetric functions play a prominent role in connecting combinatorics to representation theory of infinite dimensional Lie algebras. The partition functions of CY 3-folds can be considered as characters of representations and specifically as trace of vertex operators, twisted by one or more Casimir operators. We give a natural extension of the trace to the case when infinitely many Casimirs exist. We search a representation theoretic interpretation for the extended vertex operator. One can use a decomposition of the Fock space of level infinity into highest weight representations of Lie superalgebra $a_\infty = gl_\infty$ to break the trace into sum of the traces on the components.

The $\otimes$ summand is a joint highest weight representation of gl_∞ and a_∞ . There already existed an analogue for the Fock space of level l , namely F_l with also a similar character formula [2-4]. In fact the above Definition and Theorem is a routine generalization of the corresponding identity for F_l . In this article we extend all the details of definition of the space F_l to the infinite level and reprove the above decomposition. We use these identities to generalize certain trace formulas of vertex operators coming from string

theory.

The transformation beneath the trace is acting on ordinary Fock space $F = F_l$, also called the infinite wedge representation. The calculation uses basically the expansion of the exponential functions and calculating the trace of the resulting operator on the Hilbert space directly. The character is calculated in as a product formula. We use the decomposition formula to compute the following trace.

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Conflict of Interest

Nil

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